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Navigation and Control of Autonomous Vehicles Using Fusion of Modern Sensors

Ph.D. Thesis Summary

Autonóm járművek navigációja és irányítása korszerű érzékelők fúziójával

Ph. D. értekezés tézisei

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1 Introduction and Objectives

In the field of control theory it is a well-known opinion that the control system is only as good as its sensory system. Beside the technical development of nowadays, newer and newer sensory concepts appear. Most of these sensors can be used only in specific areas, but there are some trends which seem to be useful in wide spectrum of applications. Each device has its own advantages and disadvantages, which should be considered when they are chosen for a given task.

When a sensor has to be used in an application, it is not enough to choose the sensor properly. It is also important to use the device in a proper way, to reach the highest quality possible. Plenty of recommendations and algorithms exist how to use a sensor [3, 4]. Usually the best result is given by the combination of these solutions.

Another important aspect in the field of sensory systems is what kind of advantages and disadvantages a sensor has. It is usually necessary to combine sensors, which can eliminate the unfavorable property of the others and conversely. For example a micro-electro-mechanical system (MEMS) based accelerometer is able to measure the acceleration of a moving body directly but the velocity and the position can be derived through integration. The integrated result suffers from the drifting problem. On the other hand an image processing system can measure the position of the moving body and in some special case its velocity. But the acceleration can be calculated through derivation, which amplifies the noise of the measurement. For a complete and usable acceleration-velocity-position state vector both type of sensors should be used. Moreover, the advantages of each sensor can be raised. This approach leads to the field of sensor fusion.

The connection between the different types of sensors are described with models. The celebrated Kalman filter[5] gives an optimal solution for the actual state of the model for a given model and known covariance matrices. Therefore, the result of the sensor fusion is only as good as the used model. Hence it is a key issue to model the physical world which describes this connection as precisely as necessary.

During my PhD studies I took part in a research where the main goal was to develop an unmanned indoor quadrotor helicopter. This was a project of the Budapest University of Technology (BME) Department of Control Engineering and Information Technology (IIT). During this research one of the objectives was to develop every parts of the helicopter on our own, including the mechanical, electronic, control and informatics parts of the helicopter and the ground devices. Usually simple commercial devices were used as sensors, which were developed for different applications. The measurement results from these commercial devices suffer from some phenomenon which could have been neglected if expensive, high precision sensors were used. Most of the novel results of this thesis were inspired by the low-cost approach.

As the full system of the quadrotor helicopter is a quite unique one, there are aspects which are not typical in other researches. Therefore, some needs are emerged to develop control algorithms which can handle these special circumstances.

The main line of the quadrotor research was the indoor operation. Beside that it is also an important question to discover the possibilities of outdoor operation. In this manner I became acquainted with GPS navigation. For the unmanned control of an outdoor helicopter high precision position sensing is essential. This effectiveness can be reached by the usage of the carrier phase measurement of the GPS (CPGPS) signal.

There are already a lot of applications which use this technique. The solution is used in surveying, agricultural applications or aviation. Every fields of usage have the property that the initialization of the application don't require to run in hard real-time. This is because the carrier phase approach leads to the integer ambiguity problem, which relates to the integer number of carrier cycles between the receiver and the satellite. The typical solutions for this problem needs a huge computational capacity or need some minutes of measurements to give a reliable result[2].

In the field of mobile robotics these non-real-time solutions cannot be applied. On the other hand the typical solutions use only the GPS measurement data. It can be shown that in some special cases other sensors of mobile robots (unmanned vehicles) can be used which makes the solution of the integer ambiguity problem faster, can even be executed in real-time. One can realize that these special cases are also important in robotic navigation.

1.1 Research Goals

My research is in close connection with the quadrotor project of the BME-IIT. According to the project three main areas appeared where the common solutions are unsatisfactory. The first one is the field of the MEMS based inertial sensors (IMU) and magnetometers. The raw output of these sensors are usually unreliable (because of the error sources of the sensors), therefore some kind of calibration method is required which can ensure reliable values of the measurements. The common solutions in the literature discuss only a restricted part of the error sources or use expensive device for precise calibration. According to practice it can be said that the typically neglected error sources have a significant influence on the whole system. Therefore the first goal of this thesis is to give calibration methods for these sensors which can handle a large variety of error sources and do not require additional devices for calibration.

The second area which is concerned in this thesis is the field of state estimation and quadrotor control. In this case the motivation is also the quadrotor project, where the common state estimation solutions have disadvantages because of the low-cost approach of the project. The goal is to develop robust sensor fusion and state estimation algorithms, which are able to determine the state of the helicopter. It is also a requirement that the solutions be general as they could be used in other vehicle navigation problems. According to the results of the state estimation, the development of the control of the quadrotor helicopter is also required.

The third field of the research is connected to the precise outdoor navigation, where the carrier phase measurement based GPS solutions are required to use. The goal in this field is to present algorithms which are able to solve the integer ambiguity problem supported by the other sensors of an autonomous vehicle. The requirements are that the solutions should meet the hard real-time requirements of the vehicle navigation, therefore the initialization phase of the integer ambiguity solution has to be fast.

1.2 Research Methodology

The goals of the research contain practical problems, therefore the solutions should be able to use in practice. In this approach most of the methods are developed on the base of real measurements and every solution is tested in real circumstances.

By this way the required measurement should be collected during the research. According to the quadrotor project, an IMU was already available for the first experiments. In latter phases the IMU was redesigned and the calibration methods were tested for this device as well.

Since the whole sensory system of the quadrotor helicopter was accessible during the project, real measurements of the helicopter movement could be obtained. These data were useful in the development and the test of the state estimation algorithms. It was also important that the helicopter has weight limitations therefore every algorithm should operate on the base of embedded processors. In case of some microprocessors the rapid prototype design could be used which has support in MATLAB/Simulink, Real Time Workshop environment. Other microcontrollers have to be programmed in C.

The control algorithms are also developed and tested in embedded processor environments. In the first phase of control design a simulation environment was used (MATLAB/Simulink) and the methods were evaluated during real flights.

The outdoor GPS based navigation algorithms are also developed using real measurements. To get these data a data acquisition system was developed, which were mounted on a car and a sail-plane to collect measurement data. This data acquisition system is an embedded solution on the bases of a real-time Linux. In this part of the research the measurement data sets were collected during real vehicle movements. The data processing and navigation algorithms run offline. These offline algorithms are designed in the way that they can easily adopted into embedded systems.

2 Summary of the New Scientific Results

2.1 Calibration of the accelerometer, angular velocity sensor and magnetometer

The calibration of the inertial sensors and magnetometers are necessary, because the factory calibration is not precise enough (because of the unhandled error sources) for robotic navigation. In this thesis group separate methods are presented for accelerometer, angular velocity sensor and magnetometer calibrations.

Comparing to other solutions it can be said that methods presented here handle all of the typical error sources of the sensor. It is important to emphasize, that these methods do not use any additional tools, therefore they can be applied in any sensory system, even in low-cost solutions.

The accelerometer calibration is based on a robust, ellipsoid fitting method using SVD technique. By this way the main scaling, misalignment and bias errors of the accelerometer can be compensated. The method is extended with temperature dependency compensation. As the calculated transformation has a freedom in rotation, two different types of solutions are presented to handle this uncertainty. Since the bias like errors heavily depend on the actual state of the environment, two methods are also presented for calibration before startup .

The angular velocity sensor calibration has two steps. In the first phase the bias is compensated, and the effect of the acceleration is identified. The second phase handles the gain and misalignment errors. A temperature compensation extension is also presented.

The magnetometer calibration method is based on a similar ellipsoid fitting solution as in case of the accelerometer. The temperature compensation is introduced. The properties of the magnetic field make possible to calibrate a magnetic sensor during the movement of the vehicle. Two different types of calibration methods are presented for two different type of vehicles (for a car and for an airplane).

The variance analysis for every calibration method is also elaborated. This approach will also be important in the case of the state estimation in the later thesis groups.

Thesis Group 1. *I developed novel methods for the calibration of a magnetometer and an inertial measurement unit (IMU) containing 3D accelerometer and 3D angular velocity sensor. These methods do not suppose any additional tool, therefore they can be used in a wide field of applications. Beside the calibration of the sensor parameters, the covariance matrices of the measurements are also obtained.*

Related publications: [S1, S7, S8, S10, S12, S13]

Thesis 1.1. *Using the measurements of an accelerometer I developed an algorithm, which is able to calculate the gain, offset and misalignment error parameters of the sensor. I gave* *an extension which determines the temperature dependency of the parameters. I introduced two different methods which can calculate the initial offset and the covariance matrices of the measurements before the start of the vehicle.*

Thesis 1.2. *I developed a method for angular velocity sensor calibration, which is able to determine the offset and the acceleration dependency. I developed an extension, which is able to calculate the scaling and misalignment error parameters of the sensor. All of the methods are extended with the determination of the temperature dependency. I gave a method for the determination of the initial offset and the covariance matrices of the measurements before the start of the vehicle.*

Thesis 1.3. *I gave a method for the calibration of the magnetometer. The algorithm can determine the gain, offset and misalignment error parameters of the sensor up to one scaling factor. I gave an extension which can handle this unknown dependency. I also gave an extension for temperature dependency determination. I developed two different methods for two different types of vehicles, which can calibrate the sensor in the case of vehicle movement too. I also gave the method for the covariance matrix determination of the measurements.*

2.2 Quadrotor state estimation and control

The inner state of a system can be determined by two ways. They can be measured directly, or they can be estimated by the observation of the input and the output of the system. This thesis group presents state estimation solutions for the second case.

The goal of the state estimation is to determine the full state of vehicle movement such as orientation, angular velocity, position and velocity. The state estimation is based on the kinematic model of the moving body therefore these solutions can be used for any vehicle type.

For the orientation estimation firstly the RPY (roll-pitch-yaw, Euler angle) representation based approach was used. This method can handle the orientation estimation problem, but requires huge computation capacity. Therefore a quaternion based orientation representation was applied with the kinematic model belonging to it. This solution can handle the turnaround problem if some refinements are introduced in the Kalman filter.

For the position estimation a kinematic model based solution is presented which determines the acceleration, velocity and position in the body frame. This can be used for identification and testing purposes. On the other hand it requires the angular acceleration. It is usually hard to produce, therefore in the second variant this problem was handled by estimating the state variables in the sensory frame and in a last step only the position and velocity are transformed to body frame.

The test system of the state estimation algorithms is a quadrotor helicopter. The structure of the helicopter can be seen in Fig. 1. The vision processing based position and orientation measurements suffer a significant time delay. Therefore a state estimation method is presented which can handle this time delay.

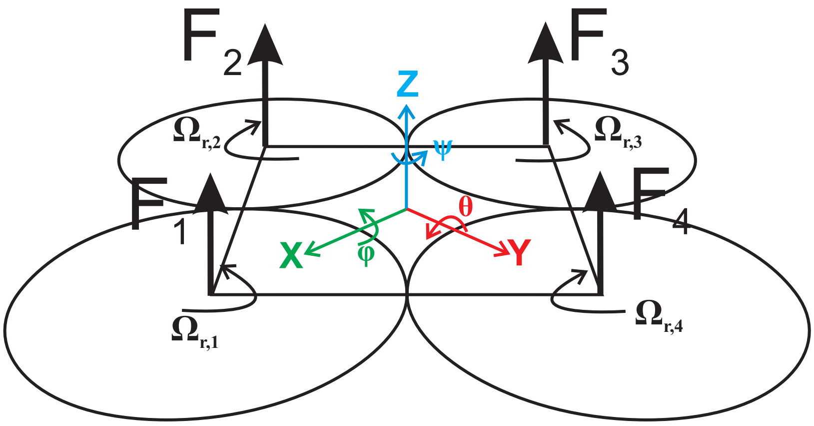

Figure 1: The structure of the quadrotor helicopter

Based on the usage of the state estimation, different types of control algorithms are designed for the helicopter. In the first approach existing algorithms are examined[1]. In the latter investigations a linear control is tested which establishes the requirement of more complex algorithms. For normal operation a nonlinear input/output linearization based solution is presented. For situations where the position and orientation determination system is unavailable (for example because of occlusion the markers), a robust control method is designed based on the H_∞ theory to perform safe landing in emerging situations.

Thesis Group 2. *I developed Kalman filter (KF) and extended Kalman filter (EKF) based state estimation methods and sensor fusion techniques for the integration of the inertial measurement unit (IMU) and external 3D position and orientation measurement system, which can be applied in the control system of ground and aerial vehicles to determine the state variables of kinematic quantities. I developed control algorithms which use the results of state estimation for the control of a quadrotor helicopter. The control algorithms and their embedded realizations use linear LQ design, nonlinear input/output linearizations and* H_∞ *synthesis.*

Related publications: [S1, S2, S3, S6, S7, S9, S12, S13, S14]

Thesis 2.1. *I developed a state estimation algorithm which is able to estimate the position and orientation of a moving vehicle and tolerates the time varying error of the inertial sensors. The orientation estimation is based on the quaternion representation, which gives more reliable and robust state estimation than the common RPY (Euler-angle) representation.*

Thesis 2.2. *In the case of image processing based position and orientation measurement methods the time delay of the measurement cannot be neglected. I developed a method which* *is able to handle the time delay of the sensors. The method assures a reliable state estimation of an indoor quadrotor helicopter and makes possible the development of precise control algorithms.*

Thesis 2.3. *I developed a nonlinear input/output linearization based control algorithm which is able to control an indoor quadrotor helicopter. The control algorithm uses the output of the state estimation. I realized the control algorithm on the embedded computer of the helicopter and demonstrated its performance during real flights.*

Thesis 2.4. *In the case of autonomous vehicles (UAVs) the unavailability of sensors cannot be ruled out. In this case the vehicle should safely reach a parking position. I developed a control algorithm for the quadrotor controller, which can be used in emergency situations. The unavailability of the vision system is handled. I realized the control algorithm on the board of the helicopter and demonstrated its applicability in simulated emergency situations during real flights.*

2.3 Carrier phase based GPS navigation

The outdoor navigation requires a sensor system, which can precisely determine the position and orientation of the vehicle. The wide-spread solution is the usage of the GPS. The common (3-10 meters precision) GPS techniques are not precise enough for position and orientation control. The sub-decimeter precision in positioning has great importance in the case of unmanned vehicles performing collision avoidance or motion amongst corridors. One aim of the research is to create the necessary tools for such tasks of unmanned vehicles. The application of the results on the field of vehicle control may be a part of future researches.

For high precision navigation the carrier phase differential GPS approach is used. This solution measures the actual state of the carrier signal. Since the wavelength of this signal is about 19cm, it is possible to produce position information with a sub-decimeter accuracy. With this precision it is also possible to determine the orientation of the vehicle. The state of the carrier signal in a given time instant and the structure of the differential GPS is shown in Fig. 2.

The carrier phase based techniques lead to the problem of the integer ambiguity determination, whose goal is to determine the integer number of carrier cycles between the satellite and the receiver:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{N}, \quad \mathbf{x} \in \mathbb{R}^3, \quad \mathbf{N} \in \mathbb{Z}^{m-1}$$

where $\mathbf{y}$ is a measurement vector, $\mathbf{A}$ and $\mathbf{B}$ are known system matrices, $\mathbf{x}$ and $\mathbf{N}$ are unknown variables.

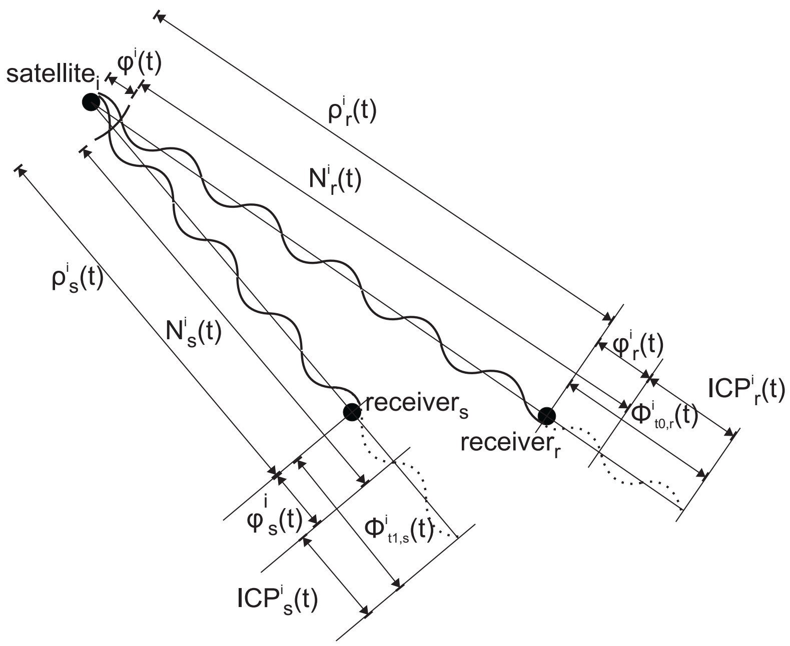

Figure 2: Structure of the carrier phase differential GPS

The common solutions [2, 6] of this problem have disadvantages which makes hard to apply them in general vehicle navigation (they can be used only in special circumstances).

The presented methods use the typical sensors which can be found on the board of a vehicle. In the first approach two GPS antennas are fixed on the vehicle, and the accelerometer and the magnetometer were used to solve the integer ambiguity problem. The main task is to determine the vector (baseline) between the antennas in the local North-East-Down (NED) coordinate system. In this case the integer ambiguity can be reliably determined using a single (or a few) measurement, which an advantage comparing to concurrent methods. When the integer ambiguity is already known, the baseline can be calculated. Using two GPS antennas and the magnetometer the 3D orientation of the vehicle can also be calculated. The magnetometer aided orientation determination has a singularity when the baseline and the magnetic vector are parallel. This can be a real problem in some circumstances. To handle this problem, a third GPS antenna is introduced.

When the orientation of the vehicle is already determined, special kinematic variables of the vehicle can be calculated. The calculation method of the angle of attack and the sideslip angle is presented. Based on these information development of identification and control algorithms are possible.

For the position determination an additional ground station (base station) is introduced. The navigation parameters are calculated relative to this base station. It is shown that the information about the relative displacement has no contribution to the double differentiated integer ambiguity.

In this case the common integer least square solution methods (LAMBDA, MLAMBDA) are used, but the developed methods take advantage of the fact that there are at least two GPS antennas on board of the vehicle and the integer ambiguity is already known from the orientation determination. In this way the position determination can be more reliable than the other solutions, and the integer ambiguities can be determined much faster (in the experiments only a single measurement was enough).

The continuous navigation requires handling the incoming and outgoing satellites and the case when there are not enough visible satellites for navigation. A method is presented to handle the change of the satellites. A two stage extended Kalman filter based solution is developed to produce navigation information. They handle the problem when the GPS measurements are eventually not available. This methods take advantage of that the inertial sensors have much smaller sampling time than GPS receivers, therefore the estimated position and orientation is available with the sampling time of the inertial sensors.

Thesis Group 3. *I developed navigation methods for a sensor platform containing carrier phase GPS receivers, inertial sensors and magnetometers. The methods are able to reliably determine the position and orientation of a moving vehicle. The presented approaches eliminate the disadvantage of the concurrent methods, the long initialization period. Based on the methods I developed state estimation algorithms, which use the carrier phase measurements and other sensor measurements for the robust position and orientation determination of a moving vehicle. The position's precision falls in subdecimeter domain which is necessary for solving critical maneuvers including collision avoidance and motion amongst corridors of unmanned vehicles (UGVs,UAVs).*

Related publications: [S4, S5, S8, S10, S11]

Thesis 3.1. *I developed a method for the integer ambiguity determination problem in the case of two GPS receivers fixed on the body of a vehicle. The method uses also the measurements of the accelerometer and the magnetometer. The method has the advantage that it is able to reliably determine the integer ambiguity using only a few measurements.*

Thesis 3.2. *I developed methods for the 3D orientation determination of a vehicle using either a set of two GPS antennas and a magnetometer or a set of three GPS antennas. Using these methods special kinematic quantities of the moving vehicle ,like angle of attack and sideslip angle, can also be determined.*

Thesis 3.3. *I developed a method for integer ambiguity resolution in the sense of precise position determination. This approach operates on the bases of the information of a base station and two moving receivers. The method can decrease the initialization time of the integer ambiguity determination.*

Thesis 3.4. *I developed an extended Kalman filter based sensor fusion algorithm, which operates on the bases of multiple carrier phase GPS receivers, inertial and magnetic sensors.*

The method can determine the position, velocity, orientation and angular velocity of a moving vehicle in a robust way. The method can handle the unavailability of the GPS sensors.

3 Applications

The results of the thesis groups are close connection with practical problems. The main motivation was the quadrotor helicopter project of the BME-IIT, which was the main application field of the developed algorithms. The calibration methods of the first thesis group are used on this platform. Based on the calibrated sensor data the state estimation and the control methods of the second thesis group were also tested and presented on the quadrotor platform. The position result of a position hold task is shown in Fig. 3.

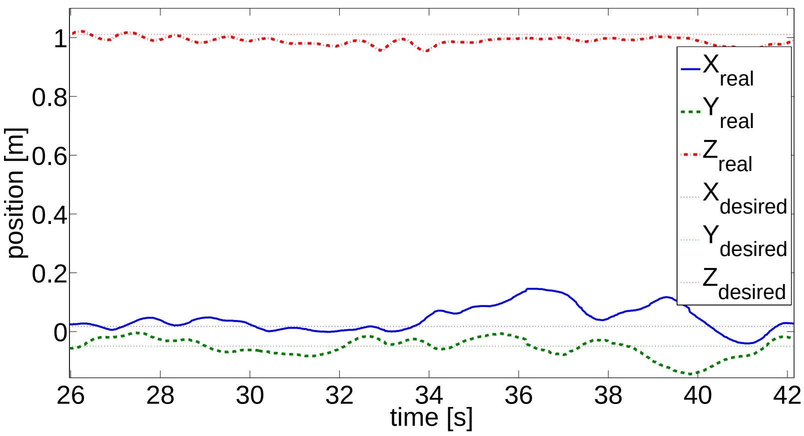

Figure 3: Helicopter position during position hold

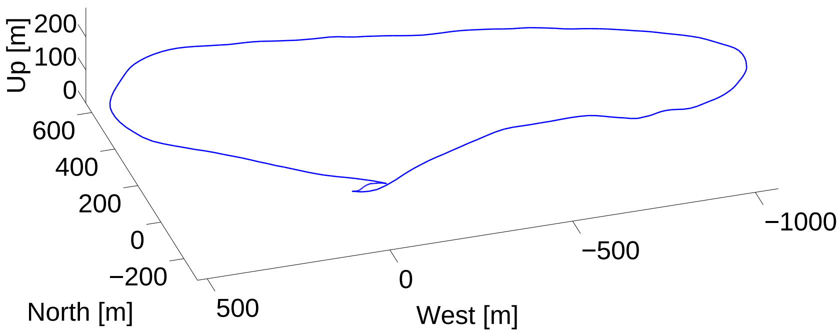

Figure 4: Path of the sail-plane

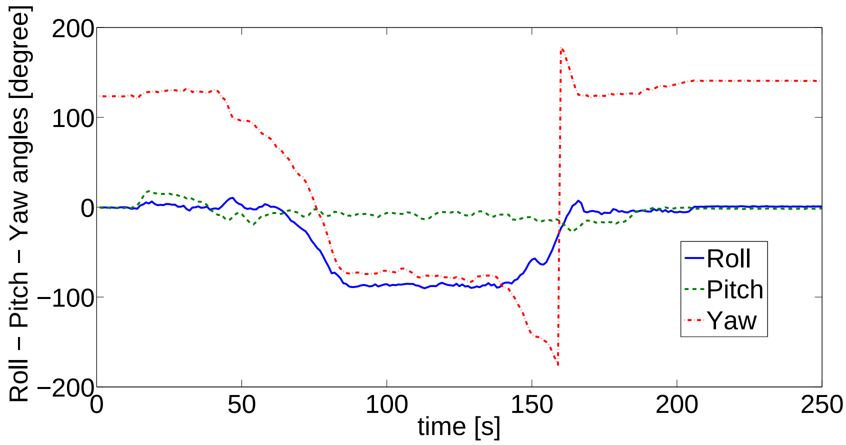

Figure 5: RPY angles during the sail-plane flight

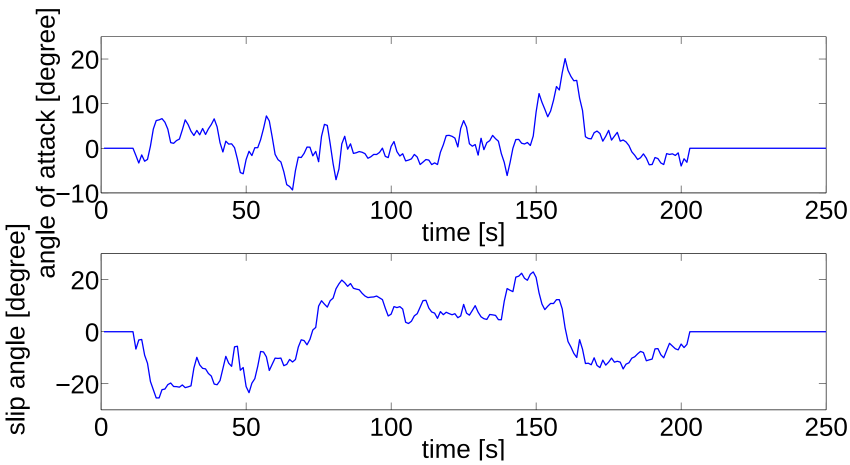

Figure 6: Angle of attack and side slip angle calculated from the 3D orientation

This research was supported by the Hungarian National Research Program "Advanced control theory and artificial intelligence techniques of autonomous ground, aerial and marine robots" under grant No. OTKA K 71762.

The results of the outdoor navigation part of the research is presented on the movement of a car and a sail-plane. In this case the state estimation algorithms of the second thesis group are adopted for outdoor environment, which shows the wide-usability of the developed methods. The result of the car and the sail-plane movement presents that the outdoor navigation algorithms of the thesis group 3 can be used in a large variety of outdoor vehicles, especially in case of unmanned ground (UGVs) and aerial (UAVs) vehicles.

The path of a sail-plane flight is shown in Fig. 4. According to this measurement the RPY angles of the 3D orientation are presented in Fig. 5. These angles are calculated from the GPS and magnetic measurements. The angle of attack and the slip angle calculated from the 3D orientation is shown in Fig. 6. The take off is performed at 10 seconds and the landing was at 205 seconds.

The calculation of the angle of attack and side slip angle is based on the transformation between the NED and the BODY frame. It is a general model which can be used for cars and airplanes as well. In this approach the angle of attack and the side slip angle are determined relative to the frame of the vehicle. If the airplane has precise wind (airflow) measurement sensors, their data can also be considered.

This work was connected to the scientific program of the "Development of quality-oriented and harmonized R+D+I strategy and functional model at BME" project. This project is supported by the New Hungary Development Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002).

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Own publications

[S1] L. Kis and B. Lantos. Szenzorfúzió alkalmazása beltéri autonóm négyrotoros helikopteren. *Hadmérnök 4:(4) pp. 191-204*, 2009.

[S2] L. Kis and B. Lantos. Quadrotor control based on partial sensor data. In Proceedings of: *19th IEEE International Workshop on Robotics in Alpe-Adria-Danube Region. Budapest, Hungary*, pp. 43–48, 2010.

[S3] L. Kis and B. Lantos. State estimation and control of an indoor quadrotor helicopter. In Proceedings of: *XVII. Magyar Repüléstudományi Napok. Budapest, Magyarország, ISBN: 978-963-313-032-2*, pp. 1–12, 2010.

[S4] L. Kis and B. Lantos. Aided Carrier Phase Differential GPS for Attitude Determination. In Proceedings of: *IEEE/ASME International Conference on Advanced Intelligent Mechatronics Proceedings. Budapest, Hungary, ISBN: 978-1-4577-0837-4*, pp. 778–783, 2011.

[S5] L. Kis and B. Lantos. INS Aided Carrier Phase Differential GPS System for Precise Navigation. In Proceedings of: *20th International Workshop on Robotics in Alpe-Adria-Danube Region. Brno, Czech Republic*, pp. 108–115, 2011.

[S6] L. Kis and B. Lantos. Quadrotor control based on partial sensor data. *International Journal of Mechanics and Control 12:(1) pp. 43-50*, 2011.

[S7] L. Kis and B. Lantos. Sensor fusion and actuator system of a quadrotor helicopter. *Periodica Polytechnica - Electrical Engineering, 2009:(53/3-4) pp. 139-150*, 2011.

[S8] L. Kis and B. Lantos. INS and magnetic sensor aided carrier phase differential GPS for attitude determination. In Proceedings of: *Proceedings of the 5th WSEAS World Congress on Applied Computing Conference (ACC'12). Faro, Portugal, 2012.05.02-2012.05.04*, pp. 106–111, 2012. ISBN: 978-1-61804-089-3.

[S9] L. Kis and B. Lantos. Time-delay extended state estimation and control of a quadrotor helicopter. In Proceedings of: *20th Mediterranean Conference on Control & Automation MED 2012: Conference Digest. Barcelona, Spain, 2012.07.03-2012.07.06.*, pp. 1560–1565, 2012.

[S10] L. Kis and B. Lantos. GPS based attitude determination using embedded magnetometer calibration. In Proceedings of: *21st Mediterranean Conference on Control and Automation 2013.06.25-28 Crete, Greece - accepted 2013.04.15*, 2013.

[S11] L. Kis and B. Lantos. High precision multiple antennas GPS positioning using carrier phase technique and sensor fusion. *Periodica Polytechnica - Electrical Engineering, Submitted for acceptance 2013.02.12*, 2013.

[S12] L. Kis, Z. Prohaszka, and G. Regula. Calibration and testing issues of the vision, inertial measurement and control system of an autonomous indoor quadrotor helicopter. In Proceedings of: *RAAD 17th International Workshop on Robotics in Alpe-Adria-Danube Region, Ancona, Italy, ISBN: 978-88-903709-0-8*, pp. 1–10, 2008.

[S13] L. Kis, Z. Prohaszka, and G. Regula. Calibration and testing issues of the vision, inertial measurement and control system of an autonomous indoor quadrotor helicopter. *International Journal of Mechanics and Control 10:(1) pp. 29-38*, 2009.

[S14] L. Kis, G. Regula, and B. Lantos. Design and hardware-in-the-loop test of the embedded control system of an indoor quadrotor helicopter. In Proceedings of: *6th Workshop on Intelligent Solutions in Embedded Systems. Regensburg, Germany, ISBN: 978-3-00-024989-1, DOI: 10.1109/WISES.2008.4623300*, pp. 35–44, 2008.

References

[1] S. Bouabdallah and R. Siegwart. Full control of a quadrotor. In *International Conference on Intelligent Robots and Systems*, 2007.

[2] X. Chang and C.C. Paige. An algorithm for combined code and carrier phase based GPS positioning. *BIT Numerical Mathematics*, 43:915–927, 2003.

[3] Jay Farrell and Matthew Barth. *The Global positiononig System and Inertial Navigation*. McGraw-Hill, 1998.

[4] Mohinder S. Grewal, Lawrence R. Weill, and Angus P. Andrews. *Global Positioning System, inertial navigation and integration*. Wiley, 2007.

[5] R.E. Kalman. A new approach to linear filtering and prediction problems. *Transactions of the ASME-Journal of Basic Engineering*, 82(D):35–45, 1960.

[6] P.J.G. Teunissen. Least-square estimation of the integer GPS ambiguities. In *IAG General Meeting pp. 1-16*, Beijing, China, 1993.