MŰEGYETEM 1782

Budapest University of Technology and Economics

Initial moment gradient load effect on the Vibration and Dynamic stability of a spatial thin walled structure

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2013

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Introduction

In engineering science, the stability of structures is one of the researchers desire. Many structures failures, either static or dynamic, have been attributed because of structural instability. The loading nature defines the problem's nature to be solved. Structures may be subjected to static, dynamic or even stochastic load what will be dealt with in this dissertation is the dynamically loaded structure [1] [2] [3] [4].

In mathematical point of view, a fundamental property of a dynamical system, which is the qualitative behavior of trajectories that is unaffected by C^1- small perturbations is called structural stability. Although a well-known theory in stability field, Lyapunov stability, speaks about considering perturbations of initial conditions for a fixed system.

Speaking about stability returns to almost two hundred years ago where stability analysis was sparked in Euler's mind in the forth decade of eighteen centuries. He advised a solution for buckling of an elastic column. Until the end of the 19^{th} century several types of problem who's related to the fundamental linear elastic were solved.

The ideas of structural stability in 1960s were applied by Stephen Smale, and his school for hyperbolic dynamics. Marston Morse and Hassler Whitrey began a parallel theory for stability which made a key part of singularity theory. Their job was developed by René Thom.

A structure's stability can be in trouble if it is under periodic load. In 1893 Liapunov worked on the behavior of dynamic instabilities for having general stability. In other words, it means a structure motion can be stable if any possible small changes being in initial condition, or better to say that there shall be a few changes in the response [5].

While a structure is under a dynamic load(s), the system is called *parametrically excited system* in brief. The parametric instability problems are not limited to the mentioned situation. The steady- state motions of nonlinear dynamical systems can be categorized in this field also. The main aim for analyzing the parametrically excited systems is to find the regions in parametric space which the system is going to be unstable. The later domains have been known as dynamic instability regions.

Numerical techniques (procedures) have been suggested widely for seeking the solution in later mentioned problems. One of these numerical methods whose has been applied for several types of structures e.g. circular or rectangular plates and in this text as well is series expansions.

State of art

Beams are widely being used in public vehicles, for instance buses frame, trains and etc. as a fundamental element in chassis structure. These elements are carrying different type of loading i.e. static and dynamic. Static loads are related to the weight of the structure, engine, and passengers. Dynamic loads, which are much more important than the previous one, are related to the external excitation effect on beams, such as rotary parts or road irregularities [6] [7] [8] [9] [10]. Many machines and structural members can be modeled, as beams with different geometries, for example beams with uniform cross-section, tapered and twisted beams. These components may have different boundary conditions depending on their applications.

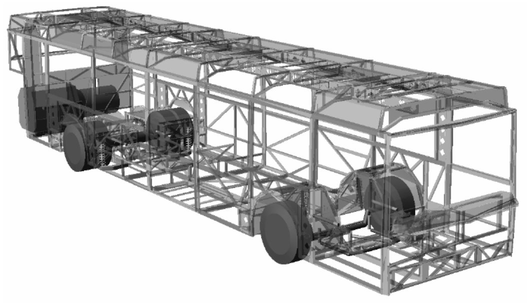

Fig. 1 Integral bus body under frame

Some researchers put their interest on seeking the stability regions of a structure in recent years as like following were selected from a mass of literatures. In general some of them focused on radial, axial and stochastic loading, whereas the other decided to work on different material type such as functional materials.

Öztürk and sabuncu on 2005 worked on static and dynamic stabilities of laminated composite cantilever beam having linear translation and torsional spring as elastic supports. The beam was subjected to periodic axial loading. They applied Euler beam and finite element method and used energy expressions in conjunction with Bolotin approach. In the study they obtained various disturbing frequency ranges in which the beam being unstable [11].

Liew et.al applied periodic axial force on rotating cylindrical shells and by the help of Ritz method and Bolotin's first approximation, the dynamic stability was investigated. Mathieu-Hill equation is obtained for Ritz energy minimization procedure [12].

Kumar and Mohammed in 2007 studied on dynamic stability of columns and frames subjected to axially periodic loads. They used finite element method with two nodes beam element for their analysis. Stability regions of columns were determined by decomposing the periodic loading into various harmonics by use of Fourier series expansion. They also applied direct integration of motion equations for discrete system. Newmark method was carried out to verify the results [13].

In 2008, Bazhenov et.al worked n the possibility of stabilizing dynamic states, which occurred by deterministic periodic parametric loading [4]. Sakar and Sabuncu in 2008 published their research results which were dealt with applying a finite element model for static and dynamics stability analysis of airfoil which is pre-twisted and it is rotating [14].

Saravia et.al studied dynamic stability behavior of a thin-walled composite beam which is rotating in 2011. They applied finite element method in their research. The investigation was based on Bolotin method for an axial periodic load. The regions of instability are obtained and expressed in non-dimensional terms [15].

Sabuncu and his colleagues in 2011 used a finite element method to obtain out-of-plane stability analysis of thin curved beams with tapered cross section. Beams were subjected to uniformly distribute radial loading. They used Bolotin approach to analyze dynamic stability and speak about first unstable regions. In this study out-of-plain vibration and buckling had discussed as well [16].

In 2012 Yang and fang, investigated the stability of an axially moving beam constituted by fractional order material subjected to parametric resonances. They applied Newton's second law and the fractional derivative Kelvin constitutive relationship. It was assumed that time dependent axial speed is changing harmonically, about a constant mean velocity [17].

In 2007 Vörös introduced seven degrees of freedom for finding the warping effect on buckling and vibration of stiffened plates [18]. He followed his research in 2009 to concentrate on the warping effect on the coupling of torsional bending coupling while a beam is initially loaded by uniform bending moment at its ends [19]. Questions which had been opened up to the present research were; *firstly if a beam is initially loaded by linear gradient moment at the ends and warping is considering as well, what would happen for natural frequencies*. *Second question related to the effect of this type of loading on the structure stability regions*. According to questions which this survey is going to answer them, *Initial moment gradient load effect on the Vibration and Dynamic stability of a spatial thin walled structure*, was selected as the title of the research.

The first step was to assume a thin walled prismatic beam with a constant cross section area along the length, as like what my supervisor has assumed in his latest research [20]. Using virtual work principal whilst the warping is taken into account for deriving motion equations leads to three differential equations. One of these equations is about the deflection of the beam and it is the uncoupled one. In this research the other two equations, which are coupled, were considered.

This research can be chopped into two different parts. The first part goes to find the first question which was mentioned earlier and tried to elaborate the effect of initial moment loading when it is changing between uniform and asymmetric one on natural frequencies and critical buckling moment of the beam. Then by using these values and helping of Mathieu-Hill equation the stability regions were obtained as the second part of the survey. In brief, it means that second part of the studying shows this type of loading effect on instability curves.

For plotting graphs, a computer programs by Maple software were applied. Some thin walled[1] cross sections were selected as examples for preparing graphs. Due to this point that the load type effect should be investigated, graphs were given in either dimensionless or semi non-dimension format and tried to make them independent from the beam material. Here it has worth to be noted that in practice for using these graphs the material limitations should be considered.

As a quick glance, the following procedure was used for achieving main goals of the current survey:

- Discuss about the coupled bending torsional frequency of a beam that is under a gradient moment load.
- Finding the effect of Reylaigh damping on the coupled bending torsional frequency while a gradient initial moment is taken into account.
- Illustrating the stability boundaries for a beam while excited by parametric gradient moment.

[1] There is no clear distinction between thin and thick sections, sometimes the rule (t_{max} / d) < 0.1 is used, where d is some other overall sectional dimension. The cross section of thin walled beams can be divided into two main groups; closed or open cross sections. Typical closed sections are round, square, and rectangular tubes. Open sections are I-beams, T-beams, L-beams, and etc. Thin walled beams are useful in engineering because of their bending stiffness per unit cross sectional area is much higher than that for solid cross sections such a rod or bar. In this way, stiff beams can be achieved with minimum weight.

Formulation

Stability concept

Taking into account a system by the following characteristic equation [21],

$$\dot{x} = f\left(x,t\right) \qquad f\left(0,t\right)=0$$
(1)

Here x nominate the state vector of the system, and $f(0,t)$ represents the initial condition. The goal is to find steady state solutions while $\dot{x}=0$. If one assumes that (1) is a linear system, this linearity causes the steady state solution being unique. When $\dot{x}=0$, it can be seen that the steady states of the system are not being change regarding to the time, while there will be no excitations.
In sense of Lyapunov theory, (1) stability solution should be defined as follows; if $x(t;x_0)$ is a solution for dynamic system (1), with initial condition $x(t)|_{t=0} = x_0$, the solution will be stable if for a given $\varepsilon>0$ there exists such $\delta>0$ where,

$$\left\|x\left(t;x_0\right)\right\|<\varepsilon \qquad \forall t>0 \quad , \quad if \qquad \left\|x_0\right\|<\delta\left(\varepsilon\right) \tag{2}$$

It has to be mentioned here that ||•|| is a proper vector norm for the given dynamical system as like(1).
The system will be called asymptotically stable if,

$$\left\|x\left(t;x_0\right)\right\|\rightarrow 0 \qquad while \quad t\rightarrow\infty \tag{3}$$

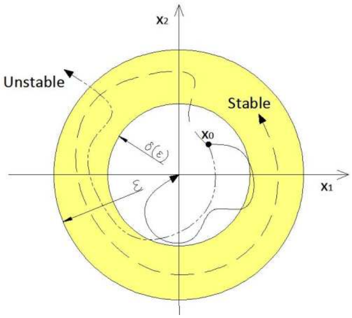

Fig. 2 Lyapunov stability

If the start point for the system being in *δ(ε)* region, x_0, and subsequently the previously mentioned solution $x(t;x_0)$ remains inside *ε* region for any time, the system will be called stable. Meanwhile $x(t;x_0)$ goes to the space origin by increasing the time, $t \to \infty$, the system shall be known as an asymptotically one. While speaking about finite dimensional system, all norms will be equivalent. It means that when a system such (1) is stable in an apt norm, it is stable in any other norms as well. In infinite dimension systems, the stability must be defined regarding to a specific norm. For instance in an elastic continuum, the energy norm is the appropriate one,

$$\|u\|^2 = \frac{1}{V}\int_V u\ u\ dV \qquad \|\dot{u}\|^2 = \frac{1}{M}\int_V \rho\ \dot{u}\ \dot{u}\ dV \tag{4}$$

Here $u(x,t)$ is the displacement field and $\dot{u}(x,t)$ is the velocity. *ρ*, *V* and *M* are density, volume, and mass respectively. Due to Koiter, the elastic continuum is stable if given *ε* and *ε'*, there exist such *δ(ε,ε')* and *δ'(ε,ε')* which,

$$\begin{gathered} \|u\| < \varepsilon \ \ and \ \ \|\dot{u}\| < \varepsilon' \quad \forall t > 0 \\ then \quad \|u(x_0,0)\| \le \delta \quad and \quad \|\dot{u}(x_0,0)\| \le \delta' \end{gathered} \tag{5}$$

Preliminary equations

According to Saint-Venant's theory of free torsion, the cross-section will not be remain as a plain generally and points can move freely in the direction of the rod and the torsion's angle changes linearly with constant rate. By restricting the torsional warping with internal or external constraints, torsion rate will also being changed along the beam. The impeded torsion was developed by Valsov [22]. The Bernoulli-Vlasov theory will be implemented because of the main objective of this research which is constraint torsion effect. In the updated Lagrangian (UL) approach applied in this research, system quantities are referred to the last known equilibrium configuration, here to be called as initial state. In this work, the basic assumptions are as follows: the beam member is straight and prismatic, the cross-section is rigid in its plane but is subjected to torsional warping, rotations are large but strains are small, the material is homogeneous, isotropic and linearly elastic. The initial moment is acting on the strongest axis.

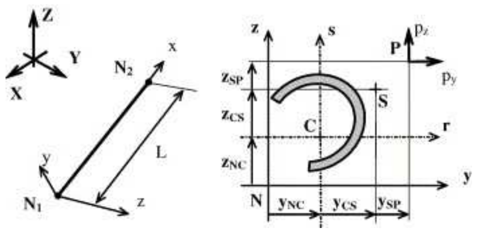

Fig. 3 Beam element local systems and eccentricities

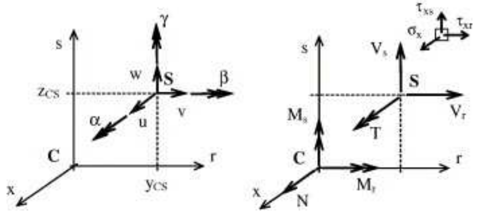

Fig. 4 Local displacement parameters and stress resultants

Using linearized virtual work principal for beam structure subjected to initial stress(6), motion equation of the beam structure can be derived. $\delta\Pi_L$,$\delta\Pi_{Gi}$ and $\delta\Pi_{Ge}$ are elastic strain energy with respect to initial stresses, changing in potential energy with respect to initial stresses, the second order effects of eccentric initial loads respectively. $\delta\Pi_M$ is the virtual work of inertia forces and δW gives the virtual work of external loads [20] [19].

$$\delta\left[\Pi_L + \Pi_{G1} + \Pi_{G2} + \Pi_{Ge} - R\right] = 0 \tag{6}$$

Modal analysis

As mentioned earlier the first step after deriving the necessary equations by virtual work method to investigate the effect of special loading type (initial gradient moment) on frequency and critical buckling moment of the structure.

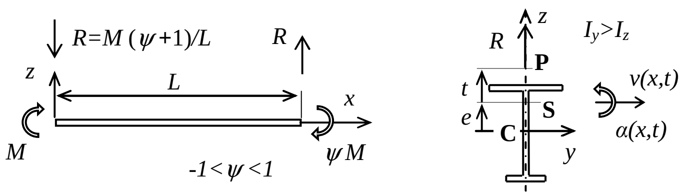

Fig. 5 Displacement, Rotations and coordinates system (sample cross section)

Ritz method was taken into account to define trail functions such as (7) for the lateral displacement *v(x,t)* and twisting *α(x,t)* of a considered mono symmetric cross section to satisfy the boundary conditions, In the case of simply supports at each end[2].

$$v(x,t)=\sum_{i=1}^{n}F_i(t)\,f_i(x) \quad , \quad \alpha(x,t)=\sum_{i=1}^{m}V_i(t)\,g_i(x) \tag{7}$$

Replacing (7) into (6) and did some simplification leads to motion equation of the beam in matrix format. $\underline{\underline{M}}$, $\underline{\underline{S}}$ and $\underline{\underline{K_e}}$ are mass ,stability and stiffness matrices. *M* is the initial moment acting on the beam ends (Fig.5)

$$\underline{\underline{M}}\;\underline{\ddot{X}}+\left(\underline{\underline{K_e}}+M\,\underline{\underline{S}}\right)\underline{X}=0 \tag{8}$$

Without losing the generality of the solution, Rayleigh damping in the following form of can be added to the motion equation(8).

$$\underline{\underline{D_a}}=2\;\xi\;\underline{\underline{M}} \tag{9}$$

By defining ζ as the ratio of damping,

$$\xi=\zeta\omega \quad and \quad 0<\zeta<1 \quad \Rightarrow 0<\xi<\omega \tag{10}$$

and considering under damped situation ($0<\zeta<1$), the relation between un-damped and damped solution can be expressed by following mathematical transformation:

$$\underline{X}(t)=e^{-\xi\,t}\,\underline{q}(t) \tag{11}$$

[2] Fork like support prevents torsional rotation and allows free warping.

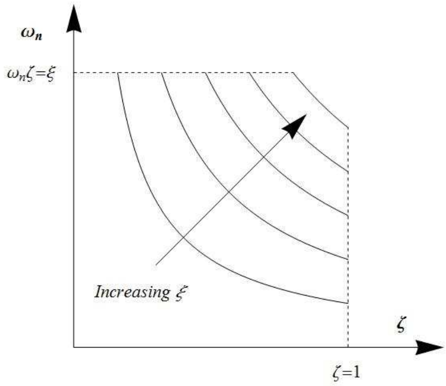

Fig. 6 Frequency vs. damping ratio for different damping coefficient

Final form of equation of motion for an assumed beam in (Fig.5) will be:

$$\underline{\underline{M}}\,\underline{\ddot{q}}(t)+\left(-\xi^2\underline{\underline{M}}+\underline{\underline{Ke}}+M\,\underline{\underline{S}}\right)\underline{q}(t)=0 \qquad (12)$$

Frequency

For obtaining the frequency it is enough to assume that the beam is free of load (free vibration). It means that the initial moment, *M*, must be set to zero in(12). By the modal analysis method, natural frequencies can be determined. Here according to the characteristic of the loading the minimal numbers were taken into account for the trail function approximation(7). Therefore two lateral bending and a torsional frequency were found for the beam.

$$\left(\omega_{b1}^{d}\right)^{2}=\left(\frac{\pi}{L}\right)^{4}\frac{EI_{z}}{\rho A}-\xi^{2} \quad , \quad \left(\omega_{b2}^{d}\right)^{2}=16\left(\frac{\pi}{L}\right)^{4}\frac{EI_{z}}{\rho A}-\xi^{2}$$
$$\left(\omega_{t1}^{d}\right)^{2}=\left(\frac{\pi}{L}\right)^{2}\frac{GJ}{\rho I_{ps}}\left[1+\left(\frac{\pi}{L}\right)^{2}\frac{EI_{\omega}}{GJ}\right]-\xi^{2} \tag{13}$$

Material properties are represented by E as young modules, *G* is called shear modules and ρ is density. A, I_z, I_{pS} and I_ω are a cross section properties, which are named cross section area, principal second moment, polar moment and warping constant respect to shear center.

Dividing the first lateral bending frequency over the first torsional one, makes it clear that up to certain value for the length the minimum natural frequency of the beam will be the torsional one and then it will be changed to lateral bending frequency in the absence of damping.

Critical Buckling

While no vibration considering for the beam, critical buckling moment can be determined from an Eigen-value problems method. As the first step the gradient moment factor, ψ, was set to minus one which represents that the beam is subjected to uniform moment loading. In this regard the critical buckling for uniform moment loading was derived, M^{d}_{cr0}. The procedure was continued for a case where the gradient moment factor was not restricted. In this case the general formula can be expressed for the critical moment, M^{d}_{cr}. There is an important parameter in Eurocode 3, moment gradient coefficient $C(\psi,\zeta)$, which gives the relation between latterly motioned critical moment. This is defined for un-damped case. What is given in this research can be called the improved form of it that include damping as well.

$$\left(M_{cr}^{d}\right)^{2}=C^{2}\left(\psi,\xi\right)\left(M_{cr0}^{d}\right)^{2} \tag{14}$$

While the damping ratio is set to zero (un-damped), critical buckling moment for uniform loading, M_{cr0}, can be given by using un-damped frequencies[3].

$$M_{cr0}^2 = \rho^2 A I_{pS} \left(\frac{L}{\pi}\right)^4 \omega_{b1}^2 \omega_{t1}^2 \qquad (15)$$

Forced vibration

The vibration of the beam had to be dealt with while it is subjected to initial gradient moment loading. In this way the external moment should be defined somehow that varies from zero (free vibration) to fully load[4] (sound of the structure). With this introduction the external moment has defined as(16). μ, is steady state moment load factor.

$$M = \mu M_{cr0}^d C_d \quad , \quad C_d = C(\psi, \zeta) \qquad (16)$$

Substituting (16) in (12) and using(13), will lead to characteristic equation of the structure, which is sixth order polynomial in this case. Making the later equation dimensionless respect to first un-damped lateral bending frequency in bending mode or first un-damped torsional frequency in torsional mode gives the frequency distribution versus loading parameters, μ and ψ[5].

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[3] ζ=0 in (13)
[4] buckling
[5] μ *is* steady state moment load factor and the gradient moment factor is ψ.

Stability analysis

After obtaining the fundamental frequencies and critical buckling moment of the structure (Fig.4), stability regions and borders should be determined for it. In this level of the research as it has been mentioned in introduction dynamic stability was spotlighted only. For this reason, first of all the initial moment had to replaced with a time dependent one as follows:

$$M = M_{cr}\left(\lambda + \kappa\, Cos\left(\Omega t\right)\right) \tag{17}$$

λ is static buckling moment percentage and κ is for dynamic one. λ and μ are changing between zero and one. Critical moment designated by M_{cr}. considering(17), (9) and (8) returns (18) , which is known as a *Mathieu-Hill*.

$$\underline{\underline{M}}\ \underline{\ddot{X}} + \underline{\underline{D_a}}\,\underline{\dot{X}} + \left(\underline{\underline{K_e}} + M_{cr}\left(\lambda + \kappa\, Cos\left(\Omega t\right)\right)\underline{\underline{S}}\right)\underline{X} = 0 \tag{18}$$

There are several possible approximated solution techniques in this field such as *Bolotin* based on *Floquet*'s theory, *Galerkin* method, the *Lyapunov* second method, asymptotic techniques by *krylov*, and perturbation and iteration are taken into consideration as well. One of the most used processes is to be assumed a periodic function. Among lots of periodic functions in time the sinusoidal series has been applied by lots of researchers.

Un-damped case

One of the most used processes is to be assumed a periodic function. Among lots of periodic functions in time the sinusoidal series has been applied by lots of researchers. In present study, this time dependent periodic function is taken into account based on Brown [23] with *2T* period (first region of stability). Approximated periodic solution, which is advised for, is [24]:

$$\underline{X} = \sum_{k=1,3,5}^{\infty} \underline{a_k} Sin\left(\frac{k\Omega t}{2}\right) + \underline{b_k} Cos\left(\frac{k\Omega t}{2}\right) \tag{19}$$

Having (19) and (18), and also keeping in mind that trigonometric functions are independent from each other causes (18), can be rearranged into two different determinant equations with respect to the (19) coefficients which have to be equal to zero for having non-trivial solution and obtaining instability borders.

$$\det\left(-\left(\frac{\Omega}{2}\right)^2 \underline{\underline{M}} + \underline{\underline{K_e}} + M_{cr}\left(\lambda \pm \frac{\kappa}{2}\right)\underline{\underline{S}}\right) = 0 \tag{20}$$

A dimensionless parameter, ϕ, should be defined as the ratio of input frequency over either first lateral bending or first torsional frequency for making plots. Returns to(18), makes it clear that in the absence of damping the static buckling moment percentage, λ, and gradient moment factor, ψ, effects on structure stability by using instability graphs were needed to be considered.

Damped case

The last but not least factor which has effect on the stability border is damping coefficient, ξ. Here is a point which must be cited is the stability regions are found by(21).

$$\det\begin{bmatrix} a & c \\ c & b \end{bmatrix} = 0 \tag{21}$$

$$a = -\left(\frac{\Omega}{2}\right)^2 \underline{\underline{M}} + \underline{\underline{K_e}} + M_{cr}\left(\lambda - \frac{\kappa}{2}\right)\underline{\underline{S}}$$

$$b = -\left(\frac{\Omega}{2}\right)^2 \underline{\underline{M}} + \underline{\underline{K_e}} + M_{cr}\left(\lambda + \frac{\kappa}{2}\right)\underline{\underline{S}} \qquad (22)$$

$$c = \Omega \xi \underline{\underline{M}}$$

New Scientific Results

Assumptions

A thin walled beam model with symmetrical cross section (shear center and cross section are coincided) was taken into account[6]. Basic assumptions which this study was founded on their basis are: the beam member is straight and prismatic. Cross section's area is constant along the length, and the cross-section is rigid in its plane but is subjected to torsional warping, rotations are large, strains are small, and the material is homogeneous, isotropic and linearly elastic. The beam was sustained with two simply supported fork like constraints at each ends that allow free warping and prevent torsional rotation.

Initial concentrated moments were acting on end points of the beam (theoretically on surfaces where geometrical boundary conditions were applied) on the strongest principle axis. In this text

[6] Upon this assumption four cross sections in two families (CS1, CS2, CS3 and CS4) were introduced.

while these moments acting axes direction are same, the condition is named asymmetrically loading. As another case the uniform loading was used for a situation that end moments acting axes direction are opposite.

Coupled frequencies and critical buckling moment

By the help of Ritz method and potential energy principle, then applying increment operator a set of equations was derived. The matrix form of these equations that have been obtained is known as the motion equations of the structure. Considering the modal analysis technique for solving an Eigen-value problem the characteristic equation of the beam was returned, beam's natural frequencies were found in the absence of external moment. Regarding to the ratio of first lateral bending frequency over first torsional one, two separated mode could be defined for the beam with previously mentioned conditions. If this ratio being greater than one it was called dominantly torsional mode while this ratio is lower than one the beam mode is known as a dominantly bending one. The critical buckling moment was found as well. A moment gradient coefficient, $C(\psi)$ was defined as the ratio of critical buckling moment over the uniformly loaded critical buckling moment. On the last but one step characteristic equation was modified by the help of natural frequencies, critical buckling moment and steady state moment factor, μ (represents the share of critical buckling moment on the element) forced vibration analysis was done.

I.a) The optimal selections for Ritz terms are two terms for lateral deflection and one term for torsional one. Higher terms have no significant effect on moment gradient coefficient, C(ψ) [25].

I.b) Considering dominantly bending mode, while the beam is subjected to uniform moment, causes a coupling in the frequency of structure [25].

I.c) When asymmetric moment is acting on the beam, the critical steady state moment factor, μ_c, was defined. The frequency equals to the first lateral bending one up to μ_c and then it will be coupled by increasing the steady state moment factor value up to one (full load) [25].

I.d) For cross sections which have dominantly torsional mode the coupling occurs for any value of moment gradient parameter ψ, and steady state moment load factor μ.

Damped Coupled frequencies and critical buckling moment

Rayleigh damping effect on structure's frequencies of the structure and shape modes was discussed to modify latterly derived equation for un-damped case, by adding the damping term to the matrix form equation. Defining the damping ratio, ζ, helped to express equations in dimension less mode, which is more suitable for comparing the results. The generality of the solution was not changed in this sense. Using a trail exponential equation as a mathematical transformation between damped and un-damped case and applying modal analysis for Eigen-value problem, the damped frequency and critical moment were found. Substituting of last two founded equations in characteristic equation made it possible to investigate the frequency and shape mode of a beam while damping is also taken into account.

Thesis II

II.a) Moment gradient coefficient, $C(\psi)$ was improved to $C(\psi,\zeta)$ by considering the damping ratio, ζ [26].

II.b) Selecting higher values for the Rayleigh damping ratio leads, the damped Moment gradient coefficient, $C(\psi, \zeta)$ being shifted upward [26].

II.c) The critical steady state moment factor, μ_c, which had been given, was modified to μ_c^d to cover up damping ratio effect.

II.d) Frequency and mixed mode ratio distribution's nature versus moment gradient parameter ψ, and steady state moment load factor μ remained same. But the amplitudes were decreased in both dominantly bending and torsional one [26].

II.e) Mixed mode factors amplitudes were enlarged by increasing the damping coefficient, ζ while bending mode is taken into account [46]. Meanwhile this ratio is decreased in comparison with undamped case, when the torsional mode is the dominant one.

Dynamic stability analysis (Un-damped)

For investigating instability regions of the beam structure, the matrix form of the beam's characteristic equation of motions was considered. As far as the dynamic stability was selected to be dealt with, the initial moment loading should be multiplied by a periodic time dependent function. The latterly mentioned equation form is known as a Mathieu-Hill equation. In the present study the first instability regions of first stability approximation due to its importance were spotlighted. In this way a trigonometric function with full period was suggested to be as a possible solution. By replacing the trail function into the modified motion of equation with periodic time dependent initial load, two independent series were derived. Series could be separated due to sine and cosine terms which were independent functions. As far as these series contain infinite terms, a limitation should be selected for achieving the optimal solution. Moment gradient parameter ψ and static buckling moment percentage λ, effects were shown on instability regions. For making simplifying graphs, frequency ratio, ϕ (the ratio of input frequency over either first lateral bending or torsional one regarding to the beam's mode i.e. bending or torsional) was introduced.

Thesis III

III.a) Having more than two terms in series of trail sinusoidal function has no significant effect on the first instability regions of first stability approximation [27].

III.b) By selecting the dominantly bending mode, for the asymmetric moment, the beam is stable for any value of frequency ratio, ϕ, while it is changing around the first instability region [27].

III.c) Instability regions exist for any amount of moment gradient parameter, ψ, if dominantly torsional mode is considered.

III.d) Increasing the share of static buckling moment percentage λ, in initial time periodic moment load causes the beam being more instable.

Dynamic stability analysis (damped)

The damping effect on stability regions was studied as the last part of this research. The solution process included a series of trigonometric functions with full period. By selecting different values for Rayleigh damping ratio, ζ, and static buckling moment percentage λ, in initial time periodic moment load, these parameters influence on instability regions were elaborated.

Thesis IV

IV.a) While the Rayleigh damping coefficient, ζ is increased the stability of the beam with simply supported hinged at ends, subjected to time dependent non-uniform moment is improved [28].

IV.b) Considering the Rayleigh damping into account, has no influence on static buckling moment percentage, λ, effect on instability of the beam. Higher amounts of λ, reduces the stability of the beam.

Results utilization

The present study has been done as a fundamental research on the basis of mechanical modeling, mathematical and engineering calculations. It was followed by computer programming for evaluating equations. Results can be useful for industries that are needed to deal with periodic loadings in their product's designing process. Particularly, public transportation vehicle structures producers, which are using beam elements. Current outcomes will be also valuable for commercial car chassis producer, airspace and cutting tool machine designers industries to have a better understanding of loading type effect.

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