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INVESTIGATION OF MULTI-VALUED, HYSTERESIS-TYPE
NONLINEARITIES IN NUMERICAL FIELD PROBLEMS

PhD Thesis Booklet

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1. Introduction

The effective analysis of various kinds of problems containing complicated nonlinearities in the form of hysteresis operators can be accomplished by the efficient integration of the achievements of the fields of engineering sciences, applied mathematics and information technology with the application of innovative approaches and solutions. The mathematical representation of hysteresis operators and hysteresis-type phenomena plays an important role in many disciplines, and the application of these operators is not restricted to the field of engineering. Besides, that the most widely known examples of hysteresis belong to the behavior of ferromagnetic materials, the hysteresis, as a more general phenomenon appears on many other fields of engineering and various other disciplines, like social sciences or biology. Because of the reason above, the phenomenological description of hysteresis can be especially advantageous, thus it can lead to a more insightful understanding of the fundamentals of hysteresis-type phenomena in general.

In order to a hysteresis operator be applicable in an actual computation, an implementation has to be constructed, concerning, that most of the engineering problems, which require the application of a hysteresis operator can only be handled numerically, the analytical solution of partial differential equations containing hysteresis operators cannot be obtained, at best the existence and uniqueness can be ascertained.

In my thesis I deal with the phenomenological description of hysteresis, and also the implementation possibilities of these kinds of operators into field calculation problems. Within this field, I carry out analytical and numerical investigations concerning the behavior and stability of the hysteresis operator, furthermore I examine the stability problems of the numerical solution of a field calculation problem containing a hysteresis operator, and I propose a suggestion in order to improve the efficiency of the solution procedure. Lastly, I demonstrate the applicability of the developed hysteresis model through a few selected case studies, and an actual industrial application.

2. The applied methods, goals and results

I built up most of my thesis on a basis of theoretical and phenomenological considerations. I have applied here the most important results, and widely used mathematical apparatus of the relevant fields; and by the aid of these tools and results I have made propositions regarding new methods in these areas.

I have mostly constructed my implementations using the Matlab[®] environment, and I have also carried out the numerical investigations concerning the material developed in the theses in Matlab[®], while in the case of more complex finite element implementations I have used the features of the COMSOL[®]-Matlab[®] environment. For the analytic, symbolic computations I

have applied the Maple[®] computer algebra system.

2.1. ODE-based hysteresis operator

During the construction of the hysteresis operator developed, one of the most important points was the compact mathematical representation, and the possibility of easy implementation. Another major point was to provide a general description of hysteresis phenomenon, which are not coupled too tightly to any concrete area of application. Based on the above aspects, the established ODE-based operator meets the design goals, furthermore the identification of the model, - which can be defined as a multi-parameter optimization problem - is relatively simple. The developed model describes hysteresis phenomenon on the basis of statistical considerations, and the hysteresis operator has a so called local memory structure.

The results of model simulation show very good agreement with measurement results (error is less than 5%), even in the case of very tiny minor loops, which is particularly impressive because the shape of the minor loop is described by a mere scalar parameter in the model. The validation of the operator had been carried out based on actual laboratory measurements.

The vectorial extension of the model is also promising, since it is basically the two-dimensional version of the statistical description introduced for the scalar case; and as the examples show, the model possesses the most important characterizing features of vectorial hysteresis (saturation property, loss property), and shows good qualitative agreement with results from literature.

Thesis:

I have developed a general phenomenological scalar hysteresis model by the aid of a statistical approach based on simple statistical considerations of the behavior of simple bistable units (domains, clusters) of the media, which resulted in a versatile model of hysteresis having useful properties, furthermore I have also developed a possible vector extension of the model, and I have established an efficient measurement-based identification method validated by actual measurements, which proved to be very accurate [8, 3, 9, 7, 6].

2.2. Stability analysis of the hysteresis operator and the implementation

Hysteresis operators with local memory usually features the accommodation property, which practically means, that in the case of relatively small minor loops corresponding to periodic excitations become stable only after a few transient cycles. During the investigation of the operator from the stability point of view, the goal was the determination of the exact location of the stabilizing transient

minor loops, since such result was not available in the literature. In order to carry out this investigation, I have described the solutions corresponding to periodic excitations in a properly constructed phase-space, where the behavior of the transient loops can be analyzed by the aid of an appropriately selected Poincare section. The result of the investigation is a Poincare map, from which the exact location of the stable minor loops can be directly calculated in view of the amplitude of the excitation.

During practical implementations, the examinations and results concerning the stability of numerical procedures containing a hysteresis operator can be really important, since stability is a fundamental requirement for any numerical method. In this regard the investigations are worth to be carried out based on the results concerning stability of numerical methods and nonlinear fixed-point iterations. The hysteresis, as a multi-valued nonlinear operator with memory can raise extreme difficulties from the stability point of view. During the iterative solution of implicit nonlinear systems of equations a global stability condition derived from the Banach fixed-point theorem can be applied, but it does not necessarily lead to the most efficient (fastest) method of solving the problem. Based on the investigation of the Lyapunov space of parameters characterizing the iteration and the hysteresis operator, such a conclusion can be drawn, that the decisive part of parameter space is filled with parameter sets corresponding to stable solutions, even in the case of selecting a higher damping constant (in order to speed-up the iteration), than the one suggested by the global stability condition. On the basis of these examinations I have proposed a suggestion for the local (parameter dependent) selection of the damping constant, this way the iteration procedure shown in the example becomes more than on order of magnitude faster, compared to the original (global stability condition) version.

Thesis:

I have carried out the thorough analysis of the hysteresis operator concerning its stability properties by the method of Poincare-sections in an appropriately constructed phase space, and as a result of the analysis I was able to derive a symbolic expression for the exact stabilizing location of the accommodating minor hysteresis loops. I have also analyzed the stability properties of the numerical solution of a diffusion problem with hysteresis, and after the examination of the Ljapunov-space of the parameters of the iteration corresponding to the numerical solution I have suggested a method for the cost-effective calculation of an appropriate damping constant ensuring the convergence of the iteration [10, 11, 5, 4].

2.3. Application of the operator in field calculation, case studies

The final part of the work consists of the demonstration of the implementation and application possibilities of the operator through case studies. One set of the problems presented here deals with the hysteresis modeling of ferromagnetic media, concerning the problem of magnetic penetration depth; on the other hand, a problem considered from the field of two-phase flow and dynamic phase transition. It is shown, that the developed ODE-based hysteresis operator can be coupled to the system of conservation equations of the original problem in the form of an extra equation, as an implicit state function.

In the case of two-phase flow with phase transition, the developed hysteresis operator perfectly fits to the conservation equations of the convective transport of energy, representing the latent heat released/absorbed during the first order vapor/liquid phase transition, and this way the metastable behavior of the phase transition is described by the hysteresis operator. This approach is not only advantageous from the numerical point of view (elimination of the ‘infinitely large gradient’ due to the step-like phase change), but it also describes the underlying physics of phase transition more precisely.

In addition to the theoretical results and case studies, I have established a computational core for the simulation of two-phase flow with phase transition in the form of a reusable component. This enables to carry out simulations concerning two-phase flow problems in any kind of high level computational environment, which is capable of the application of .NET-based components.

Thesis:

I have derived a formulation for the coupling of the developed hysteresis operator to the equations of a convection dominated fluid-flow problem, and I have implemented the developed hysteretic flow model into a two-phase flow network simulation software package as a computational core, which can be modularly embedded into higher level computational environments as a reusable component. I have demonstrated the applicability and versatility of the developed hysteresis model in two case studies, from the field of magnetics and two-phase flow with dynamic phase transition respectively. In the studies I have applied the finite element method as primary numerical tool for the investigation of the behavior of the hysteresis operator developed [1, 14, 2, 12, 13].

3. New Scientific Results

3.1. Thesis 1.

I have developed a general phenomenological scalar hysteresis model by the aid of a statistical approach based on simple statistical considerations of the behavior of simple bistable units (domains, clusters) of the media, which resulted in a versatile model of hysteresis having useful properties, furthermore I have also developed a possible vector extension of the model, and I have established an efficient measurement-based identification method validated by actual measurements, which proved to be very accurate [8, 3, 9, 7, 6].

- 1.1. I have constructed an ODE-based scalar hysteresis model by the aid of statistical approach of hysteresis phenomena, which resulted in an improved variant of general Duhem-like models of hysteresis [8]. In the developed model I have introduced a mathematical description of the hysteresis curves, which enables of shaping the minor hysteresis loops independently from the major curve offering large versatility to the model, while preserving the advantageous properties of the closed form analytical formulation.
- 1.2. I have developed a two-step, measurement-based identification procedure of the model, applying linear combination of probability density functions for precisely shaping the major hysteresis curve by optimization, and a one-parameter minimization for approximating the shape of the minor loops [3, 9]. The identification of minor loops can be accomplished by either first order reversal curves or concentric minor loops.
- 1.3. I have proposed a possible vector extension of the established scalar model, which provides a reasonable description of vectorial hysteresis phenomena in two-dimensions [9, 7, 6]. The model captures the most important features of vectorial hysteresis, obeys the general fundamental physical principles of vector hysteresis, and shows good agreement with known experimental results.

3.2. Thesis 2.

I have carried out the thorough analysis of the hysteresis operator concerning its stability properties by the method of Poincare-sections in an appropriately constructed phase space, and as a result of the analysis I was able to derive a symbolic expression for the exact stabilizing location of the accommodating minor hysteresis loops. I have also analyzed the stability properties of the numerical solution of a diffusion problem with hysteresis, and after the examination of the Ljapunov-space of the parameters of the iteration corresponding to the numerical solution I have suggested a method for the cost-effective calculation

of an appropriate damping constant ensuring the convergence of the iteration [10, 11, 5, 4].

- 2.1. I have investigated the stability of the ODE-based scalar hysteresis model developed in Thesis 1, concerning the behavior of the accommodating minor loops. I have constructed an appropriate three-dimensional phase space for the representation of periodic solutions and by the aid of this representation I have derived a Poincare-map, from which the exact location of the stable minor loop can be obtained [10].
- 2.2. I have examined a nonlinear diffusion problem with hysteresis by the Lyapunov-method and based on the results I have developed a method for the selection of the damping constant of the nonlinear iteration applied. A one dimensional diffusion problem and its numerical solution has been analyzed, paying particular attention to the fixed-point iteration resulting from the implicit nature of the iteration scheme in nonlinear material. I have proven, that the unstable behavior of the inner fixed-point iteration originates from bifurcation or - in extreme cases - chaotic behavior of the attractor of the iterative map [11, 5, 4]. Based on the examination of the parameter space of the iteration I have developed a damping constant calculation method to speed up the iteration significantly, thus the average number of iterations needed is more than an order of magnitude lower than in the case of the original damping constant [11].
- 2.3. I have analyzed the general structure of Picard-type iterations for diffusion-type PDE containing a hysteresis operator, and I have derived a general approach for the estimation and runtime control of the damping constant of the iteration. I have shown for the one-dimensional case, that the global stability condition for the damping constant of the iteration is the same, regardless of the hysteresis model for both of the discretization methods (FDTD Yee-algorithm, FEM) applied, and I have pointed out, that the local selection of the damping constant during the nonlinear iteration is more efficient, than applying a global stability condition throughout the whole numerical solution of the problem.

3.3. Thesis 3.

I have derived a formulation for the coupling of the developed hysteresis operator to the equations of a convection dominated fluid-flow problem, and I have implemented the developed hysteretic flow model into a two-phase flow network simulation software package as a computational core, which can be modularly embedded into higher level computational environments as a reusable component. I have demonstrated the applicability and versatility of the developed hysteresis model in two case studies, from the field of magnetics and two-phase flow with dynamic phase transition respectively. In the studies I have applied the finite

element method as primary numerical tool for the investigation of the behavior of the hysteresis operator developed [1, 14, 2, 12, 13].

- 3.1. I have implemented my model in a two-phase flow field calculation problem by extending the energy conservation equation of convective flow with a source term containing a hysteresis operator representing the release/absorption of energy in the form of latent heat, according to the principle of vapor/liquid phase transition. Based on this extended formulation, I have inserted the developed hysteresis model into a finite element calculation involving vapor-liquid first order phase transition [1, 14, 2] proving the applicability of the proposed model and demonstrating the hysteretic behavior in the metastable region.
- 3.2. I have implemented the model of two-phase flow with hysteretic phase transition as the computational core of an actual network flow calculation engineering software [12, 13] providing a simulation engine in the form of reusable component for all kinds of higher level software environments, which have access to interfaces offered by .NET based reusable components.
- 3.3. I have proved by a sequence of case studies regarding penetration depth of magnetic field in nonlinear media, and two-phase flow with dynamic phase transition, that both the developed hysteresis model and the method of FEM implementation are reasonable [14], and can be applied during actual engineering field-calculation problems.

4. Further research plans

In the research work discussed in the previous chapters I have made some improvements concerning hysteresis modeling, stability analysis of hysteretic problems, and furthermore, I have presented an actual application of the developed hysteresis model in a work, corresponding to the analysis, modeling and simulation of a two-phase flow network. Since the research carried out is quite interdisciplinary, there are many areas, where the continuation of the research could be desirable and would possibly lead to promising further results.

The investigation of the role and applicability of the developed hysteresis model in the field of avalanches would be an interesting and promising research direction, since the fundamentals of the model are built on a statistical description of the collective behavior of bistable units resulting in an ‘average’ behavior in the form of a hysteresis curve. Nevertheless if the research could focus on a more general class of this ensemble of bistable units, it could lead to a possible connection to earthquake modeling, temperature induced brittle-ductile phase transitions in solids, etc.

The implementation details of the various hysteresis operators into actual numerical computations is also an important part extending the recent research,

since the practical application of a hysteresis operator in an numerical environment is far from trivial, and the design, the representation, and the programming aspects of the task are really under appreciated and worth to having considered as an independent research area connecting to the topic. The difficulties of the implementation are clearly demonstrated by the fact, that the actually widely used commercial finite element packages does not contain hysteresis operators, the nonlinear material behavior is handled only in a significantly simplified form.

The identification procedure developed for the hysteresis model could be refined further by constructing an appropriate metric for measuring the distance between measured major hysteresis curves and the CDF of normal distribution, which would have the potency of providing more insight into the fundamental properties, and origins of hysteretic behavior, which appears in many, apparently independent engineering fields and disciplines.

The stability investigation methods developed, in their actual form are valid for one dimensional discretizations. It would be a concern of further research to continue these investigations, and extend the scope of the methods established to higher dimensional cases, expecting even more general results, with potential applications in numerical computational software packages.

Investigating and improving methods of measurement and identification of vectorial hysteresis could also be a possible area of future research activity, since there are many complex field calculation problems (or complicated geometries), where the application of a scalar model of hysteresis is not satisfactory, and the intrinsically vectorial nature of hysteresis phenomenon has to be taken into account. However the measurement of vectorial hysteresis is a very resource-intensive procedure, requiring not only sophisticated measuring devices and methods, but carefully prepared samples and a precisely built single sheet tester, which can have a considerable investment demand. In spite of the difficulties, this direction of research is really useful not only from the practical point of view (efficient and reliable methods of measurement, validation of vectorial models), but it could also have a theoretical impact, enabling a deeper and more insightful understanding of hysteresis phenomena in general.

I am convinced, that the intensive application of the appliances and methods of informatics in almost every branch of science can lead to a new direction of development of computational methods, where the significant results not only achieved by the tools of abstract mathematical description, but beside of that, the innovative algorithms and data structures describing the discreteized methods could form an other important source of serious scientific results. If this kind of integration can be accomplished effectively, then it will be fruitful from the theoretical point of view, and for the practical applications as well.

Hivatkozások

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