Budapest University of Technology and Economics
Faculty of Economics and Social Sciences
Institute of Business Sciences
Department of Management and Corporate Economics

Supporting Management Decision Making with Mathematical Programming Models

Theses of doctoral dissertation

Author: Viola Tatay

Supervisor: Tamás Koltai, dr. Habil
professor

Budapest
2013.

CONTENTS

1. SCIENTIFIC BACKGROUND AND OBJECTIVES OF THE RESEARCHES

Acquiring durable and unique competitive advantages is essential in company competitiveness. Companies' long term success is determined by the strength and steadiness of the possessed competitive advantages (Porter, 2006). Companies have to differentiate themselves from their competitors in today's competition. The tools of differentiation changed dynamically in the past (Stalk-Hout, 1990). Leading companies, which are successful in cost- and quality based competitions, today focus on different time parameters. Time has become a strategic resource and time based competition is the current prevailing management paradigm. However, time cannot be a strategic resource for a long time, a new source of competitiveness is expected to appear and spread. According to Davenport (2006), quantitative approach can be the next source of competitiveness of leading companies. Consequently, development and practical application of quantitative tools can be an important part of competitiveness.

One wide field of the quantitative tools that can be applied to support practical management decisions is operations research – also called as management science or decision science. Operation research deals with optimization theory. The objective of management science is to find optimal or satisfactory solutions for processes and systems based on different quantitative methods and computational algorithms (Rapcsák, 1988). However, the scope of production management is the efficient operation of production and service systems. Production management decisions can be successfully supported by the results and models of operations research.

One of the most important fields of operations research is mathematical programming. Central problem of mathematical programming is to allocate scarce resources to various operations that is a classical managerial problem (Hillier-Lieberman, 1995). During my researches I have focused on two important areas of mathematical programming: sensitivity analysis of linear programming models and assembly line balancing modeled by binary programming problems.

Linear programming (LP) is such a holistic decision support tool that can be applied to support operational decision making relating to several functional fields of corporate operation. One of the most typical application areas of LP models is production management (Nahmias, 1997).

Solving linear programming models can contribute to successful managerial decisions in several ways. Building quantitative models may result in thorough comprehension of the problem, acquaintance of subfields and revealing of relationship between different parts (Koltai, 2001). In case of practical application of linear programming models sensitivity analysis can be as important as finding the optimal solution itself. Based on sensitivity analysis results effects of the changes of the environmental factors (prices, costs) on the optimal solution and the probable effects of managerial decisions (changes in the capacity, demand management) can be evaluated. However, commercial software packages can give misleading sensitivity analysis results if the optimal solution of a linear programming problem is degenerate (Jansen et al., 1997; Rubin-Wagner, 1990).

I do not consider sensitivity analysis results *correct from management point of view* if the sensitivity analysis results can not be used to answer management questions completely. Using not correct or misleading information can lead to unfavorable consequences. For example if a cost parameter in the objective function of a linear production planning problem is analyzed, then narrower validity range can be gained if the optimal solution is degenerate. The narrow validity range is *correct from mathematical point of view*, as outside the given validity range a new basis belongs to the optimal solution. Thus, the results are correct from mathematical point of view. However, the same production quantities may belong to the new basis – as well as to the previous one – so the production plan should not be changed in case of a small change of the cost data. Consequently, the validity range for the cost parameter is wider from management point of view than from mathematical point of view. In my doctoral thesis I use the term *correct from management point of view* to problems similar to the cost parameter case of the production planning problem. The sensitivity analysis results are correct from management point of view if the information can be used directly to support management decisions. Consequently, sensitivity analysis results misleading from management point of view are correct from mathematical point of view, however these pieces of information can not be used in management decision making.

Problem of degeneracy is well-known in the literature. Several papers and books discuss the problem (e.g. Jansen et al, 1997; Gal, 1986) and some commercial software packages can recognize degenerate problems. However, no such tool exists that can determine the correct sensitivity analysis results from management point of view. In my researches I have had the following objectives relating to degenerate linear programming problems:

- Reveal the differences between the mathematical and managerial sensitivity analysis.

- To develop a calculation method with which the correct sensitivity analysis results can be calculated.
- To illustrate the practical significance of the difference between sensitivity analysis results determined by commercial software packages and the correct results computed by the developed calculation.

Binary programming models are linear programming models with zero or one decision variables. One typical practical application area of binary programming is assembly line balancing (ALB). ALB problems occur where several indivisible work elements (tasks) are to be grouped into (work)stations along a continuous production line. Binary programming models are typically NP hard problems (Hillier-Lieberman, 1995). Important purposes of researches relating to assembly line balancing models are to simplify the models and to develop effective solving algorithms. However, today even practical size binary models can be solved in a reasonable time frame due to the development of computer and information technology of the last decades. Thus, the focus of the researches can be shifted to practice driven model formulations and to the practical application of mathematical assembly line models (Boysen et al., 2008).

Solving assembly line balancing models can provide useful information to various management decisions. Different efficiency studies of the line or the workstations and sensitivity analysis results can support managerial decisions related to the operation of assembly lines (Waters, 1996). Consideration of different worker skills may be an important part of the practical application of ALB models. There are very few papers which are dedicated to the effects of application of workers with different skill levels on the optimal solution. I have not found in the literature any paper that gives a general method which can be applied to generally model the effects of workers with different skill levels in the assignment of tasks to workstations. The objectives of my researches relating to assembly line balancing models can be summarized as follows:

- To develop different methods and managerial decision support tools that can lead to widespread practical application of assembly line balancing models.
- To develop models giving better approximation of reality.
- To illustrate the practical application of assembly line balancing models to a real corporate problem.

2. METHODS OF RESEARCH

In my researches I have analyzed problems that can facilitate practical application of mathematical programming models. Daily management decisions can be supported by the calculation methods and models I have developed. I have made sample problems and case studies to illustrate the practical applications of the developed tools.

I have begun my researches with *literature review*. I have studied the management paradigms that determined the competition among companies in the past few decades and the changes of the prevailing management paradigms. I inspected the literature relating to quantitative approach that can be the next prevailing management paradigm. I revised papers and books concerning the history, the different application areas and models of mathematical programming – especially of linear programming and assembly line balancing.

I have studied the problem of sensitivity analysis results of degenerate linear programming problems. I have examined the difference between the mathematical and the managerial sensitivity analysis. I have constructed a *model system* working in Excel environment and using LINGO optimization software that can be used to determine the correct sensitivity analysis results – from management point of view – for any linear programming problem. The first step of the construction was calculation of the correct sensitivity analysis results of a small sample problem. After receiving the expected correct results I have calculated the correct sensitivity analysis results for more case studies from the literature. In my dissertation the sample problem of Koltai and Terlaky (2000) is applied to illustrate the correct sensitivity analysis results. In the journal paper the problem of degeneracy and the difference between the mathematical and managerial sensitivity analysis are discussed in detail, however, the correct sensitivity analysis information is not calculated.

After reviewing the literature of mathematical programming models applied for balancing assembly lines, I have concluded that research objectives facilitating practical application and focusing on practical problems should be achieved. I demonstrated the application of simple assembly line balancing models and relating management issues with data of a real bicycle assembly line. To perform this investigation I have *collected and analyzed data* of one bicycle model. After observing the bicycle assembly process I have defined new goals: constructing models which gives better approximation of reality. I have created *a calculation method* that can be applied in optimization of any parameter of an assembly line if there are significant differences in skill levels of the workers.

My scientific results concerning linear and binary programming models can contribute to successful managerial decision making for leading companies in the production and service industry.

3. NEW SCINETIFIC RESULTS

Development and application of quantitative tools considering terms of practical application are very important in company competitiveness. My scientific results are related to development and application of such models.

Linear programming is one of the most frequently applied areas of mathematical programming. Sensitivity analysis is essential in linear programming as it gives information about the effects of changes of the objective function coefficients or the right-hand side parameters. Commercial software packages might not be able to provide the correct sensitivity analysis results from management point of view if the linear programming problem is degenerate. No tool exists that can determine the correct sensitivity analysis results from management point of view for degenerate linear programming models. In the first part of my researches my research goal has been to determine the correct sensitivity analysis results. Notation used in the paper is summarized at point 8.

3.1. Thesis 1

If a linear programming problem is degenerate, the validity ranges of the objective function coefficients (OFC) determined by commercial software packages may be narrower than the real ranges. The narrower range can aim management's attention to objective function coefficients in vain.

> **Thesis 1: A calculation method that can calculate easily in practice the correct validity range information for objective function coefficients– for degenerate linear programming problems – has been developed.**
>
> *(Related publications: S2, S3, S5, S8, S9, S14)*

For one objective function coefficient two linear programming problems are needed to be solved in order to get the correct validity ranges from management point of view. The allowable maximal decrease can be determined with the solution of the following linear programming model,

$$\begin{aligned} &\mathbf{A^T y} \geq \mathbf{c} - \gamma_i \mathbf{e}_i \\ &\mathbf{b^T y} = OF^* - \gamma_i x_i^* \\ &\gamma_i \geq 0 \\ &\text{Max } (\gamma_i). \end{aligned} \qquad (1)$$

The optimal solution of (1) (γ_i^-) is the allowable maximal decrease of objective function coefficient i – with fixed optimal solution. The allowable maximal increase can be determined with the solution of the following linear programming model,

$$\begin{aligned} &\mathbf{A}^{\mathbf{T}}\mathbf{y} \geq \mathbf{c} + \gamma_i \mathbf{e}_i \\ &\mathbf{b}^{\mathbf{T}}\mathbf{y} = OF^* + \gamma_i x_i^* \\ &\gamma_i \geq 0 \\ &\text{Max } (\gamma_i). \end{aligned} \qquad (2)$$

The optimal solution of (2) (γ_i^+) is the allowable maximal increase of objective function coefficient i – with fixed optimal solution. Consequently, for a linear programming problem with I objective function coefficients the solution of $2I$ LP problems is required to get the correct validity range information. This is discussed in more details in Thesis 3.

In my doctoral dissertation the production planning model of Koltai and Terlaky (2000) is used to illustrate the problems of degeneracy and the differences between the results given by the LINGO software and calculated by the constructed calculation method. Table 1 summarizes the sensitivity analysis results of the objective function coefficients of this sample problem.

Table 1.: Sensitivity analysis results of the objective function coefficients of the sample problem

OFC	Original value	LINGO software decrease	LINGO software increase	Suggested method γ_i^-	Suggested method γ_i^+
$r_{1,1}$	10	-25	5	-∞	5
$r_{2,1}$	10	-5	5	-5	∞
$r_{1,2}$	25	-5	∞	-5	∞
$r_{2,2}$	20	-5	5	-25	5
$i_{1,1}$	5	-25	5	-∞	5
$i_{2,1}$	5	-5	5	-5	∞
$i_{1,2}$	5	-25	∞	-30	∞
$i_{2,2}$	5	-25	∞	-25	∞

It can be seen from Table 1, the correct validity range information used by management is determined by LINGO software only for two out of eight objective function coefficients. I have calculated the correct sensitivity analysis results for all of the objective function coefficients. The results of sample problem can be deduced so it can be verified easily. This is discussed in detail in the doctoral dissertation.

3.2. Thesis 2

Commercial software packages may determine misleading shadow prices and narrower validity ranges for the right-hand side (RHS) parameters if the optimal solution is degenerate. In case of degeneracy two shadow prices may exist, however mathematical programming software packages give only one shadow price. The effect of decreasing or increasing a right-hand side parameter value can be different because of the left and right shadow prices. Software packages do not inform management correctly.

> **Thesis 2: A calculation method that can calculate easily in practice the correct sensitivity analysis results for right-hand side parameters– for degenerate linear programming problems – has been developed. The left and right shadow prices with their relating validity ranges can be compute with the help of the calculation method.**
>
> *(Related publications: S2, S3, S5, S8, S9, S14)*

Calculations relating to left and right shadow prices must be separated. Left and right shadow prices are differentiated with the help of perturbation (δ). Over the two perturbated LP models, two LP model is needed to be solved for each shadow prices to get the correct sensitivity analysis results. Consequently, six LP problems must be solved to calculate the correct sensitivity analysis results for one right-hand side parameter.

If the left shadow price is calculated, then constraint j of the original problem is perturbated with $\delta<0$. The allowable maximal decrease of right-hand side parameter b_j relating to left shadow price can be determined with the solution of the following linear programming model,

$$\begin{aligned} &\mathbf{Ax} \le \mathbf{b} + \delta\mathbf{e}_j - \xi_j\mathbf{e}_j \\ &\mathbf{c}^{\mathrm{T}}\mathbf{x} = OF^{-*} - \xi_j y_j^{-*} \\ &\xi_j \ge 0 \\ &\mathrm{Max}(\xi_j). \end{aligned} \tag{3}$$

The allowable maximal increase of right-hand side parameter *bj* relating to left shadow price can be determined with the solution of the following linear programming model,

$$\begin{aligned} &\mathbf{Ax} \le \mathbf{b} + \delta\mathbf{e}_j + \xi_j\mathbf{e}_j \\ &\mathbf{c}^{\mathrm{T}}\mathbf{x} = OF^{-*} + \xi_j y_j^{-*} \\ &\xi_j \ge 0 \\ &\mathrm{Max}(\xi_j). \end{aligned} \tag{4}$$

The optimal solution of LP problems (3) and (4) are the allowable maximal decrease ($n\xi_j^-$) and allowable maximal increase ($n\xi_j^+$) respectively, relating to left shadow price of right-hand side parameter b_j.

If the right shadow price is calculated, then constraint j of the original problem is perturbated with $\delta>0$. The allowable maximal decrease of right-hand side parameter b_j relating to right shadow price can be determined with the solution of the following linear programming model,

$$\begin{aligned} &\mathbf{Ax} \le \mathbf{b} + \delta\mathbf{e}_j - \xi_j\mathbf{e}_j \\ &\mathbf{c}^{\mathrm{T}}\mathbf{x} = OF^{+*} - \xi_j y_j^{+*} \\ &\xi_j \ge 0 \\ &\mathrm{Max}(\xi_j). \end{aligned} \tag{5}$$

The allowable maximal increase of right-hand side parameter b_j relating to right shadow price can be determined with the solution of the following linear programming model,

$$\begin{aligned} &\mathbf{Ax} \le \mathbf{b} + \delta\mathbf{e}_j + \xi_j\mathbf{e}_j \\ &\mathbf{c}^{\mathrm{T}}\mathbf{x} = OF^{+*} + \xi_j y_j^{+*} \\ &\xi_j \ge 0 \\ &\mathrm{Max}(\xi_j). \end{aligned} \tag{6}$$

The optimal solution of LP problems (5) and (6) are the allowable maximal decrease ($p\xi_j^-$) and allowable maximal increase ($p\xi_j^+$) respectively, relating to right shadow price of right-hand side parameter b_j.

Table 2 summarizes sensitivity analysis results of right-hand side parameters of the sample problem that is discussed in detail in the doctoral dissertation. It can be seen from Table 2, that two sided shadow prices belong to five out of eight right-hand side parameters. Only three right-hand side parameters ($D_{2,2,}$, B_2, W_2) have one shadow price. By coincidence, the validity ranges of these three right-hand side parameters were determined correctly by LINGO software. For right-hand side parameters $D_{1,1}$, $D_{2,1}$ and B_1 a shadow price valid only in a given point was given by LINGO software. A shadow price valid in a given point is important from mathematical point of view, this information ha no managerial meaning. I determined the correct sensitivity analysis results (left and right shadow prices if both of them exist and validity ranges of the right-hand side parameters) for all of the eight right-hand side parameters with the constructed calculation method.

Table 2: Sensitivity analysis of the right-hand side parameters

RHS parameter	Original value	LINGO software			Suggested method					
		y_j	ξ_j^-	ξ_j^+	y_j^- (y_j)	$n\xi_j^-$	$n\xi_j^+$	y_j^+	$p\xi_j^-$	$p\xi_j^+$
$D_{1,1}$	0	15	0	0	10	-200	0	20	0	100
$D_{2,1}$	100	15	0	0	10	-100	0	20	0	100
$D_{1,2}$	200	20	-100	0	20	-100	0	25	0	100
$D_{2,2}$	100	20	-100	100	20	-100	100	-	-	-
B_1	300	-5	0	0	-10	-100	0	0	0	∞
B_2	200	0	-100	∞	0	-100	∞	-	-	-
W_1	200	0	0	∞	-10	-100	0	0	0	∞
W_2	200	0	-200	∞	0	-200	∞	-	-	-

The computation of the correct sensitivity analysis results for all right-hand side parameters of a linear programming problem with J constraints requires the solution of $6J$ LP problems. The number of the additional LP problems can be decreased by mathematical and managerial filtering.

3.3. Thesis 3

The sensitivity analysis results determined by the commercial software packages are available with the optimal solution – no further calculation is required. The constructed method requires solution of several LP problems. For a linear programming problem with I objective function coefficients and J constraints $2I$+$6J$ LP problems must be solved to get the correct sensitivity analysis results. However, the number of the additional LP problems to be solved can be decreased. From management point of view, the number of the additional LP problems can be decreased if some sensitivity analysis results are not needed. This depends on the model and the environment of the model, universal statements cannot be formulated in this case. Some considerations are discussed in the doctoral thesis. The number of additional LP problem to be solved can be decreased from mathematical point of view also. It can be stated whether a linear programming problem is degenerate based on the sensitivity analysis results of commercial software packages. If the allowable decrease or increase of a right-hand side parameter is zero, then the LP problem is surely degenerate. If a shadow price is not at the border of its validity range – thus the allowable decrease and increase of the relating right-hand side parameter are not zero – then the shadow prices determined by the software is correct. Only the validity range information is needed to be checked, because in case of degeneracy narrower validity ranges can be determined by commercial software packages. Thus, solution of 4 LP problems can be neglected.

Thesis 3: A computational model that calculates correct sensitivity analysis results when the optimal solution is degenerate is created. In the model mathematical filtering is automated, and further possibilities for reduction of the additional linear programming models based on managerial considerations are suggested.

(Relating publications: S2, S3, S5, S8, S9, S14)

The created computational model is realized with the help of Excel spreadsheet, Visual Basic Application (VBA) macro manager and LINGO mathematical programming software. The steps of the computational model can be seen in Figure 1. The stages of the calculation are the following:

1. The first step is the solution of the primal problem. The optimal values of the decision variables required by management and the value of optima are exported to Excel.
2. The second step is the calculation of the correct sensitivity analysis results of the objective function coefficients. The allowable maximal decrease and the allowable maximal increase for I objective function coefficients must be computed. Visual Basic manages the LP problems solved by LINGO software in a FOR cycle from i=1 to $i=I$. In one cycle, first the lower bound and after that the upper bound of the validity range is calculated.
3. The third step is the calculation of the correct sensitivity analysis results of the right-hand side parameters. VBA checks for right-hand side parameters from j=1 to $j=J$ whether they are at the boarder of their validity range ($\xi_j^-=0$ or $\xi_j^+=0$). If this logical condition is true then two sided shadow prices may exist, thus all of the 6 additional LP problems are needed to be solved. At first, VBA calculates the left shadow price then the allowable maximal decrease and allowable maximal increase of the right-hand side parameter of the calculated left shadow price. After this the right shadow price with its correct validity range are calculated. If the right-hand side parameter is not at the border – namely the original logical condition is false – then just two LP problems must be solved. In this case the allowable decrease and the allowable increase are computed for right-hand side parameter j.

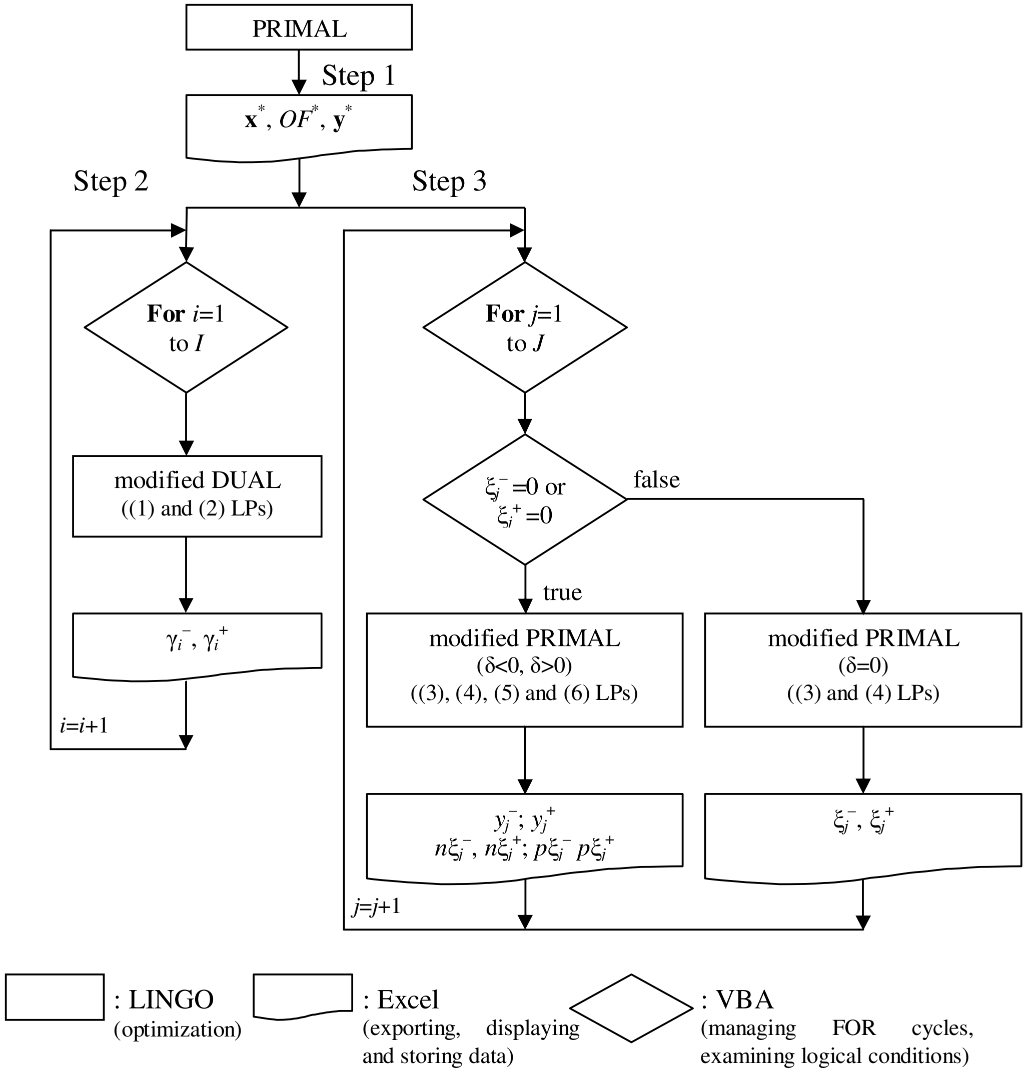

Figure 1: Realization of sensitivity analysis calculating correct results from management point of view

3.4. Thesis 4

One typical application area of binary programming models is assembly balancing that can be used to manage tasks in the production or service industry.

One of the basic assembly line balancing models is Simple Assembly Line Balancing Model-1 (SALBM-1) that minimizes the number of workstations with given production quantity and with given production time – consequently with given cycle time. The other important basic model in assembly line balancing is SALBM-2 that minimizes the cycle time, consequently maximizes production quantity for a given number of workstations.

Thesis 4: A graphical tool to determine the most efficient assembly line configuration for a given production process is developed. The optimal production quantity range for an assembly line with given number of workstations can be determined with the help of the graph.

(Related publications: S1, S6, S7, S11)

Application of *N* workstations is surely optimal if the production quantity is in the following production range:

$$Q_{\mathrm{Max}}^{\mathrm{OPT}}(N-1) < Q \le Q_{\mathrm{Max}}^{\mathrm{OPT}}(N), \tag{7}$$

where $Q_{\mathrm{Max}}^{\mathrm{OPT}}(N)$ is the maximal production quantity relating *N* workstations. If the production quantity is in the range given by (7) – calculated with the help of SALBM-2 – then the assembly line operates with maximal efficiency. More than $Q_{\mathrm{Max}}^{\mathrm{OPT}}(N)$ products can only be produced if more workstations are available, and less than $Q_{\mathrm{Max}}^{\mathrm{OPT}}(N-1)$ products can be produced with fewer workstation. Figure 2 shows the optimal production quantity for a bicycle assembly process.

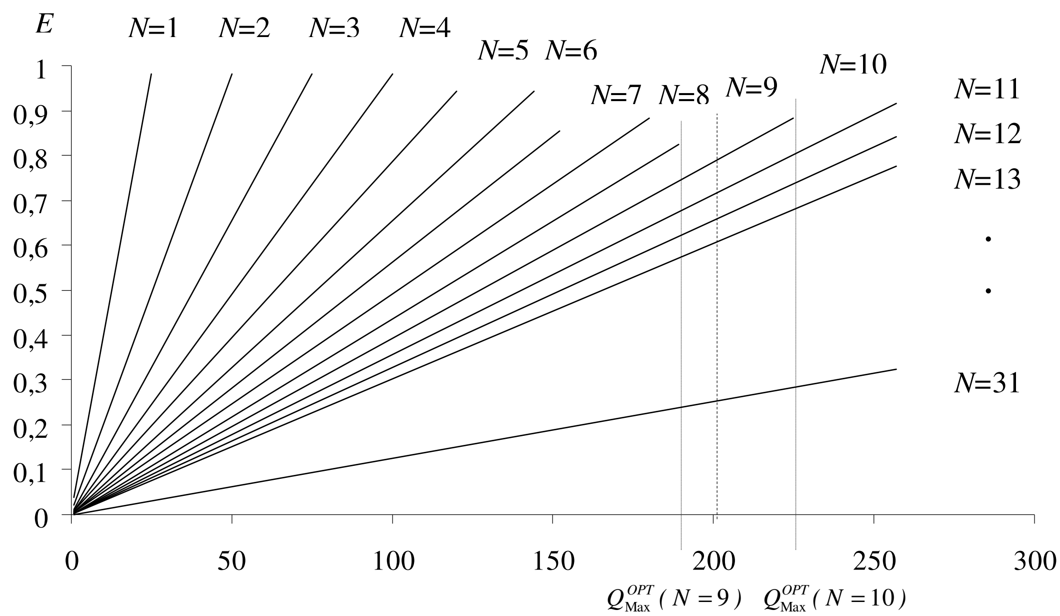

Figure 2: Line efficiency as a function of production quantity and number of workstations, and the optimal production quantity

3.5. Thesis 5

One possibility of increasing the relevance of assembly line balancing models may be the consideration of worker skill conditions. I did not find any models in the literature that take into consideration the different worker skill levels generally in the mathematical programming models of assembly line balancing.

> **Thesis 5: A modeling framework which can help to consider the effect of different workforce skill situations in the optimization of assembly lines is constructed. The number of decision variables applied to assign differently skilled workers to workstations in assembly line balancing models considering worker skill conditions can be decreased.**
>
> *(Relating publications: S4, S6, S7, S10)*

I differentiated three cases in the analysis of different skill levels. Low Skill Constraints (LSC), High Skill Constraints (HSC) and Executive Skill (ESC) constraints can be added to any assembly line balancing model, thus different worker skill situations can be modeled in the optimization of assembly lines.

In case of LSC higher skill levels are denoted by higher number. Workers with the lowest skill level (k=1) are able to perform the easiest tasks. As the worker skill level increases, more tasks can be assigned to a worker. A worker with the highest skill level (k=K) can perform any tasks. The low skill conditions are formulated as the following,

$$\sum_{m \in S_k} x_{mn} \leq z \sum_{v=k}^{K} l_{nv} \qquad n = 1,\dots,N; \quad k = 1,\dots,K \tag{8}$$

$$\sum_{n=1}^{N} l_{nk} \geq W_k \qquad k = 1,\dots,K \tag{9}$$

$$\sum_{k=1}^{K} l_{nk} \leq 1 \qquad n = 1,\dots,N \tag{10}$$

$$\sum_{m=1}^{M} x_{mn} \geq l_{nk} \qquad n = 1,\dots,N; \quad k = 1,\dots,K\,, \tag{11}$$

where x_{mn} is the binary variable of assigning tasks to workstations, S_k is the set of tasks with k level, z is a sufficiently high number, l_{nk} is the binary variable of assigning workers with skill level k to workstations and W_k is the lower bound of number of workers with skill level k.

In case of HSC higher skill levels are denoted by lower numbers. Any tasks can be assigned to workers with the highest skill level (k=1). Workers with the lowest skill level

(k=K) can perform only the easiest tasks. The high skill conditions are formulated as the following,

$$\sum_{m \in S_k} x_{mn} \le z \sum_{v=1}^{k} h_{nv} \qquad n = 1,\dots,N; \quad k = 1,\dots,K \tag{12}$$

$$\sum_{n=1}^{N} h_{nk} \le W_k \qquad k = 1,\dots,K \tag{13}$$

$$\sum_{k=1}^{K} h_{nk} \le 1 \qquad n = 1,\dots,N \tag{14}$$

$$\sum_{m=1}^{M} x_{mn} \ge h_{nk} \qquad n = 1,\dots,N; \quad k = 1,\dots,K\,, \tag{15}$$

where x_{mn} is the binary variable of assigning tasks to workstations, S_k is the set of tasks with k level, z is a sufficiently high number, h_{nk} is the binary variable of assigning workers with skill level k to workstations and W_k is the upper bound of number of workers with skill level k.

In case of ESC specialized skills are required to perform special tasks. Tasks being k level can only be performed by workers with skill level k.

$$\sum_{m \in S_k} x_{mn} \le z e_{nk} \qquad n = 1,\dots,N; \quad k = 1,\dots,K \tag{16}$$

$$\sum_{m \notin S_k} x_{mn} \le z(1 - e_{nk}) \qquad n = 1,\dots,N; \quad k = 1,\dots,K \tag{17}$$

$$\sum_{m=1}^{M} x_{mn} \ge e_{nk} \qquad n = 1,\dots,N; \quad k = 1,\dots,K\,, \tag{18}$$

where x_{mn} is the binary variable of assigning tasks to workstations, S_k is the set of tasks with k level, z is a sufficiently high number and e_{nk} is the binary variable of assigning workers with skill level k to workstations.

Using only two levels can be a special case of organizing tasks and workers into levels. In this case a worker or a task has a given property or not. This particular situation is discussed in the doctoral thesis in detail. The effect of application of workers with different skill levels on the optimal solution is illustrated by a bicycle assembly process. The process of the bicycle assembly was divided into 31 tasks and two skill levels were applied to illustrate the practical application of skill level conditions.

As binary programming models are typical NP hard problems, reduction of number of the decision variables was always an important part of researches.

The earliest workstation to which mth task can be assigned can be calculated (LJ_m). Consequently, the earliest workstation to which a worker with skill level k can be assigned is the minimum of the earliest workstations of tasks with level k,

$$LS_k = \underset{m \in S_k}{Min}\left(LJ_m\right). \tag{19}$$

The latest workstation to which mth task can be assigned can be calculated (UJ_m). Consequently, the latest workstation to which a worker with skill level k can be assigned is the maximum of the latest workstations of tasks with level k,

$$US_k = \underset{m \in S_k}{Max}\left(UJ_m\right). \tag{20}$$

The number of decision variables applied to assign workers with skill level k to nth workstation can be decreased based on (19) and (20) as the followings:

$$l_{nk} = 0 \qquad n < LS_k \text{ and } n > US_k \quad k = 1,...,K\,, \tag{21}$$

$$h_{nk} = 0 \qquad n < LS_k \text{ and } n > US_k \quad k = 1,...,K\,, \tag{22}$$

$$e_{nk} = 0 \qquad n < LS_k \text{ and } n > US_k \quad k = 1,...,K\,. \tag{23}$$

4. PRACTICAL UTILIZATION UND FURTHER RESEARCH AREAS

In my researches I have focused on managerial decision supporting by quantitative tools and my aim was to develop models that can facilitate daily management decision making. Calculating methods and models I have developed can contribute to more efficient processes in the production and service industry.

The correct sensitivity analysis results from management point of view can be calculated – for any LP problem and with the help of any tool – based on my research results stated in thesis (1) and (2). Thesis (3) gives an implementation of this calculation. With the help of my first three theses the correct sensitivity analysis results from management point of view can be computed for linear programming problems.

Theses (4) and (5) are related to practical application of assembly line balancing models. Thesis (4) describes a graphical tool that can be applied in daily managerial decision making relating to assembly lines. The optimal production quantity range and the way of efficiency improvements can be determined based on this graph and relating calculations. Thesis (5) introduces a model system for modeling different worker skill situations if there are significant differences in the worker skill levels. The number of decision variables applied to assigning workers to workstations is decreased with a logical condition. The planning and organizing different trainings for workers can be easier with the results of thesis (5).

As a result of my researches, I have developed quantitative tools to support managerial decision making. In case of the calculation of correct sensitivity analysis results, the calculation method should be implemented into a mathematical programming software. Thus, the management would have the possibility of choosing between mathematical and managerial sensitivity analysis results. My scientific results relating to assembly line balancing models can be regarded as a first step of a big research. One possibility for further researches can be the extension of mathematical programming models. The investigation of applying the results to other simple or general assembly line balancing problems can be an important question. Further research area may be studying other possibilities for modeling worker skills.

5. LIST OF PUBLICATION RELATED TO DISSERTATION

Book Chapters

S1 Tatay V. (2010): Gyártósor-kiegyenlítés alkalmazásának tapasztalatai egy kerékpárgyártó üzem példáján. *Pro Scientia Aranyérmesek X. jubileumi konferenciája*, Harvard Press, Budapest, ISBN 978-963-88289-1-0, pp. 74-79.

S2 Koltai T., Kalló N., Tatay V. (2009): Optimumkeresés bizonytalan paraméterekkel a termelés- és szolgáltatásmenedzsmentben. In Veresné Somosi M. (Ed.) : *Vezetési ismeretek III.* Miskolci Egyetem Gazdaságtudományi Kar, ISBN: 978-963-661-886-5; 978-963-661-889-6, pp. 104-115.

Web of Science Database Paper

S3 Koltai, T., Tatay, V. (2011): A Practical Approach to Sensitivity Analysis in Linear Programming under Degeneracy for Management Decision Making. *International Journal of Production Economics (IF=1,760)* , Vol. 131, No. 1, pp. 392-398.

Scopus Database Paper

S4 Koltai, T., Tatay, V. (2011): Formulation of Simple Workforce Skill Constraints in Assembly Line Balancing Models. *Periodica Polytechnica – Social and Management Sciences*, Vol. 19, No. 1, pp. 43-50.

Foreign Language Journal Paper Published in Special Journal

S5 Koltai T., Romhányi G., Tatay V. (2009): Optimalizálás bizonytalan paraméterekkel a termelés- és szolgáltatásmenedzsmentben. *Vezetéstudomány*, Vol. 60, June, pp. 68-73.

Foreign Language Conference Papers

S6 Koltai, T., Tatay, V., Kalló, N.(2011): Application of Simple Assembly Line Balancing Models to Support Quick Response Operation in a Bicycle Production Process. *3rd Rapid Modelling Conference: Rapid Modelling for Sustainability*. Leuven, Belgium, 09.12-14.2011, pp. 1-10.

S7 Koltai, T., Tatay, V. (2010): Application of Simple Assembly Line Balancing Models to Support Production Quantity Related Decisions. *16th International Working Seminar on Production Economics*. Innsbruck, Austria, 03.01-05.2010, pp. 285-296.

S8 Koltai, T., Tatay, V. (2008): Support of production management decisions by sensitivity analysis of linear production planning models. *15th International Annual EurOMA Conference*. Groningen, Netherlands, 06.15-18.2008, pp. 1-10.

S9 Koltai, T., Tatay, V. (2008): A Practical Approach to Sensitivity Analysis of Linear Programming under Degeneracy in Management Decision Making. *15th International Working Seminar on Production Economics, Pre-Prints Volume III.* Innsbruck, Austria, 03.03-07.2008, pp. 223-234.

Published Domestic Conference Papers in Foreign Language

S10 Tatay, V., Koltai, T. (2011): Supporting Production Management Decisions with Assembly Line Balamcing Models in the Presence of Skilled and Unskilled Workers. *microCAD 2011 International Scientific Conference, Economic Challenges in the XXI Century Section.* Miskolc, Hungary, 03.31-04.01.2011, pp. 125-130.

S11 Tatay, V., Koltai, T. (2010): Solving Assembly Line Balancing Models in Excel Environment to Support Production Management Decisions. *microCAD 2010 International Scientific Conference, Economic Challenges in the XXI Century Section*. Miskolc, Hungary, 03.18-20. 2010, pp. 165-170.

S12 Inczédy, K., Koltai, T., Tatay, V. (2010): The Comparison of the Optimal and Real Operation in a Yeast Production Plant – A Case Study. *microCAD 2010 International Scientific Conference, Economic Challenges in the XXI Century Section*. Miskolc, Hungary, 03.18-20. 2010, pp. 159-164.

S13 Koltai, T., Tatay, V. (2009): Application of Fuzzy Parameters in Production Planning Models. *microCAD 2009 International Scientific Conference, Economic Challenges in the XXI Century Section*. Miskolc, Hungary, 03.19-20.2009, pp. 141-146.

S14 Koltai, T., Tatay, V. (2008): The effect of degenerate LP sensitivity analysis results on management decision making. *microCAD 2008 International Scientific Conference, Economic Challenges in the XXI Century Section*. Miskolc, Hungary, 03.20-21.2008, pp. 27-32.

Published Domestic Conference Papers

S 15 Tatay V., Koltai T.: Menedzsmentdöntések támogatása munkások különböző képzettségeit figyelembe vevő gyártósor-kiegyenlítési modellekkel. *„Hitel, Világ, Stádium" jubileumi konferencia*, Sopron, 2010.

S 16 Tatay V., Koltai T. (2010): Gyártósor-kiegyenlítés alkalmazásának tapasztalatai egy kerékpárgyártó üzem példáján., Pro Scientia Aranyérmesek X. jubileumi konferenciája, Budapest, 2010.09.30.- 2010.10.03.

Other publication related works

Scientific Students' Association Conference Papers

S17. Tatay V., Degenerált lineáris programozási feladatok érzékenységvizsgálata menedzsmentalkalmazásoknál

– Conference of Scientific Students, BUTE, GTK, Management Section (1st prize, Award of Pro Progressió Foundation), 2007. November

– XXIX. National Conference of Scientific Students, Economics Section, (1st prize) 2009. April (published abstract in English and in Hungarian)

S18. Tatay V., Menedzsmentdöntések érzékenységvizsgálati problémáinak szemléltetése az AIMMS szoftverrel

– Conference of Scientific Students, BUTE, GTK, Management Section (1st prize), 2006. November

– XXVIII. National Conference of Scientific Students, Economics Section (2nd prize), 2007. April (published abstract in English and in Hungarian)

Oral Presetations

S19 Koltai, T., Tatay, V.: Formulation of General Workforce Skill Constraints in Assembly Line Balancing Models, *Veszprém Optimization Conference: Advanced Algoritms (VOCAL)*, Veszprém, Hungary, December 11-14., 2010.

S 20 Koltai, T., Tatay, V., The Interpretation of Fuzzy Linear Programming Results of Production Planning Problems, *Veszprém Optimization Conference: Advanced Algoritms (VOCAL)*, Veszprém, Hungary, December 15-17., 2008.

Number of publications: 16
Number of peer reviewed publications: 5
Number of Science Citation Index publications: 1
Number of references: 20
Number of independent references: 1
Number of references in Science Citation Index: 1
Number of independent references in Science Citation Index: 1
Total impact factor: 1,760

List of references

Refer to Koltai, T., Tatay, V. (2008): A Practical Approach to Sensitivity Analysis of Linear Programming under Degeneracy in Management Decision Making. *15th International Working Seminar on Production Economics, Pre-Prints Volume III.* Innsbruck, Austria, 03.03-07.2008, pp. 223-234.

Borgonovo E., Peccati L. (2011): Finite Change Comparative Statics for Risk Coherent Inventories, *International Journal of Production Economics*, Vol. 131, No. 1, pp 52-62.

6. REFERENCES

1. Boysen, N., Fliedner, M. and Scholl, A. (2008): Assembly Line Balancing: Which Model to Use When? *International Journal of Production Economics*, Vol. 111, pp. 509-528.

2. Corominas, A, Pastor, F. and Plans, J. (2008): Balancing Assembly Line with Skilled and Unskilled Workers. *Omega*, Vol. 36, pp. 1126-1132.

3. Davenport T. H. (2006): Competing on Analytics. *Harvard Business Review*, January, pp. 97-106.

4. Gal, T. (1986): Shadow Prices and Sensitivity Analysis in Linear Programming under Degeneracy. *OR Spektrum*, Vol. 8, pp. 59-71.

5. Hillier, S. F., and Lieberman, G. J. (1995): *Introduction into Operations Research.* McGraw-Hill International Editions

6. Jansen, B., de Jong, J. J., Roos, C., Terlaky, T. (1997): Sensitivity analysis in linear programming: just be careful! *European Journal of Operational Research*, Vol. 101, pp. 15-28.

7. Koltai T. (2001): *A termelésmenedzsment alapjai I.* Műegyetemi Kiadó, Budapest

8. Koltai, T., Terlaky, T. (2000): The difference between the managerial and mathematical interpretation of sensitivity analysis results in linear programming. *International Journal of Production Economics*, Vol. 65, pp. 257-274.

9. Nahmias, S. (1997): *Production and Operation Analysis*, Irwin

10. Porter, M. E. (2006): *Versenystratégia.* Akadémiai Kiadó, Budapest.

11. Rapcsák T. (1988): Az operációkutatás kialakulásáról és hazai helyzetéről. Magyar Tudomány, 4. sz., pp. 259-266.

12. Rubin, D. S., Wagner, H. M. (1990): Shadow Prices: Trips and Traps for Managers and Instructors. *The Institute of Management Sciences*, Vol. 20, pp. 150-157.

13. Stalk, G. Jr., Hout T. M. (1990): *Competing Against Time: How Time Based Competition is Reshaping Global Markets*, New York, Free Press

14. Waters, D. (1996): *Operations Management*, Addison-Wesley Publishing Company.

7. NOTATION

Indeces:

i	–	decision variables of a linear programming problem (i=1,…,I)
j	–	row of a linear programming problem (j=1,…,J)
m	–	index of tasks (m=1,…,M)
n	–	index of workstations (n=1,…,N)
k	–	index of the skill levels (k=1,…,K)

Parameters:

$\mathbf{A}$	–	coefficient matrix with a_{ji} elements
$\mathbf{A}^\mathrm{T}$	–	transpose of $\mathbf{A}$ with a_{ij} elements
$\mathbf{b}^\mathrm{T}$	–	vector of the right hand side parameters with b_j elements
$\mathbf{c}$	–	vector of the objective function coefficients with c_i elements
M	–	number of tasks in assembly line balancing models
N	–	number of workstations in assembly line balancing models
LJ_m	–	the earliest workstation of the mth task
UJ_m	–	the latest workstation of the mth
LS_k	–	the earliest workstation to which workers with k skill level can be assigned
US_k	–	the latest workstation to which workers with k skill level can be assigned
Q	–	production quantity
$Q_{Max}^{OPT}(N)$	–	maximal production quantity related N workstation
z	–	sufficiently high number
W_k	–	number of workers with skill level k

Sets:

S_k	–	tasks belonging to skill level k

Decision variables:

$\mathbf{x}$	–	decision variables of the primal problem with elements x_i
$\mathbf{x}^*$	–	optimal values of the decision variables of the primal problem with elements x_i^*
$\mathbf{y}$	–	decision variables of the dual problem with elements y_j
$\mathbf{y}^*$	–	optimal values of the decision variables of the dual problem with elements y_j^*
OF^*	–	optimal value of the objective function
$\underline{\mathbf{e}}_i$	–	unitvector with I elements and e_i=1; e_k=0 for $k \neq i$
$\underline{\mathbf{e}}_j$	–	unitvector with J elements and e_i=1; e_k=0 for $k \neq$
δ	–	pertubation of the right-hand side parameter
y_j^-	–	left shadow price of the right hand side parameter b_j (δ<0) (SP^-)
y_j^+	–	right shadow price of the right hand side parameter b_j (δ>0) (SP^+)
γ_i	–	change in the objective function coefficient c_i
γ_i^-	–	allowable decrease of the objective function coefficient c_i
γ_i^+	–	allowable increase of the objective function coefficient c_i
ξ_j	–	change in the right-hand side parameter b_j

$n\xi_j^-$	–	allowable decrease related to the left shadow price of right-hand side parameter b_j
$n\xi_j^+$	–	allowable increase related to the left shadow price of right-hand side parameter b_j
$p\xi_j^-$	–	allowable decrease related to the right shadow price of right-hand side parameter b_j
$p\xi_j^+$	–	allowable increase related to the right shadow price of right-hand side parameter b_j
ξ_j^-	–	allowable decrease of right-hand side parameter b_j
ξ_j^+	–	allowable decrease of right-hand side parameter b_j
x_{mn}	–	binary decision variable; x_{mn}=1, if the *m*th task is assigned to the *n*th workstation, otherwise x_{mn}=0
l_{nk}	–	binary decision variable; l_{nk}=1, if *k* skilled worker is assigned to the *n*th workstation, otherwise l_{nk}=0
h_{nk}	–	binary decision variable; h_{nk}=1, if *k* skilled worker is assigned to the *n*th workstation, otherwise h_{nk}=0
e_{nk}	–	binary decision variable; e_{nk}=1, if *k* skilled worker is assigned to the *n*th workstation, otherwise e_{nk}=0