PhD Thesis booklet

Entanglement in open quantum systems and many body simulations by tensor networks

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Background

The theory of open quantum mechanical systems emerges inevitably in the fields like quantum optics, quantum chemistry or the latest 'quantum biology' where the interactions between the system of interest (usually in non-equilibrium) and the environment play a crucial role. The so-called weak coupling and singular coupling limits give a Markovian semigroup description of the system interacting with its environment. This approximation can be derived with the assumption that the typical variation time of our system is much larger than the environment correlation time. In quantum information, quantum communication and computation, open quantum systems play an important role, because one wants to describe the decoherence of a prepared system, as well as prepare multipartite entangled states in the presence of the noisy environment.

Simulations of many body quantum systems are also discussed in my PhD thesis. More precisely, based on the concept of entanglement entropy, efficient numerical algorithms have been developed recently to simulate ground states of local Hamiltonians. These methods are justified for one-dimensional systems but are also extended to higher dimensions. They provide polynomially scaling algorithms and not hindered by the sign problem arising in Quantum Monte Carlo simulations applied to frustrated spin systems or fermions. Thus, they provide a promising tool for the better understanding of interacting quantum systems but there is still a need for their developments and clarifications.

Objectives

Regarding open quantum dynamics I wanted answer questions about the entangling capacity of a heat bath. I also tried to characterize the asymptotic states of multipartite systems, since there were rigorous results about the asymptotic states in case of one or two qubits but no exact results for more, and study the possibility of asymptotic entanglement. These investigations could be interesting not only from a mathematical point of view, but they could give answers about the feasibility of prepared entangled states in practice which is a key ingredient of quantum computation and communication.

Motivated by the recent success of many body simulations, I tried to apply some of the new ideas, test the algorithms and also give some contribution to the field. I planned to perform a study of different types of phase transitions simulated by these new algorithms and maybe find better methods than the previously used ones. As the simulations of more than one-dimensional systems provide the biggest challenge, I wanted to develop some new approaches for this problem too.

Thesis Statements

1. A sufficient condition on the structure of the Kossakowski-Lindblad master equation has already been given such that the generated reduced dynamics of two qubits results entangling for at least one among their initial separable pure states. We studied to which extent this condition is also necessary. Further, I contributed in finding sufficient conditions for bath-mediated entanglement generation in higher dimensional bipartite open quantum systems and gave numerical evidence for a bath that does not entangle two qubits but does entangle pair of qubits. Ref. 1.

2. We studied the asymptotic entanglement of three identical qubits under the action of a Markovian open system dynamics that does not distinguish them. I identified the algebra of invariant operators and showed that one could not construct the asymptotic states by a conditional expectation as in the two qubit case. I gave however certain classes of states of which one can determine the time evolution thus their asymptotic states too. We also showed that by adding a completely depolarized qubit to a special class of two qubit states, by letting them reach the asymptotic state and by finally eliminating the added qubit, can provide more entanglement than by direct immersion of the two qubits within the same environment. Ref. 2.

3. An open bipartite quantum system with dissipative Lindblad type dynamics is considered. In order to study the entanglement of the stationary states, we developed a perturbative approach. I applied it to the physically significant case when a purely dissipative perturbation is added to the unperturbed generator which by itself would produce reversible unitary dynamics and showed that indeed, there exists dynamics with stationary states on the boundary of the separable states which become entangled when a suitable dissipative perturbation is added. Ref. 3.

4. I used the finite-entanglement scaling of infinite matrix product states (iMPS) to explore phase transitions in the ground states of one-dimensional local Hamiltonians. I proposed a scaling ansatz for the correlation length of a non-critical system in order to explore infinite order transitions. This universal method provides considerably less computational costs compared to the finite-size scaling method . To this end, I studied possible MPS-based algorithms to find the ground states of the transverse axial next-nearest-neighbor Ising (ANNNI) model in a spin chain with first and second neighbour interactions and frustration. The ground state has four distinct phases with transitions of second order and one of supposedly infinite order, the Kosterlitz-Thouless transition. To explore phase transitions in the model, I investigated general quantities such as the correlation length, entanglement entropy and the second derivative of the energy with respect to the external field, and test the finite-entanglement scaling. The results show good agreement with previous studies of finite-size scaling using DMRG. The most peculiar phase in the model is the so-called floating phase, a narrow region amid the antiphase and the paramagnetic field. Its existence at large frustration parameters κ is an open question, my results suggest it still exists at $\kappa = -5$ but it is not indicated for $\kappa = -10$ with the applied maximal bond dimension $D = 80$. Ref. 4.

5. I constructed an algorithm to simulate imaginary time evolution of translationally invariant spin systems with local interactions on an infinite, symmetric tree. The state is described by symmetric iPEPS and I use translational invariant operators for the updates at each time step. The contraction of this tree tensor network can be computed efficiently by recursion without approximations and one can then truncate all the iPEPS tensors at the same time. The algorithm is a generalization of the one in and it is very well conditioned and stable due to the symmetry. The computational

cost scales like $O(D^{q+1})$ with the bond dimension D and coordination number q, much favourable than that of the iTEBD on trees . Studying the transverse-field Ising model on the Bethe lattice, we find a second order phase transition, surprisingly with finite correlation lengths. Ref. 5.

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