Parameter estimation of dynamical systems

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1 Introduction

The motivation behind this work is to provide a mathematical statistical theory and computational algorithms to estimate certain parameters in dynamical models of various kinds. The main problems originate from biological, chemical models and from telecommunication network technologies.

In case of telecommunication networks, quality of service problems is a top priority issue. There are several mechanisms to address these and one fundamental component in many of them is the logic that determines the rate of the signaling or packet flow. The mathematical problem here is to estimate the intensity of a stochastic point process.

The first part of this work is about a novel method that is proposed to estimate the intensity of inhomogeneous point processes. While keeping good statistical properties, this recursive method is simple, efficient and fast compared to other state-of-the-art methods and thus can be used for real-time intensity estimation. The mathematical background of the method is discussed and theorems are proposed to support the findings in simulation results for given examples. It is very important that in practical cases, better capacity utilization can be achieved using the proposed method.

Signaling capacity is often a bottleneck of telecommunication networks. In many cases an optimization of signaling is necessary, — and a method to do this is proposed, — but not sufficient because the network gets overloaded anyhow in certain emergency cases e.g., earthquakes, floods, fire, etc. Call priority handling in load control algorithms is then essential.

In the second part of this work, the mathematical background of Token Bucket, as a main tool for call admission control, has been investigated and propositions are given that helped construct a new and efficient algorithm to set the parameters of Token Bucket in order to achieve the desired behavior of these systems.

The third part is inspired by the problem of call gapping in an environment where different traffic classes of the same priority level are over-loading the network and thus the flow has to be filtered. I propose complex call gapping mechanisms and a corresponding abstract mathematical model to describe the behavior in each case. Although the model is elementary, it reveals some important new aspects of the nature of call gapping algorithms.

The technical solutions are patented and implemented in real-time telecommunication traffic control systems of Ericsson.

One of the classical application fields of dynamic models in biological and chemical sciences is the description of chemical reactions. Particularly in reaction kinetics, parameter estimation is the method we use to identify and calibrate the model we build i.e., to estimate the value of certain constants using measurements.

Two essentially different models are in the center of the interest of this discussion. One is the *global deterministic* model which is described with continuous deterministic differential equations and the other one is the *global stochastic* one which uses the techniques of discrete state continuous time stochastic processes.

Common solutions for parameter estimation uses linear regression techniques. This technique needs lots of measurements and requires many computations.

In part four, the *global deterministic* models are investigated. A neural network method is proposed that, — once trained, — can be used for fast parameter estimation of linear and nonlinear systems with partial measurement and with a huge number of parameters. Then, a theoretical method, using matrix inversion, is investigated and extended to be able to handle a larger class of differential equations. Some propositions are given what to expect from the estimation with this method in case of certain models with finding links with the classical linear regression parameter estimation methods using the stochastic version of the deterministic differential equations.

In part five, another popular model to describe biological and chemical reactions is the (continuous time discrete state) *global stochastic* model (to the theory of which,—among other,—Rényi made an important contribution) that uses Markov processes to describe the behavior, is examined. The parameter estimation of such models has been an open question with limited results. I propose a method to estimate state transitions of a special class of models which can have infinite states but the number of possible transitions from one state to another should be limited. This method uses either the intensity estimation method proposed in the first part or any good enough state-of-the-art intensity estimation method.

All methods are applied to models of biological and chemical reactions. Ignition, combustion models, the Volterra-Lotka equations, the Brusselator dynamics, the Michaelis-Menten reaction, the Butadiene transport in the human body and some other case studies are investigated, simulated, modeled in order to support the real benefit of the proposals.

2 New Results

I will present my new results in this section. With the word *thesis* I refer to those important assertions that I conclude from the new methods I define, and the lemmas, propositions and theorems I prove.

2.1 A method to estimate the intensity of non-homogeneous point processes

As a starting point, Brémaud's definition based on martingales is considered as a definition of the intensity of a point process. Fundamentals of maximum likelihood intensity estimation is given by Ogata [Ogata (1978)].

The motivation behind this part origins from the telecommunication networks. The main problem is to find a good description for the speed of the signaling and data traffic. There are several models to target this. Classical approaches introduced by Erlang [Erlang (1917)] treats the traffic events as the times of the jumps of a Poisson-process. Another popular technique to describe the dynamic behavior of the intensity function assumes that it is driven by a hidden Markov-chain. In this case the iterative Expectation

Maximization algorithm is the most common tool to identify these models, which aims at successively approximating the maximum likelihood estimate for the latent parameters (see e.g., [Dempster et al.(1977)]). The self exciting Hawkes-process, which is also popular to describe dynamics in finance, seems to be a good characterization of signaling traffic because it is based on a Poisson process but assumes that certain events generate new, additional ones thus modifying the intensity of the observed process. However, in most of the applications a simple statistics, the average number of events during a certain period of time, is used to give an estimate for the intensity. The objective is to give a more efficient one what is also computationally feasible in real time systems.

A recursive intensity estimate is proposed that can be used to estimate the intensity of non-homogeneous point processes regardless of the type of hidden model assumed. Statistical properties are discussed, theorems and simulations are presented to prove the usability of the proposed method.

Thesis 1 A family of statistics is proposed to efficiently estimate the intensity of time inhomogeneous point processes.

Definition 1 (Recursive Intensity Estimate (RIE)) *Assume the traffic has been observed at t_i time instances and let $\Delta t = t_n - t_{n-1}$ be the time elapsed from a previous event. Now the*

$$\begin{aligned}\hat{\lambda}(t_n;T) &:= \max\{1/T, (T\hat{\lambda}(t_{n-1};T) - (t_n - t_{n-1})\hat{\lambda}(t_{n-1};T) + 1)/T\},\\ \hat{\lambda}(t_0;T) &:= 0 \end{aligned} \tag{1}$$

statistics is the Recursive Intensity Estimate of a point process.

2.1.1 Results for the properties of the estimate

To develop the theory behind this estimate I have introduced some notation for the conditional bias and variance. The most important results are related to these quantities and are given by the following theorems.

Proposition 1 (The bias of $\hat{\lambda}$.) *The bias of the estimate $\hat{\lambda}(t_n;T)$ in case of a point process, given that for the gaps Δt_i between the events are $\mathbb{E}[\Delta t_i] \equiv \mathbb{E}[\Delta t] = \text{const.} \in \mathbb{R}^+$, is the following:*

$$\hat{B}_n[T|F_s] = \frac{1}{T(1-\mathbb{P}[T]) + \mathbb{E}[\Delta t|\Delta t < T]} - \mathbb{E}[\lambda(t_n)|F_s].$$

Similar results can be given for the variance of the estimate:

Theorem 1 (Zero variance in limit.) *For any process for which* $\mathbb{E}[\Delta t_i] = \mathbb{E}[\Delta t] = \text{const.} < +\infty$ *and* $\mathbb{E}[\Delta t_i^2] = \mathbb{E}[\Delta t^2] = \text{const.} < +\infty$*, the variance goes to zero in limit i.e.,* $\lim_{T\to\ +\infty} \hat{\text{VAR}}_{+\infty}[T] = 0$ *almost surely.*

The sketch of the proof is similar for the two theorems and is the following:

- The estimate $\hat{\lambda}(t_n)$ defined by Definition 1 is Markovian.
- We can have a recursion on the conditional biases.
- We arrive at a geometric series for which the quotient is strictly smaller than 1 if the conditions of the theorems are met.
- The proofs are completed with using convergency criteria and some calculations.

I would like to pinpoint one interesting finding that I proved as part of the proofs. The following lemma turned out to be extremely important for both the bias and the variance and though it seems to be simple it is not so obvious in general:

Lemma 1 *Let* $F[T]$ *be the CDF of* Δt *where* $\Delta t < T$ *and suppose that* $\mathbb{E}[\Delta t|\Delta t < T], \mathbb{E}[\Delta t^2|\Delta t < T], ..., \mathbb{E}[\Delta t^n|\Delta t < T], i = 1, 2, ..., n$ *exists, then*

$$\sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) (-1)^{i+1} \mathbb{E}[\Delta t^i|\Delta t < T]/T^i < 1.$$

Proof. To obtain the proof, first observe that $F[T] \leq 1$, then we can write

$$\sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) (-1)^{i+1} \int_0^T \frac{s^i}{T^i} \frac{f_t(s)}{F_t(T)} \mathrm{d}s =$$
$$= \int_0^T (1 - \frac{s}{T})^n \frac{f_t(s)}{F_t(T)} \mathrm{d}s < \int_0^T 1 \frac{f_t(s)}{F_t(T)} \mathrm{d}s = 1$$

where the inequality holds because $0 < s \leq T$.■

2.1.2 Results for Poisson process

The proposition before gives the bias as $n \to +\infty$ for given T values. In the case of a homogenous Poisson process the greater the T value, the better the estimate is and the estimate is unbiased as $T \to +\infty$. These properties will be shown to be important for applications.

Proposition 2 *In case of the Poisson process the t_{n-1}-conditioned bias of the estimate $\hat{\lambda}(t_n; T)$ converges to 0 as $T \to +\infty$ almost surely.*

Sketch of the proof: Observe the closed form of the bias in limit. Obviously, both $T(1 - \mathbb{P}[T])$ and $\mathbb{E}[\Delta t | \Delta t < T]$ should be monotonously increasing in T.

Proposition 3 (Higher conditional expectation.) *The conditional expectation of the $\hat{\lambda}$ statistics is always higher than the intensity of the Poisson process: $\lambda \leq \mathbb{E}[\hat{\lambda}(t_n, T)|F_n]$.*

This proposition is important for the applications and can be used for call admission control algorithms if we require to give priority for emergency calls.

2.1.3 Comparing the recursive estimate to the ones currently in use

Thesis 2 The proposed statistics has similar asymptotical and better one-step properties than the family of classical maximum likelihood ones in certain cases because it uses only the most up-to date information available.

In practical implementations, the Periodic Intensity Estimate, defined below, is used to estimate the traffic rate so I have compared mine with the following statistics inspired by the maximum likelihood estimate of a stationary Poisson process:

Definition 2 (Periodic Intensity Estimate (PIE))

$$\bar{\lambda}(t_n; T) := \frac{N(t_n) - N((t_n - T) \wedge 0)}{t_n \wedge T}$$
$$\bar{\lambda}(0; T) := 0. \qquad (2)$$

I have examined the statistical properties of this estimate using the same notations and tools and the following proposition is given that is easy to check.

Proposition 4 *Conditioned on any* $F_s, s \leq t_n - T$, *the statistics* $\bar{\lambda}$ *gives an unbiased estimate of intensity* λ *whenever* $\lambda' \in L_1$ *and* $\mathbb{E}[\int_{t_n-T}^{t_n} u\lambda'(u)\mathrm{d}u|F_s] = 0$ *for all* $t_n \in \mathbb{R}$.

It is important that if the conditions of the above theorem are not satisfied (e.g., $\lambda(t) = 1 + \sin(t)$ or $\lambda(t) = 2ct$), then the estimate we get is biased.

The two definitions (for RIE and PIE) are getting closer as $T \to +\infty$.

Proposition 5 *For every* t_n: $\bar{\lambda}(t_n; +\infty) = \hat{\lambda}(t_n; +\infty)$.

The following theorem is a key because it compares the absolute biases of the $\bar{\lambda}$, $\hat{\lambda}$ estimates.

Theorem 2 *Suppose that we have an unbiased estimate of the intensity up to time* F_{n-1} *and the value is* λ_{n-1}. *Also suppose that* $\mathbb{P}[\Delta t < T] = 1$. *The average intensity of the process up to the probabilistic time* t_n *is defined by the following equation* $\lambda_n = \frac{1}{t_n - t_{n-1}} \int_{t_{n-1}}^{t_n} \lambda(u)\mathrm{d}u$. *Then the following statements are true:*

1. *The* F_{n-1}*-conditioned bias of the estimate* $\bar{\lambda}$ *(i.e.* $\bar{B}_n[T|F_{n-1}]$*) at* t_n *is zero if and only if* $\lambda_{n-1} = \lambda_n$ *for any parameter* $0 < T$;
2. *The* F_{n-1}*-conditioned bias of the estimate* $\hat{\lambda}$ *(i.e.* $\hat{B}_n[T|F_{n-1}]$*) at* t_n *is zero if and only if* $\lambda_{n-1} = \lambda_n$ *or* $\lambda_n = 1/T$ *for any parameter* $0 < T$;
3. *Given that* $\lambda_{n-1} \neq \lambda_n$, $\hat{B}_n[T|F_{n-1}] \leq \bar{B}_n[T|F_{n-1}]$ *if and only if* $\lambda_n \leq \frac{1}{2}(1/T)$.

About the proof:

- Both statistics can be given with a recursive definition ($\hat{\lambda}$ is given by that, but $\bar{\lambda}$ is to be modified).
- A process that is observed in discrete time (observations are discrete in practice) or, say, if we are only interested for the intensity if something happens (at t_n times), then the intensity is not unique.

- The average intensity $\lambda_n = \frac{1}{t_n - t_{n-1}} \int_{t_{n-1}}^{t_n} \lambda(u)\mathrm{d}u$ can always be defined.
- The above findings give handy tools to compare the biases and each part of the theorem is easy to prove.

I expect that the F_{n-1}-conditional variance of $\hat{\lambda}$ will be lower than that of the $\bar{\lambda}$ one at t_n at certain cases. I cannot show this but we have simulation runs that have shown smoothness in variance.

2.1.4 Token Bucket rate estimate

An estimate is presented, which is often used to determine the traffic rate, but we prove that it gives misleading result.

Thesis 3 The Token Bucket estimate can not be used for rate measurement and therefore not to be applied in fair sharing algorithms.

Definition 3 (Token Bucket based Rate Estimate)

$$\tilde{\lambda}(t_n) = \frac{\chi(t_n)}{T_j} + \max\{0, \frac{T\tilde{\lambda}(t_{n-1}) - (t_n - t_{n-1})r(t_n)}{T}\}. \tag{3}$$

Proposition 6 (The bias of $\tilde{\lambda}(t_n; T, r)$.) *Having a point process with i.i.d. increments and $\mathbb{E}[\Delta t | \Delta t < \infty]$, the bias of the estimate $\tilde{\lambda}(t_n; T, r)$ for the intensity $\lambda(t_n)$ is the following:*

$$\tilde{B}_n[T, r | F_s] = \frac{1}{T} \frac{1 - r\mathbb{E}[\Delta t | \Delta t < T]}{1 - \mathbb{P}[T]} - \mathbb{E}[\lambda(t_n) | F_s]$$

The problem here that with $T \to \ + \infty$ this bias can be any large independently from T. This statistics may not be used as an estimate for the intensity. Theoretically when $\lambda = r$ then $\tilde{\lambda} \sim \text{Uniform}(0, r)$. We will not take further discussions on this statistics but it is important that this alone should not be used ever as an intensity estimate.

2.2 Parameter setting of the Token Bucket mechanism

In this part, the Token Bucket call gapping mechanism, as a main tool for call admission control, is investigated. The mathematical background is developed using Markov chains and propositions are given to set the parameters to achieve the desired behavior of these systems.

Thesis 4 The Token Bucket call gapping (as patented by Crawford [Crawford (1980)] and standardized by ITU-T [ITU-T H.248.11]) can be modeled with a Markov chain and thus the drop probabilities can be calculated even for the multi-level version of it. It is feasible to calculate watermark settings for given requirements in telecommunication systems because efficient algorithms can be provided using the some basic findings. [CP2-IEEE-SoftCom-2009]

Definition 4 (Token Bucket call gapping strategy ($\gamma_t(r, \mathbf{W})$)) *There are discrete events,—offers to the bucket,—at times $t_0, t_1, ..., t_n$. Each offer has an assigned mark $j, j = 1..m$, called its priority. and Token Bucket makes a decision to admit or reject each. Maintain a bucket fill*

$$b(t) = \max\{\chi(t), b(t_{n-1}) - r(t_{n-1})(t_n - t_{n-1}) + \chi(t)\}, \tag{4}$$

where $\chi(t) = 1$ iff there is an offer. Admit the offer with priority j if $b(t_n) \leq W_j$, where W_j is the jth watermark of the watermark vector $\mathbf{W} = (W_1, W_2, ..., W_m)$. If the offer is admitted, the above definition is used for the next value of the bucket fill b. If the offer is rejected, then $b(t_n)$ is recalculated with $\chi(t) = 0$.

In many definitions the token generation rate for a Token Bucket can be deterministic (either discrete or continuous in time) or stochastic. In the stochastic case, the time between token generations is exponentially distributed. A Markov model is developed for the stochastic token generation case. Let $\mathbf{S} = (S_0, S_1, S_2, ..., S_{W_m})$ be the states, $\Lambda = (\lambda_1, \lambda_2, ..., \lambda_m)$ the incoming call rates of each priority groups $1, ..., m$ and μ is the token generation rate, $\mathbf{W}$ is the watermark vector. The the token bucket is modeled with the discrete state space continuous time Markov Chain $\mathfrak{M} = (\mathbf{S}, \Lambda, \mu, \mathbf{W})$.

Suppose that there are $m = 2$ priority classes and thus $m = 2$ watermarks ($W_1 = 3, W_2 = 5$) associated to two priority levels. Suppose that each of the consecutive tokens is generated according to an exponential distribution with parameter μ and new requests, with two different priority levels, arrive according to two Poisson processes with parameters λ_1, λ_2, respectively. Then the Markov chain representing the system has the following rate matrix:

$$\begin{pmatrix} -(\lambda_1+\lambda_2) & (\lambda_1+\lambda_2) & 0 & 0 & 0 & 0 \\ \mu & -(\lambda_1+\lambda_2)-\mu & (\lambda_1+\lambda_2) & 0 & 0 & 0 \\ 0 & \mu & -(\lambda_1+\lambda_2)-\mu & (\lambda_1+\lambda_2) & 0 & 0 \\ 0 & 0 & \mu & -\lambda_1-\mu & \lambda_1 & 0 \\ 0 & 0 & 0 & \mu & -\lambda_1-\mu & \lambda_1 \\ 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix}$$

Let $p_i = \mathbb{P}[S = S_i]$ the probability that the chain $\mathfrak{M} = (\mathbf{S}, \Lambda, \mu, \mathbf{W})$ is in state S_i. The conditional probability of rejecting an offer of level j is the probability being in a higher or equal state than S_{W_j}: $\mathbb{P}[\text{"Reject"}|j] = \sum_{k>W_j} p_k$ from which $\mathbb{P}[\text{"Reject" of type } j] = \mathbb{P}[\text{"Reject"}|j]\mathbb{P}[j] = \sum_{k>W_j} p_k \mathbb{P}[j]$. Then the probability of rejecting an offer is $\mathbb{P}[\text{"Reject"}] = \sum \mathbb{P}[\text{"Reject"}|j] = \sum \mathbb{P}[\text{"Reject"}|j]\mathbb{P}[j] = \sum\sum_{k>W_j} p_k \mathbb{P}[j]$.

The most important propositions follow:

Proposition 7 *The loss in the bucket with $m = 1$ (and then $\lambda = \lambda_1$) is decreasing when W raises no matter if $\lambda > \mu$ or $\lambda < \mu$.*

This proposition tells us that the with a higher watermark the loss is smaller in case of overload or underload as well.

Proposition 8 *The following intuitive statements are true:*

- $(W'_j = W_j + 1 \& \Lambda_{W_j} > 1)$
  $\Rightarrow (p'_{i \le W_j} < p_{i \le W_j} \& p'_{i > W_j} \ge p_{i < W_j})$
- $(\lambda'_j = \lambda_j + \epsilon > \lambda_j)$
  $\Rightarrow (p'_{i \le W_j} < p_{i \le W_j} \& p'_{i > W_j} \ge p_{i < W_j})$,

where $\Lambda_w = \sum_{j=1}^{w} \lambda_j$.

This proposition tells us that if the system is overloaded then raising the watermark will result that the probability of being in a higher state raises. Intuitively, a few more tokens are allowed to be consumed from the bucket so there are going to be less of them more likely. The situation is similar if the number of offers from priority level j are raised since in this case it is more likely that level j offers are banned i.e. the probability of being in a higher state is higher.

Using the findings, an algorithm is proposed to maintain the watermark settings in such a way that certain requirements are satisfied.

The requirement to satisfy defines a maximum traffic loss probability $(p_1, p_2, ..., p_m)$ per priority at a well defined incoming call rate Λ and at a distribution of incoming offer priority levels $\mathbb{P}[j]$. We can define the feasible set of watermarks for this kind of requirement:

Definition 5 (Feasible set of watermarks for fixed parameters.)

$$\mathfrak{W}_{\text{fix}}(\lambda, \mathbf{L}_{\text{fix}}) = \{\mathbf{W} : Loss[\lambda, \mathbf{W}] < \mathbf{L}_{\text{fix}}\}$$

An efficient algorithm is proposed to find the feasible watermark set for a requirement like the one above what is the following:

```
Check:=Function[j,
        For[k = m to j do
            Repeat
                Check[k];
                W_k ++
            until Loss_k < L_k
        ]
];
Run{Check[1]}.
```

From theorems 7 and 8 the loss at priority level m decreases only with raising W_m. We go through all the classes and raise the corresponding watermarks until the conditions are met. The algorithm does not test on every feasible set of watermarks but finds one very fast.

2.3 Queue-less call gapping mechanisms

Network optimization and traffic classification have always been in the center of interest of the telecommunications industry. In [J6-LTRACK-JOURNAL] and [CP4-HTE 2005], an algorithm is proposed to minimize the signalling load for IP Mobility solutions for which I made parameter optimization. Then this algorithm is further discussed in e.g., [J5-MMM-Telecom], [J5b-MMM-Hiradas*], [HTE 2006*] and [CP3-MOMM 2006], where a general model of IP mobility systems to optimize the handover strategy is proposed. The findings there led to the proposal of a novel approach of IP mobility that transforms all the logic and control from the network to the terminal when it comes to handover decision (see. e.g., [J3b-CMFS-Hiradas*], [J3-CMFS-Telecom] and [Networks 2008*]). More about the proposal and the algorithms I developed can be read
in [MOMM 2008*].

On the other hand, despite the careful planning and optimization of networks, overload situations often arise in case of network faults, wrong dimensioning or special events such as disasters, football matches, television shows, earthquakes. To handle these situations, overload control systems are introduced to the network that controls the flow of signalling messages or data packets if needed. (In case of data packets, these mechanisms are often referred to congestion control mechanisms.) To achieve good Quality of Service and network transmission characteristics, so-called, Call Gapping throttles are used to filter offers in real time without introducing delay i.e., without utilizing queues. There are cases, in which queuing is allowed, but these are out of the center of the interest here.

I have invented new methods and algorithms how to filter the offers or packets in order to meet certain requirements and how to implement this to Internet Multimedia Subsystem or to any other network. (For a reference either see my patents: [P2], [P1] and [P3] or [CP1-IARIA-InfoSys-2011]). These requirements are the maximal throughput with priority handling while providing a certain share of the capacity to different traffic classes. The requirements are similar to those for the Weighted Fair Queuing like mechanisms but delay is not allowed. A mathematical description is also given to understand the behavior of these systems and the proof that the invented methods satisfy the requirements.

The verbal form of the requirements are the following:

- **Requirement-A** *Maximal throughput with bound*: No *offer* should be rejected if there is enough available capacity in the system to serve it, but no *offer* should be admitted if there is not enough available capacity to serve it in the system.

- **Requirement-B** *Priority levels*: Each *offer* may be assigned a priority level and the *offer* with higher priority shall be admitted in favor of the one with the lower priority level.

- **Requirement-C** *Throughput share for traffic classes*: The offers can be classified and for the traffic class i the s_i portion of the capacity of the target shall be provided.

Thesis 5 Some new methods are invented: Rate Based (RB) call gapping, priority handling with Biased Estimator (BE) method, traffic class share handling with Goal Rate (GR) and Priority Raise (PR) methods.
Applied separately or together with existing solutions (Token Bucket (TB) call gapping method and priority handling with Multiple Watermark levels (MW)) a throttle can be constructed that can meet a great variety of requirements. [CP1-IARIA-InfoSys-2011]

Definition 6 (Rate Based call gapping (RB)) *Measure the rate of the traffic as if the request was admitted and if this provisional admission rate is smaller than the allowed one then admit the offer, reject otherwise.*

Definition 7 (Biased Estimator priority handling (BE)) *[P1] Take a biased estimate for which there exists a parameter of it for which the probability of under- or overestimating the rate is higher if a parameter is higher/lower.*

Definition 8 (Goal Rate calculation to provide class shares (GR)) *Class-wise measure the offered traffic and determine the desired throughput. One can set any environment on the call/sec vaules.*

Definition 9 (Priority Raise method to provide class shares (PR)) *[P1] Class-wise measure the provisional admitted traffic and modify (decrease) the priority if it would exceed some required throughput constants.*

I have several theorems and proofs to support the methods defined above. I would like to stress on one method that turned out to have the best characteristics among the examined ones.

2.3.1 A queue-less call gapping strategy

Thesis 6 Two call gapping strategies (γ_g, γ_x) are proposed that do not introduce delay to the processing and meet the maximal throughput, priority and traffic class handling requirements. Among these, γ_x has the better characteristics, that is even better than the classical Token Bucket mechanism. [CP1-IARIA-InfoSys-2011]

Definition 10 (Strategy γ_g.) *Let us suppose that at t_n an offer arrives and the system is in state $\{t_{n-1}, \hat{\rho}_i(t_{n-1}), \hat{a}_i(t_{n-1})\}$ and $c(t_n)$:*

1. *Determine priority constants i.e., calculate T_j;*

2. *update the incoming rates estimated for all i classes:* $\hat{r}(t_n)$ *with* $\chi_k(t_n) = 1$ *iff* $i = k$*, 0 otherwise;*

3. *calculate a provisional admission rate for all i:* $\hat{a}(t_n)$ *with* $\chi_k(t_n) = 1$ *iff* $i = k$*, 0 otherwise;*

4. *calculate the bounding rate for class i only:* $g_i(t_n)$*;*

5. *if* $\hat{\alpha}_i \leq g_i$ *then admit the offer and let* $a(t_n) := \alpha(t_n)$ *else reject the offer and update* $\hat{\alpha}(t_n)$ *with* $\chi_k(t) = 0, \forall k$ *(!);*

6. *continue with 1. for the next event.*

We propose to update $\hat{\rho}_i, \hat{\alpha}_i, \hat{a}_i$ *according to the following equation:*

$$\hat{\lambda}(t_n) := \frac{\chi(t_n)}{T_j} + \max\{0, \frac{T_j\hat{a}(t_{n-1}) - (t_n - t_{n-1})\hat{a}(t_{n-1})}{T_j}\},$$

where $\hat{a}$ *is an estimator, the same as defined in by 1, asymptotically unbiased for the* $\lambda(t)$ *real intensity of a point process when* $T \to +\infty$*, thus to be replaced by* $\hat{\rho}_i, \hat{\alpha}_i, \hat{a}_i$ *and indicator* $\chi_i(t_{n-1}) = 1$ *iff the offer is of type i and 0 otherwise (or further specified like in step 5). Note that the time parameter* T_j *changes in time as well according to the priority level and that the former always has to be remembered.*

To calculate the bound rate we introduce $u(t)$ *(the provisional used capacity) according to* ***Requirement-B****:*

$$\begin{aligned} u(t) &:= \sum_{\forall i} \min\{s_i c(t), \hat{\rho}_i(t)\} \\ &= \sum_{\hat{\rho}_i(t) \leq s_i c(t)} \hat{\rho}_i(t) + \sum_{s_i c(t) < \hat{\rho}_i(t)} s_i c(t) \end{aligned}$$

Thus the remaining (unused) capacity in the system is $c(t) - u(t)$*. This has to be split between traffic classes with higher incoming rate then the agreed share* $\hat{\rho}_i(t) > s_i c(t)$*. Then*

$$g_i(t) := \min\{\hat{\rho}_i(t), s_i c(t) + (\hat{\rho}_i(t) - s_i c(t))\frac{c(t) - u(t)}{\rho - u(t)}\}.$$

Then to achieve better throughput characteristics, this is further improved as follows:

Definition 11 *Rate Based Call Gapping with Bucket-type Aggregate Characteristics:* γ_x*: Take all the definition from the new call gapping mechanism* γ_g *for* $\hat{\rho}, \hat{\alpha}, \hat{a}, u, g_i$ *and define* $T_j(t) = W_j/r(t)$*. Take* W_j *and the bucket fill change definition* b *from the original token bucket* γ_t*. Perform all the steps like in* γ_g *but decide using the following constraint equation:* $\frac{b(t_n)}{W_j}\hat{a}_i(t_n) \leq g_i(t_n)$*.*

The figure shows that all the mechanisms limit the admitted offer rate while trying to keep the highest throughput. In this scenario we examine the traffic on aggregate level i.e., there is only one traffic class for which the capacity of the throttle should be maximized and limited. The capacity is 1 offer/sec for the simple simulation case while the average number of offers per sec increases from 0.8 to 2 meaning that there is a 200% load on the node.

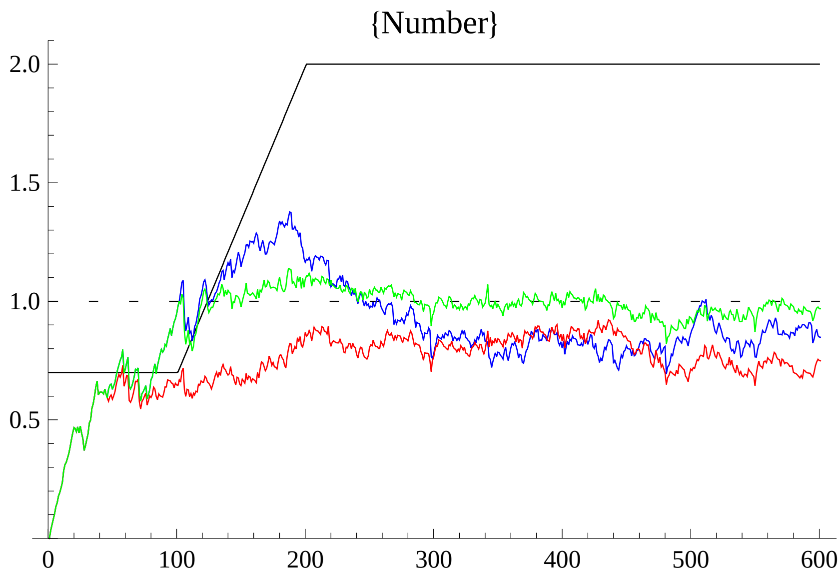

Figure 1: The new algorithm (γ_g) on aggregate level. (Black: nominal offer rate, red: the token bucket's, blue: γ_g's, green: γ_x's throughput.)

As it can be seen in Figure 1 all three mechanisms limit the admitted traffic although Token Bucket allows considerable peak at the beginning. (The size of the peak depends on the parameters we set. Here the 1 offer/sec capacity is very small compared to the watermark which is set to 10.) On the other hand, rate based call gapping seems to under-utilize the system while the joint mechanism seems to have the smoothest and also maximal throughput.

After a total 600 offers from each traffic with the same exact trajectory the results show that $\gamma_t, \gamma_g, \gamma_x$ has admitted 415, 386, 404 number of calls respectively.

The problem with the mathematical discussion of maximal throughput is that the results depend severely on the value of the offer rate and capacity. It is only possible to compare the mechanisms at given rates, which is useless for real applications.

2.3.2 Supplementary theses: priority handling with biased estimate

Thesis 7 It is possible to handle priority calls using Rate Based call gapping and Recursive Intensity Estimate with lower T parameter values for higher priority streams.

First let us observe that the RIE estimator has a negative bias for Poisson process:

Theorem 3 *For Poisson process:*

$$\frac{1}{T(1-F[T])+E[\Delta t|\Delta t<T]} > \lambda.$$

Theorem 4 *The absolute bias is higher with the smaller T thus the underestimation is strict monotonously decreasing in T for renewal processes.*

$$\frac{1}{T(1-F[T])+E[\Delta t|\Delta t<T]} < \frac{1}{T'(1-F[T'])+E[\Delta t|\Delta t<T']},$$

whenever $T < T'$.

Validation The proposed Rate Based call gapping method was first validated by simulations written in *Mathematica* [5]. The strategy is patented as part of the TISPAN (Telephony over IP and Signaling and Protocols for an Advanced Network) New Generation Networks standardization activities and implemented into Ericsson products.

2.4 Parameter estimation with *matrix inversion* method

The basic idea comes from the standard problem of estimating the parameters in linear models. Let us consider the complex chemical reaction consisting of the reaction steps

$$\sum_{m=1}^{N_c} \alpha(m,r)X(m) \rightarrow \sum_{m=1}^{N_c} \beta(m,r)X(m), r = 1, 2, ..., N_k. \tag{5}$$

The usual mass action type model of this reaction is the Cauchy problem:

$$\begin{aligned} \dot{c}_m(t) = \textstyle\sum_{m=1}^{N_c} (\beta(m,r) - \alpha(m,r))k(r) \\ \textstyle\prod_{p=1}^{N_c} c_p^{\alpha(p,r)}(t) \\ c_m(0) = c_m^0 \ (m = 1, 2, \ldots, N_c), \end{aligned} \tag{6}$$

where $c_m(t) := [X(m)](t) \quad (m = 1, 2, \ldots, N_c)$ is the concentration of the mth species, and $k_r \quad (r = 1, 2, \ldots, N_k)$ is the reaction rate coefficient of step (5). Obviously, (6) is of the form

$$\dot{c}(t) = F(c(t))k \quad c(0) = c^0 \tag{7}$$

with the linear operator on any function of c, $F(c(t)) \in \mathbb{R}^{N_k} \longrightarrow \mathbb{R}^{N_c}$ compressing the structure of the complex chemical reaction in a certain way.

Mass action kinetics is enough to ensure the linear dependence of the right hand side on the parameters to be estimated. However, it is not fully necessary: the condition underpinning our method may be satisfied in other cases, as well. Furthermore, if the right hand side is an inhomogeneous linear function of the parameters to be estimated, our method works still with a slight modification. The generalization of Eq. (7) can then be rewritten as

$$\begin{gathered} c(t_n) - c^0 = \\ \textstyle\int_0^{t_n} F(c(s))\mathrm{d}s \cdot k + \int_0^{t_n} F_r(c(s))\mathrm{d}s. \end{gathered}$$

Thesis 8 If the matrix $(\int_0^{t_n} F(c(s))\mathrm{d}s)^\top(\int_0^{t_n} F(c(s))\mathrm{d}s)$ is invertible (a pseudoinverse of $\int_0^t F(c(s))\mathrm{d}s$ can always be calculated), an estimate of k can be obtained [J4-ANN-AMCS-2007]:

$$\hat{k} = \left(\left(\int_0^{t_n} F(c(s))\mathrm{d}s\right)^\top \left(\int_0^{t_n} F(c(s))\mathrm{d}s\right)\right)^{-1} \left(\int_0^{t_n} F(c(s))\mathrm{d}s\right)^\top ((c(t_n) - c^0) - \int_0^{t_n} F_r(c(s))\mathrm{d}s).$$

Note that the basic idea of the *matrix inversion* method was introduced in [Hangos et al. (1998)] and I just provide a small extension here.

The *matrix inversion* method is rather nice since it includes only and approximation of the integral and then the parameter estimation is straightforward. The weak point of the method is the matrix inversion step and also the error the integration introduces. Therefore I carried our examinations in this direction.

2.4.1 Error estimation for the linear regression and the inverse methods

Thesis 10 Explicit form is given to estimate bounds for the variance of the samples for the most popular *linear regression* models and for the *matrix inversion* method under different models.

We have an autonomous system: $\dot{c}(t) = kf(c(t)), c(0) = c_0$ (with one variable now for the sake of simplicity) and the measurements ($\tilde{c}(t_i) = c(t_i) + \xi(t_i), \xi(t_i) \sim N(0, \xi^2)$). Suppose that $f(c(t_i) + \xi(t_i)) = f(c(t_i)) + \xi(t_i) \sim N(c(t_i), {\xi^2}_c)$ and also that they are pairwise independent i.e., the origin of the error is due only to the measurement equipment. (This is many times the case in practice.)

I consider two extremal strategies to build the model either for the classical linear regression or to calculate the integral for the *matrix inversion* method.

- **Model #1**. In this case the sample set and then the model is built up using n of the following integral approximations:

$$\tilde{c}^1(t_i) := \tilde{c}(0) + k\int_0^{t_i} f(\tilde{c}(s))\mathrm{d}s,$$

supposing we have $m \in \mathbb{R}$ measurements and $m - n + 1 \leq i \leq m$.

- **Model #2**. However, the following sample set and model could also be constructed, say, with n sample equations:

$$\tilde{c}^2(t_i) := \tilde{c}(t_{i-1}) + \int_{t_{i-1}}^{t_i} f(\tilde{c}(s))\mathrm{d}s.$$

The dynamics of the measured system can be described with the following stochastic differential equations:

$$\tilde{c}^1 = \tilde{c}(0) + k\int_0^{t_i} f(c(s))\mathrm{d}s + k\int_0^{t_i} \xi \mathrm{d}W, i \in [m-n+1, m]$$
$$\tilde{c}^1 = \tilde{c}(0) + k\int_{t_{i-1}}^{t_i} f(c(s))\mathrm{d}s + k\int_{t_{i-1}}^{t_i} \xi \mathrm{d}W, i \in [1, n].$$

The solutions of the systems are the following:

$$\mathbb{E}[\tilde{c}^1(t_i)] = c_0 a^{kt_i}, VAR[\tilde{c}^1(t_i)] = k(e^{kt_i} - 1),$$
$$\mathbb{E}[\tilde{c}^1(t_i)] = c_{t_{i-1}} a^{k(t_i - t_{i-1})}, VAR[\tilde{c}^1(t_i)] = k(e^{k(t_i - t_{i-1})} - 1).$$

Suppose that the limits of the variance of the error of integrations exist and let $\lim_{t\to\ +\infty} \xi_\int \Delta/(i-1) = \xi_\#$ and $\lim_{(t_i - t_{i-1})\to\ +0} \xi_\int \Delta/(i-1) = \xi_\Delta$. Combining these with the findings above, we get the following four variances corresponding to the four unbiased solutions:

$$\begin{aligned}
W_{1,\infty} = &\quad V_{1,\infty} + \xi_\Delta &= -k + \xi_\#, \\
v_{2,\min} + \xi_\# \geq W_{2,\infty} = &\quad V_{2,\infty} + \xi_\# &\leq v_{2,\max} + \xi_\#, \\
v_{1,\inf} + \xi_\Delta < W_{1,0} = &\quad V_{1,0} + \xi_\Delta &< v_{1,\sup} + \xi\Delta, \\
W_{2,0} = &\quad V_{2,0} + \xi_\Delta &= 0.
\end{aligned}$$

2.5 Estimating transition probabilities in discrete state space stochastic models

Thesis 10 I developed an estimation method to estimate transition probabilities in discrete stochastic Markov-models. The main idea is that the measurement of the gaps between consecutive events is not used directly but first used for intensity estimation. Using classical approaches does not estimate the intensity well enough and require too many measurements so the estimation of the parameters in the stochastic model is not accurate enough. Using the proposed Adaptive Estimator (RIE) the state-of-art Haar-Fisz transformation based intensity estimation method, the parameters are estimated with great accuracy. [J0-IEM-TECHM-2011]

Definition 12 *We consider a state based model with $m \in \mathbb{R}$ possible transition probability classes. Only a bounded number of states may be reached from any other state and m is the maximum of these transition types $T_j s$, $j = 1, ..., m$. The model can be written in the from:*

$$P_j(t) = \Delta t k_j f_j(\mathbf{s}(t)) + o(\Delta t), j = 1...m, \tag{8}$$

where $\mathbf{k} \in \mathbb{R}^m$ are the rate constants to be estimated, $f_j : \mathbb{R}^l \mapsto \mathbb{R}, j = 1, ..., m$ is a known mapping that transforms the state space to the weight of the possible state transitions. Each f_j is associated to a k_j.

We can write the equation (8) in the form: $k_j \cong P_j(t)/(\Delta t f_j(\mathbf{s}(t))), j = 1, ..., m$. Now the idea is to estimate $\Delta t = t_{n+1} - t_n$ with the intensity of the process: $\lambda(t_n) \simeq 1/(t_{n+1} - t_n)$. This is a first moment type estimate, since for every Markov model, $(t_{n+1} - t_n) \sim \text{Exp}[\lambda(t_n)]$. To estimate $\mathbf{P} = (P_1, P_2, ..., P_m)$ we can use the relative frequencies as a maximum likelihood estimate. The problem with this is that since in most of the cases the number of states outnumber the number of experiments we will have no good estimate of P_j. To overcome this problem see the definition of the estimation process:

Definition 13 *[J0-IEM-TECHM-2011] We measure the states $\mathbf{s}(t_i)$ at times t_i. Let $\mathbf{r}(t_i) = (r_1, r_2, ..., r_m)$ be defined as $r_j(t_i) = \text{dist}^{-1}(\mathbf{s}(t_{i+1}) - (\mathbf{s}(t_i) + \mathbf{T}_j))$ iff state change of type j and $\mathbf{T}_j$ represents this transition. We propose the following estimation method for the parameter $\mathbf{k}$:*

$$\breve{\mathbf{k}}_j = (\sum_{i=1}^{n} \frac{r_j(t_i)\breve{\lambda}(t_i)}{f_j(\mathbf{s}(t_i))})/(\sum_{i=1}^{n} r_j(t_i)), \tag{9}$$

orig. val.	k_1 1	k_2 0.001	k_3 1
"$1/\Delta t$"	18.580 (13.333)	0.0168 (0.0092)	18.676 (4.9235)
$\bar{\lambda}$	0.3706 (0.0001)	0.0008 (0.)	1.3377 (0.0072)
$\hat{\lambda}$	1.2431 (0.0763)	0.0011 (0.00007)	1.0674 (0.0379)
H-F	1.1213 (0.0375)	0.0013 (0.0001)	1.3594 (0.0853)

Table 1: The table depicts the estimates for the parameters where the Volterra-Lotka system was simulated with parameter values $k_1 = k_3 = 1$ and $k_2 = 10^{-3}$. The data were generated with two different simulation methods and a 2% random measurement error was added.

i.e., to take the average of the estimates at given times t_n when state change type j occurred. The estimation method is flexible in the sense that it can be used with any intensity estimation modules for λ.

This definition can be extended to all $f_j(x, y)$ functions which have an exact inverse function by its first argument i.e., $f^{-1}(f(x, y), y) = x$.

The estimation method is flexible in the sense that it can be used with any intensity estimation modules. Table 1 shows that the proposed $\hat{\lambda}$ estimate and the Haar-Fisz transformation based method are the best choices.

The maximum-likelihood estimate is easy to construct for stationary Markov chains if the necessary number of measurements are available and if the probabilities of the state transitions are comparable. Comparing the proposed method with the original maximum likelihood approach we can find the following differences that supports the relevance of my work:

- This new method works for non-stationry Markov chains as well;
- this new method requires a low number of measurements compared to the number of states e.g., parameters of a chain with hundreds of thousands of states can be estimated using hundreds of measurements;
- this new method requires only that the number of possible state transitions $\mathbf{T}_j$s should be small enough;

- the heart of the method is the estimation of the time varying intensity of a Markov process (Wald process) for which the method I proposed suits well.

The above parameter estimation may be applied for models with a huge state space but limited transition types using few measurements.

3 Application of the Results, Further work

The work has been motivated by the Quality of Service and signaling control problems of New Generation Networks. My findings contributed to the IMS overload control standardization project at Ericsson Telecommunications Hungary Research Laboratory because the proposed call gapping mechanism is better than the ones used before by means of capacity utilization and control accuracy. Some parts of the methods are therefore patented and implemented and in Ericsson products and are used on the field.

The mathematical models developed to describe the behavior of the call gapping mechanisms and the requirements on them revealed a lot of important properties of such mechanisms. E.g., the fact that no priority handling is available without either violating the requirements on maximal throughput or under-utilizing the system is intuitive but was hard to quantify before. I have proposed a model to do this.

Parameter estimation with neural networks has been applied to examine models of chemical and biological reactions especially to combustion and transport models. Once the network is trained then it can be used for on-line estimation unlike the others which require too heavy computations.

Parameter estimation is in the center of the interest of applied mathematics. Particularly, point processes are used not only to model telecommunication traffic or biochemical reactions, but also financial trends, etc. In statistics there is never a best estimator but there is always some that is better for the purpose than another. I have showed that the estimate I proposed to estimate the intensity of time inhomogeneous point processes with is the best known for my purpose. I have extended the classical mathematical apparatus for the discussions and I plan extend it even more in the future.

A number of open questions remained. It would be interesting to give an equation using $\mathbb{E}[(N(t)-N(s))^2|F_s]$ where $s<t$ analogous to $\mathbb{E}[N(t)-N(s)|F_s]=\mathbb{E}[\int_s^t \lambda(u)du]$. In the future, I want to investigate the prediction properties of the estimate I have proposed since preliminary results shows

that it can be used for prediction. Modifying a bit on the intensity estimator, I have proposed an estimate for the discretized derivate of the Wiener process and checked if the result is a white noise process. I expect much more new results in this direction soon.

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