Budapest University of Technology and Economics
Faculty of Transportation Engineering

Statistical investigation of lifetime and fatigue process of vehicle components and structures

Ph.D. Thesis

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2011.

Introduction

Importance of the topic

The typical breakdown mode of many machine components, especially of vehicle components, is the material fatigue. This is why the fatigue is an intensively investigated phenomenon since its revelation (Wöhler, ≈1860). Despite the continuous research, the technical development again and again leads to structures which catch even of public attention to the fatigue, e.g. pressurized fuselage of jet aeroplanes (1950) high speed train wheels with rubber ring under rim (2000)…

The goals of research

The most important goal of fatigue related engineering computations is the determination of a characteristic lifetime (e.g. allowed service time, inspection period), or estimation of the probability of a damage event at known conditions. Results of resource consuming fatigue experiments are needed for reaching the goals, and we always have to face numerous uncertainty. The low allowed fracture probabilities in vehicle industry (*<1%*) makes the question more difficult. In light of above facts can be seen the importance of the following efforts:

- The "similar" previous cases must be investigated, and all possible "reusable" information must be extracted from them
- The method of usage of "reusable" information must be worked out and improved.

In accordance with above statements, main goals of my work are:

- Investigation of the relationship between laboratory experiments and real operation lifetimes, taking into consideration the service load peaks.
- Elaboration of an evaluation method able to extraction of reusable information from numerous laboratory fatigue experiment.
- Evaluation of a large available fatigue test result database, primarily for characterization of scatter properties and secondarily for the shape of lifetime distributions.
- Investigation of method for usage of extracted information, application of Bayes method.
- Characterisation of practical advantages of taking into consideration of reusable information.
- Elaboration and characterisation of a probability model able for description of empirical lifetime distributions.

Method of research

During my work I have possibilities for using the following methods:

- evaluation of numerous published fatigue experiment results
- evaluation of published service experiences
- model building, theoretical deductions
- numerical procedures, stochastic simulations

Antecedents

Related to distribution of service life a methodology was given by Matolcsy, which is appropriate for practical applications and the limits of application of methodology was also given. The service load process $X(t,\omega)$ and service load bearing capacity of structure $R(t,\omega)$ must be treated as stochastic process. The condition of fracture is: $X(t,\omega) > R(t,\omega)$ The role of load process was divided by Matolcsy to fatigue effect of oscillations and to fracture due to an extreme peak. For determination of the load bearing capacity $R(t,\omega)$ „equivalent fatigue load" can be used which not contains extreme peaks. The service life can be determined by the statistics of $R(t,\omega)$ and the extreme load peaks. [Matolcsy]

Many researchers analysed large number of fatigue lifetime distribution for studying of scatter properties and / or distribution shape. [Butler] [Saunders] [Matolcsy] [Gedeon1] Theoretical results are known related to the lifetime distributions at different load levels of a Wöhler experiments. [Castillo2] [Matolcsy] [Márialigeti]

The decisions of practical importance (e.g.: safe life in service) are based on statistical estimations. A typical task is the estimation of lifetime distribution function parameters based on lifetime samples. Numerous estimation methods are known for solving this task. The of Bayes method is unique, because it is a well based mathematical method for using prior information, and leads to practical interpretable results. Bernardo describes the Bayes method as a complete paradigm which is an alternative against the frequentist statistics. [Bernardo] Due to the advantages, Cornell suggest the Bayes method for engineering applications. [Cornell]

In the Bayes method the prior information according to the quantity to be estimated are described by a probability distribution (prior distribution) which is modified by evaluation of new samples, and result is the posterior distribution of the quantity. The result of the estimation can be computed based on posterior distribution (e.g.: most likely value, expected value).

For description of lifetime distributions many type of distributions are applied by many researchers yet. Evaluation of empirical distributions and theoretical verification of some types are also known. [Saunders] [Castillo1] [Gedeon1]

New results

On service life lengths

The computation method of service life lengths was investigated in case of strictly stationary operational load process $X(t,\omega)$ and different stochastic operational load bearing models $R(t,\omega)$, based on an approach by Matolcsy [Matolcsy]. The goal was to compare the lifetime statistics of laboratory tests with operational lifetime statistics. The laboratory load processes does not contain extreme peaks but their fatigue effect is considered equivalent with real load process.

1. Thesis: If the length of the crack propagation is negligible in the whole lifetime up to the fatigue fracture of a structural component then the effect of extreme load peaks on lifetime distribution can be taken into consideration computation by the following equation:

$$\lambda^{*}(t) = \lambda_X(R_0) + \lambda_0(t) - \lambda_X(R_0) \cdot \lambda_0(t)$$

where
- $\lambda^{*}(t)$: event rate of service fracture in presence of extreme peaks of service load
- $\lambda_0(t)$: event rate of laboratory fracture in case of fatigue-equivalent laboratory load process without extreme peaks
- $\lambda_X(x)$: level crossing event rate of extreme service load peaks
- R_0: initial load bearing capacity of the component

In the practically important region of fracture probability ($<1\%$) the following approximation can be applied:

$$\lambda^{*}(t) \approx \lambda_X(R_0) + \lambda_0(t)$$

The service lifetime distribution can be estimated reliably only in the probability region where the laboratory lifetime distribution is reliably known. The reliable knowledge of the laboratory lifetime distribution has similarly large significance even in the case of parallel load bearing components.

Investigation of lifetime statistics of Wöhler experiments

Statistical evaluation of numerous Wöhler experiment was done. The relation between statistical quantities of different character was investigated. The number of Wöhler experiments is *≈180* containing *≈650* lifetime samples and the total number of specimens is *≈4800.* The investigated statistical quantities having expected value character are:

- logarithm of estimated expected value of lifetimes
- estimated expected value of the logarithms of lifetimes
- estimated scale parameter of two parameter Weibull distribution

The investigated statistical quantities having scatter character are:

- relative empiric standard deviation of lifetimes
- empiric standard deviation of logarithms of lifetimes
- reciprocal of the estimated shape parameter of the Weibull distribution

The estimation of the above six quantities was done using different methods in case of all lifetime sample. The relation between quantities of the same character (*expected value – expected value, scatter - scatter*) and of different character (*expected value – scatter*) was investigated in theoretical and empirical way. Stochastic model was fitted on the relation of quantities of different character. The investigated data was ordered in four groups according to the material of specimens. The results were very similar in case of all groups and the following two theses are valid for all investigated metallic materials.

It was found on theoretical way and proved empirically that the statistical quantities of the same character can be computed based on each other with good approximation.

2. Thesis: In case of constant amplitude experiment lifetime samples the relation between the different estimations of investigated statistical quantities of same character (*expected value – expected value, scatter - scatter*) **is approximately deterministic, compared to the stochastic relation between the estimations of investigated quantities of different character** (*expected value – scatter*)**.**

Consequence: when investigating the relation between the estimations of quantities of different character any expected value and scatter can be chosen, the different choices are practically equivalent.

According to quantities of different character the following empirical statement can be made:

3. Thesis: In case of constant amplitude experiment lifetime samples the expected value of the fitted scatter parameter of the lognormal distribution is increasing with the fitted expected value parameter beyond the lifetime region of $10^4 \ldots 10^5$.

The increasing tendency is valid between the other quantities of expected value and scatter character (2. Thesis). The expected value of the logarithm of scatter like quantities can be approximated by a constant up to a threshold lifetime and by an increasing line in case of larger lifetimes.

The available lifetime samples were evaluated from point of view of lifetime distribution shape. United large empirical samples were made for all four material groups by a method similar to Saunders method [Saunders] but the standard deviation was estimated by a Bayes method, using the most likely value of the posterior distribution. The stability of united samples was checked graphically. Sub-groups were made for different sample sizes, and united samples were computed to each subgroup and compared with each other.

Identification of deviations of theoretical standard deviation in different experiments

The theoretical („exact") standard deviation of samples containing lifetime logarithms is different in different experiments depending on circumstances. The knowledge of this dependence would be useful in practice. The estimated empirical standard deviations are random variables due to the finite sample size, and random effects partially covering the wanted information. The random effects must be removed from the scatter of estimated empirical standard deviations. The separation of effects of experiment circumstances and the finite sample size was done by numerical stochastic simulations. The scatter of theoretical standard deviation was determined which explains the scatter of estimated empirical standard deviations of known finite sized samples.

The effect of finite sample sizes depends on the shape of lifetime distribution. Firstly, the computations were made using lognormal and two parameter Weibull distributions, and later the distribution shapes of united samples were used.

4. Thesis: In case of constant amplitude fatigue experiments using similar materials (e.g.: structural carbon steels) the scatter of estimated lognormal scatter parameter can not be explained only by finite sample sizes. The theoretical standard deviations of different experiments are different.

The distribution of investigated scatter-like statistical quantities can be approximated by lognormal distribution of constant scatter parameter for a given material, not depending on the expected value of the lifetime.

Investigation of properties of estimations

The estimation of *P*-quantile values were investigated in case of normal distribution. The estimation of the *P*-quantile was made by the inverse value of normal cumulative distribution function at *P*, using the expected value and corrected empirical standard deviation as distribution parameters. If the theoretical expected value or the scatter parameter of distribution is known the estimation of the *P*-quantile values has smaller scatter. The question is: the exact knowledge of expected value or the scatter parameter reduces stronger the scatter of *P*-quantile estimation?

5. Thesis: In case of normal distribution, if $P<P_k$,then the knowledge of standard deviation reduces stronger the scatter of estimation of the *P*-quantile than the knowledge of expected value. P_k is the threshold probability. It is proved by numerical simulations that the value of threshold probability P_k hardly depends on sample size: $P_k \approx 8\%$

Consequence: because the case of lognormal distribution is very similar, in the region of probabilities under $<1\%$ the prior knowledge about the scatter parameters is more important than the prior knowledge about expected value parameters.

Structural components as systems usually can be approximated by chain-like load bearing elements of number *R*, and their lifetimes are independent random variables of identical distribution (*R*: measure of chain property). Fatigue experiment samples of these components can be evaluated in many ways:

1. The measured lifetimes can be considered as the lifetime of the component as a unit.
2. The measured lifetimes can be considered as the lifetime of weakest element and survival lifetime for other elements.

If the total number of elements *R* is known then the lifetime distributions of the elements and the whole system (component) can be converted to each other. Therefore the lifetime distributions both of the elements and the whole system can be estimated using the two methods above. The identity of the results depends on the method of distribution function estimation.

6. Thesis: It is proved in analytical way that in case of a lifetime sample, the result of estimations based on likelihood function does not change if the component is considered to a system of elements arbitrary number of *R≥1* in series connection(chain), in which the lifetime of elements are independent random variables of same distribution.

Usually, the same component can be considered as a chain of different number of elements, therefore different *R* values can be reasonable. Therefore the thesis can be formulated as a general invariance requirement for estimation methods: the result of estimation must be independent from the value of *R*. Except the estimations based on likelihood function (maximum likelihood, Bayes methods) no other method was found during my work which fulfils the stated invariance requirement without restrictions.

The Bayes method provides a coherent methodology which makes it possible to incorporate relevant initial information. [Bernardo] The Bayes method was investigated in case of estimation of fracture probability.

7. Thesis: The posterior distribution of estimated values of normal distribution function was computed using Bayes method, in case of scatter parameter of lognormal prior distribution and known expected value parameter. The estimation of normal distribution function values was done by computing the expected values of the posterior distributions in the investigated points of lifetime axis. This estimation of fracture probability can be approximated by Weibull distribution in the region of small probabilities.

Consequence: In the region of small fracture probabilities, the Bayes method gives significantly larger fracture probability than the lognormal lifetime distribution using the estimated expected value of scatter parameter. Similar effect arises in case of any lifetime distribution in the region of small probabilities: the expected value of fracture probability can be much larger than the most likely value of it.

Bayes estimation of optimal length of time in service

The method of determination of the optimal length $0 < t^* < \infty$ of time in service was investigated in case of components and structures, at the following conditions:

- The component or structure produces profit proportional with service time, the proportion factor *h(ω)* depends only on external circumstances (considered to be random), taking into account the costs of fuel and maintenance, etc.
- The failure of component or structure causes loss (e.g. accident damages), the value of loss *K(ω)* depends only on external circumstances (considered to be random).
- The probability of failure (fracture) during service time is nonzero, the time up to failure is random variable of probability distribution function *f(t)* and cumulative *F(t)*.
- When determining the optimal service time t^*, the goal is to achieve maximum expected value of net profit (profit minus loses due to service failures).

8. Thesis: The expected value of net profit during whole operation is maximised by the service time t^* which makes the following equation true:

$$\frac{M\left(f(t^*)\right)}{1-M\left(F(t^*)\right)}=\frac{M\left(h(\omega)\right)}{M\left(K(\omega)\right)}$$

where *h(ω)*: is the profit from service pro unit service time
K(ω): is the loss of a failure during service
f(t), F(t): probability distribution function and cumulative of random variable describing the service time up to failure

Stochastic model of fatigue lifetime distributions

It was pointed out in my theses that the more accurate knowledge of lifetime distributions in fatigue experiments is advantageous, especially at region of small probabilities. A new stochastic model was elaborated for describing the fatigue lifetimes, which can be considered as a common generalization of widely used distribution types.

9. Thesis: A new stochastic model was elaborated for describing the fatigue lifetimes, which composes the total lifetime as a sum of random variables describing the length of different phases of the fatigue process. One property of this model is that the distribution of random variables describing the lifetime components is the distribution of a chain containing elements of lognormal lifetime distribution. Other property

of this model is that the dependence of lifetime components in probability sense is adjustable by a parameter of the model. As special case, the model contains the normal, the lognormal, the two and three parameter Weibull distributions and the gamma distribution.

The question of dependence of components was investigated in the introduced model. The measure of the dependence significantly influences the distribution, if the other parameters are fixed. But by proper selection of the other parameters, a distribution of independent components can be approximated even by deterministic dependent lifetime components. The application of deterministic dependent lifetime components makes the numerical and analytical treatment much easier.

Numerical parameter identification method was elaborated and implemented as a computer code, which able to fit the new model to empirical samples. The usability of the model was proved by using the united large samples.

Publications

Publications in close relation to thesis

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[2] **Kovács K.**: Bayesian Estimation of Fatigue Lives, Proceedings of VSDIA 2008, p. 771-780, ISBN 978 963 313 011 7

[3] **Kovács K.**: Bayesian Methods for Evaluation Fatigue Tests and their Application for a Glider Airplane Góbé, Periodica Polytechnica (www.pp.bme.hu), Transportation Engineering, 37/1-2 (2009), 57-64.

Publications in various topics according to thesis

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