TP Model Transformation Based Sliding Mode Control and Friction Compensation

Ph.D. Thesis Booklet

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Budapest, 2011.

1 Preliminaries and scientific background of the research work

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Multi-objective nonlinear control theory

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In the last decade, the representation of identification models in system theory has changed significantly. The origins of the paradigm shift can be linked with the famous speech given by D. HILBERT in Paris in 1900. HILBERT listed 23 conjectures, hypotheses concerning unsolved problems which he believed would provide the biggest challenge in the 20th century. According to the 13th conjecture there exist continuous multi-variable functions which cannot be decomposed as the finite superposition of continuous functions of less variables [42, 43, 45]. In 1957 ARNOLD disproved this hypothesis [4], moreover, in the same year, KOLMOGOROV [57] formulated a general representation theorem, along with a constructive proof, where the functions in the decomposition were one dimensional. This proof justified the existence of *universal approximators*. KOLMOGOROV's representation theorem was further improved by several authors (SPRECHER [91] and LORENTZ [71]). Based on these results, starting from the 1980s, it has been proved that universal approximators exist among the approximation tools of biologically inspired neural networks and genetic algorithms, as well as fuzzy logic [14, 19, 23, 47, 61, 78, 95, 102]. In this manner, these approximators have appeared in the identification models of system theory, and turned out to be effective tools even for systems that can hardly be described in an analytical way.

One of the most fruitful developments in the world of linear algebra and linear algebra-based signal processing is the concept of the Singular Value Decomposition (SVD) of matrices. The history of matrix decomposition goes back to the 1850s. During the last 150 years several mathematicians—Eugenio Beltrami (1835–1899), Camille Jordan (1838–1921), James Joseph Sylvester (1814–1897), Erhard Schmidt (1876–1959), and Hermann Weyl (1885–1955), to name a few of the more important ones—were responsible for establishing the existence of the singular value decomposition and developing its theory [93]. Thanks to the pioneering efforts of Gene Golub, there exist efficient, stable algorithms to compute the singular value decomposition [40]. More recently, SVD started to play an important role in several scientific fields [26, 75, 98]. Its popularity also grew in parallel with the more and more efficient numerical methods. Due to the development of personal computers it became possible to handle larger-scale, multi-dimensional problems, and there is a greater demand for the higher-order generalization of SVD for tensors. Higher Order SVD (HOSVD) is used efficiently in independent component analysis (ICA) [65], as well as in the dimensionality reduction for higher-order factor analysis-type problems—thus reducing the computational complexity [64]—to name a few examples. The HOSVD concept was first published as a whole multi-dimensional SVD concept in 2000 [66], and the Workshop on Tensor Decompositions and Applications held in Luminy, Marseille, France, August 29–September 2, 2005 was the first event where the key topic was HOSVD. Its very unique power in linear algebra comes from the fact that it can decompose a given N-dimensional tensor into a full orthonormal system in a special ordering of singular values, expressing the rank properties of the tensor in order of L_2-norm. In effect, the HOSVD is capable of extracting the very clear and unique structure underlying the given tensor. The Tensor Product (TP) model transformation is a further extension to continuous N-variable functions. It is capable of extracting the fully orthonormal and singular value ordered structure of the given function. Note that this structure cannot be analytically achieved, since there is no general analytic solution for the HOSVD. The TP model transformation was also extended to linear parameter-varying (LPV) models in 2003. It generates the HOSVD of LPV models. To be specific: it generates the parameter-varying combination of Linear Time-Invariant (LTI) models that represents the given LPV model in such a way that: i) the number of the LTI components are minimized; ii) the weighting functions are univariate functions of the parameter vector; iii) the weighting functions are in an orthonormal system for each parameter; iv) the LTI systems are also in orthogonal position; v) the LTI systems and the weighting functions are ordered by the singular values.

In conclusion, the TP model transformation finds the clear well defined and unique structure of the given LPV model. This cannot be achieved via analytical derivations. Thus the result of the TP model transformation was termed as the HOSVD-based canonical form of polytopic or LPV models in 2006 [7, 8].

The appearance of *Lyapunov-based stability criteria* made a significant improvement in the control theory of nonlinear systems. This change of the viewpoint was invoked by the reformulation of these criteria in the form of *linear matrix inequalities*, in the early 1990s. Herewith, the stability questions of control theory were given in a new representation, and the feasibility of Lyapunov-based criteria was reinterpreted as a convex optimization problem, as well as, extended to an extensive model class. The pioneers GAHINET, BOKOR, CHILAI, BOYD, and APKARIAN were responsible for establishing this new concept [2, 3, 16, 27, 29, 35, 38, 53, 76, 85]. The geometrical meaning and the methodology of this new representation were developed in the research group of Prof. József BOKOR. Soon, it was also proved that this new representation could be used for the formulation of different control performances—in the form of linear matrix inequalities—beyond the stability issues together with the optimization problem. Ever since, the number of papers about linear matrix inequalities guaranteeing different stability and control properties are increasing drastically. BOYD's paper [17] states that it is true of a wide class of control problems that if the problem is formulated in the form of linear matrix inequalities, then the problem is practically solved.

In parallel with the above research and thanks to the significant increase in the computational performance of computers, efficient numerical mathematical methods and algorithms were developed for solving *convex optimization* problems—thus linear matrix inequalities. The breakthrough in the use of convex optimization in practical applications dates back to the introduction of *interior point methods*. These methods were developed in series of papers [52], and have real importance in connection with linear matrix inequality problems in the work of Yurii NESTEROV and Arkadii NEMIROVSKI [77]. Today, these methods are used in "everyday" engineering work, and it turns out to be equally efficient in cases when the closed formulation is unknown. In consequence, the formulation of analytical problems has gained a new meaning.

It is well-known that a considerable part of the problems in modern control theory necessitate the solution of *Riccati-equations*. However, the general analytical (closed formulation) solution of multiple Riccati-equations is unknown. In turn—with the usage of numerical methods of convex optimization—we consider solved those problems today that require the resolution of a large number of convex algebraic Ricatti-equations, in spite of the fact that a result of the obtained solution is not a closed (in classical sense) analytical equation.

> In conclusion, the most advantageous property of the new, convex optimization based representation in control theory is that it is possible to easily combine different controller design conditions and goals in the form of numerically manageable linear matrix inequalities [17]. This makes it possible to solve numerous (complex) control theory problems with remarkable efficiency.

This is especially true of Lyapunov-based analysis and synthesis, but also of optimal LQ control, H_∞ control [28, 39, 94], as well as minimal variance control. The linear matrix inequality-based design also appeared in other areas such as estimation, identification, optimal design, structural design, and matrix-sizing problems. The following enumeration lists further problems that can be managed and solved in a representation using linear matrix inequalities: robust stability of linear time-invariant systems with uncertainty (μ-analysis) [80, 92, 105], quadratic stability [18, 46], Lyapunov-based stability of parameter-dependent systems [37], the guarantee of constraints on linear time-invariant system inputs, state variables, and outputs, or other goals [17], multi-model and multi-objective state-feedback control [5, 9, 17, 21, 54], robust pole-placement, optimal LQ control [17], robust H_∞ control [36, 48], multi-goal H_∞ synthesis [21, 54, 74], control of stochastic systems [17], weighted interpolation problems [17].

Friction compensation of mechatronic systems

Friction is omnipresent and a constant issue in any mechatronic system, in high precision applications as servo drives [100] or pneumatic cylinders [101] for instance. Friction is highly nonlinear and may result in steady state errors, poor performance, it can highly reduce the efficiency of machines. Control engineers are faced with the problem of modeling friction phenomena and reducing undesirable effects by mechanical ways or by control techniques. It is therefore important for control engineers to understand friction phenomena. A proper model for friction could provide relief. However, mechanisms of friction itself are still not fully understood and accurately modeled. Simple linear friction models do not perform well in solving this problem. Nonlinear approaches have also been proposed more or less successfully, many of them being based on empirically collected data. It has become obvious that nonlinear behavior cannot be modeled using linear models. Friction has been studied extensively in classical mechanical engineering. The availability of new precise measurement techniques has been a good driving force, with the computational power available today, it is in many cases possible to deal effectively with friction.

Early classical friction models are described by static mappings between velocity and friction forces which depend on the sign of the velocity, such as Coulomb friction and viscous friction [44]. However, these friction models are used in mechatronic system modeling even nowadays for the sake of simplicity.

On the other hand, many of the interesting properties observed in systems involving friction cannot be explained by static models alone. In addition, any friction compensation based on static maps defeats at very low velocities. A common dynamic friction model is the LuGre model, it offers a regularization of Coulomb friction at a velocity crossing zero, includes stiction without oscillations in stick and also reproduces the Stribeck effect and frictional lag was introduced in [24, 25] by CANUDAS et al. More recently, other newer models are elaborated and discussed in literature; like the Leuven model [1] and the generalized Maxwell-slip (GMS) friction model [1] by LAMPAERT et al. These newer models can be limited by their more complex implementation and identification process of generally greater number of parameters. The LuGre model, on the other hand provides a sufficient integrated view of friction and it is well suited for implementations such as control applications [11]. In Hungarian respective, KOZMA's key research in the area of tribology and friction is well-known [15] and the most up-to-date research is carried out by PÁLFI, who defines a numerical Finite Element Method (FEM) model of the friction hysteresis of sliding rubber within the generalized Maxwell friction model [81]. PÁLFI's research fits well in the modern trend of numerical modeling.

The two main directions of friction compensation are model-based friction compensation and model-free friction compensation; model-based strategies are the dominant approach ([24, 79]). Adaptive friction compensation was proposed in [33, 67], robust adaptive friction compensation is applied in [96]. Learning control based friction compensation is introduced in [22], [56] deals with time optimal friction compensation. Model-based robust control for friction compensation is designed in [72], while neural networks are applied for friction compensation in [62, 63, 84, 86, 106]. Observer-based friction compensation is proposed in [32], partial state-feedback in [10, 73, 99, 103] and LMI-based multi-objective friction compensation is applied in [55].

Sliding mode control

Sliding mode control of variable structure systems has a special role in the field of robust control. On one the hand, the exact description of sliding mode needs advanced mathematics, which was established by FILIPPOV in [30], [31] in the early sixties. On the other hand, it is quite easy to implement in most engineering systems ([70] and [82]), a simple relay is satisfactory in most cases. The main utility of sliding mode in control design problems is to decouple the highly coupled nonlinear dynamics and to desensitize the performance to variations of the unknown system parameters.

However, despite the theoretical predictions of superb closed-loop system performance of sliding mode, some of the experimental work indicated that sliding mode has limitations in practice, due to its need for a high sampling frequency to reduce the high-frequency oscillation phenomenon about the sliding mode manifold, collectively referred to as ”chattering”.

Standard sliding mode control has seen extensions in depth and in breadth by researchers to incorporate new techniques since its introduction. These extensions include higher order sliding mode control (HOSMC) [68], dynamic sliding mode control (DSMC) [60], terminal sliding mode control (TSMC) [49] and recently Integral sliding mode control [69]. Sliding sector was introduced by [34], another approach of sliding sector is proposed by KORONDI in [59, 104]. The extended techniques retain the main advantages of SMC and also provide better accuracy in addition to chattering removal.

The systematic sliding manifold design for linear systems was proposed by UTKIN in [97]. As an extension of that method, various linear control design methods based on state feedback (pole placement, LQ optimal and H_∞) were proposed for optimal sliding manifold design. In recent years, LMI-based sliding surface design became very popular for systems with time delay and uncertainties [20, 83, 87]. Besides linear static sliding surfaces [12] uses a shifting sliding surface, [51] applies rotating sliding surface and [50] discontinuous sliding surface for sliding mode control. A nonlinear sliding surface is used in [6]. A theoretical polytopic sliding surface was introduced by GOUAISBAUT et al. in [13, 41]. However in practical implementation they used linear sliding surface. SILVA et al. in [88–90] described mismatched and matched uncertainties in polytopic form and used LMI-based convex optimization to obtain the sliding surface.

1.1 Goal of the dissertation

Based on the Introduction we can conclude that there is a trend of utilizing neural networks, genetic algorithms and fuzzy logic for system identification and modeling, which leads to the case that elements of a control system (controlled plant, controller, observer, additive model error, nonlinear friction etc.) may not be always designed and given in a form that is common in modern control theory, but as hybrid elements, for example some elements may be given in analytical form, some as discrete identification data set, while some others in a soft computing form (as fuzzy logic, neural network, genetic algorithm etc.). To verify stability of such hybrid system in which we have a min-max Mamdani fuzzy observer and a Takagi-Sugeno fuzzy controller for example, may be a very challenging task. If genetic algorithms and neural networks are involved too, the stability test is even more difficult. On the other hand, soft-computing techniques have several important benefits such as the online tuning of parameters etc. A possible way to solve the stability test of hybrid systems is to define the control system elements by a common polytopic structure, since the stability test of systems given in the same polytopic structure can be solved easily and systematically in a routine fashion. Therefore, during my research work, one of my goals was to extend TP model transformation in a conceptual level in order to bring all the elements of the control system to a common polytopic structure.

Friction is an unpleasant phenomenon of mechatronic systems, which originated many diverse approaches for friction modeling and compensation. My second goal during my research was to introduce friction models and friction compensation to the unified common polytopic structure concept via TP model transformation and to investigate and analyze the applicability and feasibility of the methodology.

My third goal was to introduce TP model transformation based LMI convex optimization and convex hull manipulation based control design methodology to sliding mode control design for qLPV models and to investigate and analyze the applicability and feasibility of the methodology.

Therefore, my comprehensive goals in details are as follows:

- Extension of TP model transformation to describe from a representation point of view hybrid multi-component systems (which can consist soft-computing-based identifications etc.) with a common polytopic structure, establishing the possibility of effective convex hull manipulation and verifying the stability of multi-component systems systematically, based on stability verification methodologies applicable for polytopic systems. An important objective is to develop reliable and numerically appealing algorithm for extending the TP model transformation.

- The goal is to propose a new methodology and viewpoint for representing friction models, in this case focusing on uniform multi-objective polytopic representation, which is fit to polytopic LMI-based control design. This leads to the result that friction compensation can be formulated as convex optimization problem composed in the form of linear matrix inequalities. As part of the methodology, my aim is to show that the finite element HOSVD-based canonical form and the convex finite element TP models of the most widespread friction models exist and the TP model transformation generates the minimal number of linear time-invariant systems.

  - The aim is to check the effectiveness of the proposed methodology by an academic problem. Experimental measurements of a benchmark system with nonlinear friction are carried out for this purpose.
  - The goal is to apply TP model transformation based friction compensation methodology for a prototypical aeroelastic wing section, which is an up-to-date control engineering problem with complex dynamics and description.

- qLPV forms have appeared in the solution methodologies of nonlinear Sliding Mode Control, which forms are very close to the trends of modern control theory. The goal is examine the application possibilities and the advantages of applying Tensor Product model transformation control design methodology to sliding mode control design. The specific goal is to design the sliding surface and sector in a polytopic representation, where various convex hull generations of the sliding surface and sector can be described. Since the type of the convexity significantly influences the LMI-based design, the aim is to introduce this optimization tool to sliding mode control (SMC) besides LMI-based multi objective optimization.

The linear matrix inequality-based controller design was well-researched in the last decade, therefore its validity and applicability analysis is not set as a goal in this dissertation.

2 Methodologies used in the dissertation

2.1 Brief introduction to Tensor Product model transformation of qLPV models

Definition 2.1 (qLPV model)**.** Consider the quasi Linear Parameter Varying State Space model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) \tag{1}$$
$$\mathbf{y}(t) = \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{D}(\mathbf{p}(t))\mathbf{u}(t),$$

with input $\mathbf{u}(t) \in \mathbb{R}^m$, output $\mathbf{y}(t) \in \mathbb{R}^l$ and state vector $\mathbf{x}(t) \in \mathbb{R}^k$. The system matrix

$$\mathbf{S}(\mathbf{p}(t)) = \begin{pmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t)) \\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t)) \end{pmatrix} \tag{2}$$

is a parameter-varying object, where $\mathbf{p}(t) \in \Omega$ is time varying $N-$dimensional parameter vector, where $\Omega = [a_1, b_1] \times [a_2, b_2] \times .. \times [a_N, b_N] \in \mathbb{R}^N$ is a closed hypercube. $\mathbf{p}(t)$ can also include some elements of $\mathbf{x}(t)$, in this case (2) is termed as quasi LPV (qLPV) model. Therefore, this type of model is considered to belong to the class of non-linear models.

Definition 2.2 (Finite element polytopic model)**.**

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{r=1}^{R} w_r(\mathbf{p}(t))\mathbf{S}_r. \tag{3}$$

where $\mathbf{p}(t) \in \Omega$. $\mathbf{S}(\mathbf{p}(t))$ is given for any parameter vector $\mathbf{p}(t)$ as the parameter varying combinations of LTI system matrices $\mathbf{S}_r \in \mathbb{R}^{(k+m)\times(k+l)}$ called LTI vertex systems. The combination is defined by the weighting functions $w_r(\mathbf{p}(t)) \in [0, 1]$. By finite we mean that R is bounded.

Definition 2.3 (Finite element TP type polytopic model)**.** $\mathbf{S}(\mathbf{p}(t))$ in (3) is given for any parameter as the parameter-varying combination of LTI system matrices $\mathbf{S}_r \in \mathbb{R}^{(k+m)\times(k+l)}$.

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{i_1=1}^{I_1}\sum_{i_2=1}^{I_2} .. \sum_{i_N=1}^{I_N} w_{n,i_n}(p_n(t))\mathbf{S}_{i_1,i_2,..,i_N}, \tag{4}$$

applying the compact notation based on the previous chapters we have:

$$\mathbf{S}(\mathbf{p}(t)) = \mathcal{S} \mathop{\boxtimes}_{n=1}^{N} \mathbf{w}\left(p_n\left(t\right)\right) \tag{5}$$

where the (N+2) dimensional coefficient tensor $\mathcal{S} \in \mathbb{R}^{I_1\times I_2\times\cdots\times I_N\times(k+m)\times(k+l)}$ is constructed from the LTI vertex systems $\mathbf{S}_{i_1,i_2,\ldots,i_N}$ (5) and the row vector $w_n\left(p_n\left(t\right)\right) \in [0, 1]$ contains one variable and continuous weighting functions $w_{n,i_n}(p_n(t))$,$(i_n = 1 \ldots I_N)$.

Remark 2.1. : TP model (5) is a special class of polytopic models (2), where the weighting functions are decomposed to the Tensor Product of one variable functions.

2.2 Brief introduction to sliding mode control design

The design of a sliding mode controller consists of three main steps. First, the design of the sliding surface the second step is the design of the control law which holds the system trajectory on the sliding surface and the third and key step is the chattering-free implementation.

2.2.1 STEP I. Sliding surface design

A single input, multi output linear time invariant (LTI) system is considered, which has to be given in regular form.

Definition 2.4 (Regular form of LTI systems)**.** An LTI system is considered regular if it is given in the following form:

$$\begin{gathered}\begin{pmatrix}\dot{\mathbf{x}}_1\\ \dot{\mathbf{x}}_2\end{pmatrix} = \begin{pmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{A}_{21} & \mathbf{A}_{22}\end{pmatrix}\begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{pmatrix} + \begin{pmatrix}0\\ \mathbf{B}_2\end{pmatrix}\mathbf{u}\\ \mathbf{y} = \mathbf{C}\begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{pmatrix} + \mathbf{D}\mathbf{u}\end{gathered} \tag{6}$$

with input $\mathbf{u} \in \mathbb{R}$, output $\mathbf{y} \in \mathbb{R}^l$, state vector $\mathbf{x}_1 \in \mathbb{R}^{k-m}$, $\mathbf{x}_2 \in \mathbb{R}^m$ and $\mathbf{B}_2 > 0$. In this case, input signal $\mathbf{u}$ acts upon state vector $\mathbf{x}_2$ only. The reference signal is supposed to be constant and zero.

Definition 2.5 (Sliding surface)**.** The sliding surface s of the sliding mode, where the control has discontinuity, is designed in a k-dimensional space, where k is the number of state variables. The sliding surface can be given in the following form as a linear combination of state variables as:

$$\mathrm{s} = \mathbf{x}_2 + \mathbf{F}\mathbf{x}_1 = 0, \tag{7}$$

where $\mathrm{s} \in \mathbb{R}$ and $\mathbf{F} \in \mathbb{R}^{k-m}$ is the ”surface vector”. The aim is to keep the system’s trajectory on the sliding surface.

Corollary 2.1. *When sliding mode occurs (*$\mathrm{s} = 0$ *and* $\mathbf{x}_2 = -\mathbf{F}\mathbf{x}_1$*), the design problem of the sliding surface can be regarded as a linear state feedback control design for the following subsystem:*

$$\dot{\mathbf{x}}_1 = \mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{x}_2 \tag{8}$$

In (8), $\mathbf{x}_2$ *can be considered as the input of the subsystem with state matrix* $\mathbf{A}_{11}$*. A state feedback controller* $\mathbf{x}_2 = -\mathbf{F}\mathbf{x}_1$ *for this subsystem gives the switching surface of the whole VSS controller.*

In sliding mode, the system’s behavior is given by the differential equation:

$$\dot{\mathbf{x}}_1 = (\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{F})\mathbf{x}_1. \tag{9}$$

Remark 2.2. The condition of sliding mode control is that (9) is stable. This means that several state-feedback design techniques (pole placement, LQ optimal, frequency shaped method and H_∞) can be used for sliding surface design, as long as they stabilize (8) in the form of (9). The main problem is that these methods are not suitable for a nonlinear system which is more challenging. The solution can be the Tensor Product model transformation.

2.2.2 STEP II. Control law design

We use the Lyapunov stability criterion to ensure that the system is asymptotically stable and that it remains in the sliding mode ($\mathbf{s} = 0$). The simplest control law which can lead to stable sliding mode is the relay:

$$\mathbf{u} = M \cdot sign(\mathbf{s}) \tag{10}$$

Definition 2.6 (Equivalent control signal)**.** If sliding mode exists then there is a continuous control, a so-called "equivalent" control, $\mathbf{u}_{eq}$, which can hold the system on the sliding manifold. It can be calculated from $\dot{\mathbf{s}} = 0$:

$$\mathbf{u}_{eq} = -\frac{(\mathbf{A}_{21} + \mathbf{F}\mathbf{A}_{11})\mathbf{x}_1 + (\mathbf{A}_{22} + \mathbf{F}\mathbf{A}_{12})\mathbf{x}_2}{\mathbf{B}_2} \tag{11}$$

Remark 2.3. In practice, there is never perfect knowledge of the whole system and its parameters. Only $\hat{\mathbf{u}}_{eq}$, the estimation of $\mathbf{u}_{eq}$, can be calculated. Since $\mathbf{u}_{eq}$ does not guarantee the convergence to the switching manifold in general, a discontinuous term is usually added to $\hat{\mathbf{u}}_{eq}$.

$$\mathbf{u}_{eq} = \hat{\mathbf{u}}_{eq} + M \cdot sign(\mathbf{s}) \tag{12}$$

2.2.3 STEP III. Chattering free implementation, Sector sliding mode

The chattering in the basic sliding mode control is essentially due to the requirement that the system state must stick to the switching surface. There are several solutions for elimination of chattering. Many scientific papers con be found on this topic, but this section focuses on sector sliding mode control.

An important approach to reduce chattering is the **sector sliding mode**, which can be extended for TP model transformation-based sliding mode control. To implement the proposed approach, two sliding surfaces are defined first:

$$\mathbf{s}_r = \mathbf{x}_2 + \mathbf{F}_r\mathbf{x}_1 = 0 \tag{13}$$

where $r = 1, 2$.

The two sliding surfaces divide the whole state space into three regions defined as:

Definition 2.7 (Sliding sector)**.**

$$\begin{aligned} R_1 &= \{\mathbf{x} \mid \mathbf{s}_1(\mathbf{x}) > 0 \quad and \quad \mathbf{s}_2(\mathbf{x}) > 0\} \\ R_2 &= \{\mathbf{x} \mid \mathbf{s}_1(\mathbf{x}) < 0 \quad and \quad \mathbf{s}_2(\mathbf{x}) < 0\} \\ R_3 &= \{\mathbf{x} \mid \mathbf{s}_1(\mathbf{x})\mathbf{s}_2(\mathbf{x}) \leqslant 0\} \end{aligned} \tag{14}$$

where region R_3 is the sliding sector.

The sliding surface of the system is given by the following equation:

$$\mathbf{s} = \mathbf{x}_2 + \mathbf{F}\mathbf{x}_1 = 0 \quad where \quad \mathbf{F} = \frac{\mathbf{F}_1 + \mathbf{F}_2}{2}. \tag{15}$$

This leads to:

$$\mathbf{s} = \frac{\mathbf{s}_1 + \mathbf{s}_2}{2}. \tag{16}$$

The control strategy of the proposed modified sliding mode control method is

$$\mathbf{u} = \mathbf{u}_{eq} + \mathbf{u}_d, \tag{17}$$

where $\mathbf{u}_{eq}$ is the continuous "equivalent" control signal given in Definition 2.6.

Definition 2.8 (Discontinuous control signal)**.**

$$\mathbf{u}_d = -M \cdot sign\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2}\right) \qquad if \quad \mathbf{x} \in R_1 \cup R_2 \tag{18a}$$

$$\mathbf{u}_d = -M \frac{\mathbf{s}_1 + \mathbf{s}_2}{|\mathbf{s}_1| + |\mathbf{s}_2|} \qquad if \quad \mathbf{x} \in R_3 \tag{18b}$$

$\mathbf{u}_d$ is a relay type non-continuous input signal which has the role of disturbance rejection and compensation of the error of $\mathbf{u}_{eq}$ coming from parameter uncertainties. Inside the sector it is a continuous signal in order to decrease chattering.

3 Scientific Results of the Dissertation

3.1 Research results of the dissertation

3.1.1 Unified TP type polytopic concept of tensor functions and control system elements, Multi TP model transformation

I extended the TP model transformation to simultaneously transform a set of functions into common type TP functions. I defined this extended TP model transformation as multiple TP model transformation. The multiple TP model transformation is capable of transforming a set of functions to the polytopic representation over a common weighting function system. Multi TP model transformation is able to handle scalar, vector or even tensor functions.

Multiple TP model transformation conserves all the benefits as single TP model transformation, such as construction of HOSVD-based canonical form, possibility of constructing and manipulating polytopic convex hulls, ensures the tradeoff between complexity reduction and accuracy, has a reliable and numerically appealing/tractable way, which means that not only analytical functions, but functions given in other soft-computing form (e.g. as a fuzzy logic based, a neural network or a genetic algorithm) can also be transformed.

Based on the multiple TP model transformation I established the concept of representing multi-component and hybrid complex qLPV systems in a uniform polytopic structure. The components of such system may be available in an analytical form, in the form of a numerical data set or in a soft computing form (e.g. as a fuzzy logic based model, a neural network or a genetic algorithm). I proved that such systems can be transformed to uniform type polytopic forms using the multiple TP model transformation, regardless of whether or not the components are given in the same or different types of representation. I proved that by this concept convex hull manipulation can be realized in a uniform and systematical way efficiently for multi-objective control performance optimization in a wide class of control engineering problems.

Due to its special nature as control system element I gave a special focus on reference signal compensation of complex systems. I gave an example which shows that the reference signal compensation can be determined from the LTI vertex points of the system in certain cases if the complex hybrid system is transformed to uniform polytopic form.

3.1.2 Friction compensation of mechatronic systems based on TP model transformation control design methodology

I investigated friction separately from the system, because friction phenomenon is a special element of control systems. In order to continue the investigation I examined the most commonly used friction models.

I proved that the most commonly used friction models (Coulomb friction, Coulomb with viscous friction, Stribeck and LuGre friction models) can be defined as finite element TP type polytopic models.

I provided the HOSVD-based canonical form of these friction models. I proved that the Coulomb friction, Coulomb with viscous friction and Stribeck friction models can be reconstructed from minimum two LTI vertex systems, while the LuGre friction model can be reconstructed from minimum six LTI vertex systems in case of TP type polytopic forms. I showed that the reconstruction is equivalent to the analytically derived form of the models, the error of reconstruction is in the order of magnitude of 10^{-16}.

I proved that the minimal number of LTI vertex systems is the same in case of convex TP type polytopic models. I provided the SNNN, CNO and IRNO type convex TP type polytopic forms for these friction models.

I showed that the uniform polytopic representation of control system elements of the first thesis does not lead to an explosion in the number of LTI systems in the case when the friction and the system model can be treated separately. I proved that if the friction and the system model are additive the number of weighting functions is summed up.

The results of this and the previous thesis were applied for friction compensation of an academic problem, a real DC servo drive with planetary gear. I gave a dynamical qLPV/LMI-based multi-objective control (such as asymptotic stability and decay rate) design methodology. I showed that the nonlinear friction model is additive to the linear model of the DC servo drive. The resulting qLPV model can be reconstructed from 2 LTI vertex systems, which equals to the minimal number of vertex points of the friction model.

I proved that the convex polytopic forms of the DC servo drive with CNO type weighting functions satisfies the conditions of the TP model transformation-based controller design and within the transformation space Ω the Lyapunov stability criterion formulated in terms of LMIs is satisfied. Utilizing this controller design technique I derived controllers, which guarantee multi-objective control performances such as asymptotic stability and a specified decay rate.

The results of the previous theses were applied for friction compensation of a complex problem of 2 DoF prototypical aeroelastic wing section. I gave a solution to friction compensation of a 2 DoF prototypical aeroelastic wing section, which is an up-to-date, real, complex control engineering problem with a dynamical qLPV/LMI-based multi-objective control (such as asymptotic stability and decay rate) design.

I provided the HOSVD-based canonical form of the 2 DoF prototypical aeroelastic wing section model. I proved that the qLPV model of the 2 DoF prototypical aeroelastic wing section can be reconstructed from 24 vertex systems and the friction is not additive in this case. The number of LTI vertex systems of the 2 DoF prototypical aeroelastic wing section model in case of linear friction models is 6. Adding two Coulomb friction phenomena to the 2 DoF prototypical aeroelastic wing section model increases the number to $6 \times 2 \times 2 = 24$, thus the increase is equal to the number of LTI vertex systems of the Coulomb friction model.

I proved that the minimal number of LTI vertex systems is the same in case of convex TP type polytopic models. I provided the CNO type convex TP type polytopic form for 2 DoF prototypical aeroelastic wing section model.

I proved that the convex polytopic forms of the 2 DoF prototypical aeroelastic wing section model with CNO type weighting functions satisfies the conditions of the TP model transformation-based controller design and within the transformation space Ω the Lyapunov stability criterion formulated in terms of LMIs is feasible. Utilizing this controller design technique I derived controllers, which guarantee control performances such as asymptotic stability and a specified decay rate.

3.1.3 Sliding Mode Control of qLPV Systems, Sector SMC

The goal in this thesis was to make use of the Tensor Product model transformation control design methodology in sliding mode control design. More specifically, the goal was to design the sliding surface and sector in a polytopic representation, where LMI-based multi-objective control design is applicable. Various convex hull generations of the sliding surface and sector can be described by TP model transformation. Since the type of the convexity significantly influences the LMI-based optimization, the aim was to introduce the TP model representation as an optimization tool for sliding mode control (SMC) design.

The qLPV model of the system is transformed to the regular form as:

$$\begin{pmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}(\mathbf{p}(t)) & \mathbf{A}_{12}(\mathbf{p}(t)) \\ \mathbf{A_{21}}(\mathbf{p}(t)) & \mathbf{A}_{22}(\mathbf{p}(t)) \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \mathbf{B}_2(\mathbf{p}(t)) \end{pmatrix} \mathbf{u}$$

I examined the case where all elements of the system matrix can have nonlinearities i.e., a nonlinearity can be found in subspace $\dot{\mathbf{x}}_1 = \mathbf{A_{11}}(\mathbf{p}(t))\dot{\mathbf{x}}_1 + \mathbf{A}_{12}(\mathbf{p}(t))\dot{\mathbf{x}}_2$.

I proposed TP model transformation-based polytopic sliding surface design for subspace $\dot{\mathbf{x}}_1 = \mathbf{A_{11}}(\mathbf{p}(t))\dot{\mathbf{x}}_1 + \mathbf{A}_{12}(\mathbf{p}(t))\dot{\mathbf{x}}_2$. Through the proposed method I applied multi-objective control design methodology for sliding surface design. Based on UTKIN's statement that the sliding surface determines the dynamic behavior of the whole system I introduced polytopic LMI-based multi-objective control design to sliding mode control of qLPV models.

I developed convex hull manipulation based optimization of the sliding surface design besides LMI based optimization. Therefore, I defined several different types of sliding surfaces which are suitable for convex hull manipulation based optimization.

I designed the equivalent control signal of SMC in such way as to stabilize the subsystem.

I extended sector SMC for classes, where the sliding surface is given in convex polytopic form. I proposed the polytopic sector to be designed in such a way that the sector bounding surfaces remain at a constant distance from the polytopic sliding surface at every time instance, thus keeping the width of the sector constant.

I proved that polytopic sector SMC of qLPV systems with nonlinearity inside subspace $\dot{\mathbf{x}}_1 = \mathbf{A_{11}}(\mathbf{p}(t))\dot{\mathbf{x}}_1 + \mathbf{A}_{12}(\mathbf{p}(t))\dot{\mathbf{x}}_2$ is structurally stable inside and outside the polytopic sector, namely the Lypunov stability criterion is always satisfied.

I compared classical polytopic SMC and sector polytopic SMC by simulation. I designed multi-objective LMI-based polytopic sliding surface with asymptotic stability and decay rate goals. I showed that sector polytopic SMC control achieved the desired results without chattering, whereas classical polytopic SMC can only be achieved at the cost of a high chattering phenomenon.

I proved with simulations that the sector SMC is not sensitive to the width of the sector, thus a rather wide sector can be successfully applied and there is no need for additional conditions concerning the width of the sector.

4 Theses

Based on the research results of the dissertation my scientific results can be concluded in the following theses:

Thesis 1: [P–18, 27]

TP model transformation can be extended to Multiple TP model transformation conserving all the benefits of single TP model transformation, the extended Multiple TP model transformation is able to simultaneously transform a set of functions (scalar, vector or tensor) and multi-component and hybrid complex qLPV systems into a common, uniform polytopic structure, it is able to fit multi-component and hybrid complex qLPV systems to qLPV/LMI-based multi-objective control design and convex hull manipulation based optimization effectively and stability verification of such systems can be achieved systematically in a routine fashion.

Thesis 2: [P–5, 19–22, 24, 29]

Friction compensation of mechatronic systems can be fit in the modern TP model transformation based control design methodology, during which the most commonly used friction models (Coulomb friction, Coulomb with viscous friction, Stribeck and LuGre friction models) can be defined as finite element TP type polytopic models. The uniform polytopic representation of control system elements and friction models does not lead to an explosion in the number of LTI systems in the case when the friction and the system model can be treated separately, the number of weighting functions is summed up if the friction and the system model are additive.

Thesis 3: [P–3, 6, 16, 17, 23, 30, 31]

Tensor Product model transformation control design methodology can be effectively applied in sliding mode control design for qLPV systems (any element of the system matrix can have nonlinearity) and by applying TP model transformation LMI-based multi-objective control optimization and convex hull manipulation based optimization is available for sliding surface design thus directly defining the dynamics of the control, additionally, the SMC designed in such a way fulfills the Lyapunov stability criterion and polytopic sector SMC of qLPV systems is capable of chattering-free implementation.

Acknowledgements

The research was supported the Hungarian National Development Agency and the National Science Research Fund (OTKA K62836), the National Research and Technology Agency, (ERC_09) (OMFB-01677/2009) (ERC-HU-09-1-2009-0004 MTA SZTAKI) and Control Research Group of Hungarian Academy of Science for their financial support.

The research was supported by HUNOROB project (HU0045), a grant from Iceland, Liechtenstein and Norway through the EEA Financial Mechanism and the Hungarian National Development Agency.

This work is connected to the scientific program of the "Development of quality-oriented and harmonized R+D+I strategy and functional model at BME" project. This project is supported by the New Széchenyi Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002).

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