Department of Control and Transport Automation
Faculty of Transportation Engineering
Budapest University of Technology and Economics
Budapest, Hungary

Systems and Control Laboratory
Computer and Automation Research Institute
Hungarian Academy of Sciences
Budapest, Hungary

Advanced Freeway Traffic Modeling and Control

Linear Parameter Varying Concepts

Overview of Ph.D. dissertation
written by

Tamás Luspay

Supervisors:
József Bokor D.Sc.
Balázs Kulcsár Ph.D.

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Motivation

Dynamics of freeways is a complex process, therefore the mathematical description and its control are challenging tasks. In order to mimic various observed phenomena non-linear models are needed. The so called macroscopic modeling approach originates from the early 1960s and evolved through the past decades. According to its development, nowadays dynamical models are able to reproduce traffic behavior with high accuracy. These models offer not only a deeper understanding of the process but also a valuable tool for analysis and design of traffic control.

Dynamical model based traffic control appeared during the 1980s. It became conspicuous soon that successful design should tackle with the physically limited nature of the involved variables. The design trade-offs, known from non-linear systems- and control theory, came up also in traffic design. The lack of systematic design methods for constrained non-linear systems resulted in either simplifying the model or increasing the real-time numerical computation.

Recognizing these trade-offs in non-linear systems- and control theory, the Linear Parameter Varying (LPV) framework has been emerged in the early 1990s. In the last two decades LPV methodology became one of the most successful and promising direction, from both theoretical and application point of views, of the post-modern era of systems and control theory. Observing the success, effectiveness and richness of the developed LPV methodology, it is considered as a suitable tool for freeway traffic modeling and control. The dissertation aims to introduce new innovative ideas and tools for freeway systems according to the state of the art in systems- and control theory.

Overview - Contributions to the state of the art

The novel contributions of the thesis are organized into three coherent parts.

Parameter-dependent models of freeway traffic

Chapter 3 introduces the parameter-varying modeling concept for freeway systems. The generic reformulation of the well-known non-linear model [Pay71, Whi74] into a Linear Parameter Varying structure is established in the chapter. Besides the generic qLPV model, the affine parameterized, as well as polytopic representations are also developed in the first part. No approximations have been introduced during the derivation, consequently the obtained models are considered as exact reformulation, preserving the numerical accuracy of the non-linear model. In order to ease the computational complexity, approximated models are also derived. Simulation examples validate the proposed parameter-varying modeling concept for freeway systems.

Analysis of freeway traffic models

The analysis of the obtained parameter-dependent models is carried out in Chapter 4 by using advanced system theoretical methods. A set-theoretical framework is proposed and developed to investigate the effect of physically limited on-ramp control algorithms. The identification of traffic regions, where ramp metering can prevent the further congestion (and therefore traffic breakdown) is determined by introducing the notion of robust controlled positive invariant set. An iterative algorithm has been established for the outer approximation of such invariant set for the local ramp metering problem. Furthermore, adopting the notion of t-step robust controllable sets, congested traffic situations are characterized for which there exists a limited on-ramp sequence which subsequently dissolves traffic congestion. Set-theoretic algorithm is given to compute these sets and characterize traffic situations respectively.

Control of freeway traffic

Based on the developed parameter-dependent model description, a novel ramp metering technique takes place in Chapter 5. The traffic regulation problem is reformulated by using parameter varying methodology. A convex optimization problem is obtained by considering the rejection of the effects of disturbances on the section's throughput. Physical limits on the control input are handled through an indirect saturation setup. The control task is then formulated as a formal induced $\mathscr{L}_2$ norm minimization problem. A dynamical controller is proposed and solution is obtained as a convex optimization problem subject to Linear Matrix Inequality (LMI) constraints. Numerical examples are given to illustrate how the developed parameter-dependent controller is able to suppress shock waves (wide moving jams).

Basic notions, methods and tools

The fundamental concepts and mathematical notions used along the thesis are discussed below.

Definition 1 (Linear Parameter Varying system) *The dynamics of a discrete-time Linear Parameter Varying (LPV) system is written as follows [Wu95]:*

$$x(k+1) = A(p(k))x(k)+E(p(k))d(k)+B(p(k))u(k), \tag{1a}$$
$$z(k) = C_1(p(k))x(k)+D_{11}(p(k))d(k)+D_{12}(p(k))u(k), \tag{1b}$$
$$y(k) = C_2(p(k))x(k)+D_{12}(p(k))d(k)+D_{22}(p(k))u(k). \tag{1c}$$

Variables $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $d \in \mathbb{R}^{n_d}$ are the state-, input and disturbance vectors respectively. Furthermore, $y \in \mathbb{R}^{n_y}$ denotes the output vector and z is the performance output vector $\in \mathbb{R}^{n_z}$. The scheduling parameter vector is denoted by $p \in \mathbb{R}^{n_p}$ and used for capturing non-linearities. In case of state dependent scheduling function the LPV system is called *quasi-LPV*.

Special class of LPV systems is the ones with *affine parameter dependency*:

$$A(p(k)) = A_0 + \sum_{i=1}^{n_p} p_i(k)A_i. \tag{2}$$

Definition 2 (Polyhedral set) *A convex polyhedral set is a set of the form:*

$$\mathscr{P}(F,g) = \{x : Fx \le g\} = \{x : F_i x \le g_i,\ i = 1,2,\dots,s\}, \tag{3}$$

where F_i denotes the i-th row of the $s \times n$ dimensional matrix F and g_i is the i-th component of the $s \times 1$ dimensional vector g [BM08].

Definition 3 (Polytope) *Polytope is a bounded polyhedral set [BM08].*

Definition 4 (Polytopic system) *Polytopic systems are Linear Time Varying (LTV) systems in the form:*

$$x(k+1) = A(k)x(k)+E(k)d(k)+B(k)u(k), \tag{4a}$$
$$z(k) = C_1(k)x(k)+D_{11}(k)d(k)+D_{12}(k)u(k), \tag{4b}$$
$$y(k) = C_2(k)x(k)+D_{21}(k)d(k)+D_{22}(k)u(k), \tag{4c}$$

with restricting the system matrices to belong to the pre-specified polytope Ω:

$$\begin{bmatrix} A(k) & E(k) & B(k) & C_1(k) & D_{11}(k) & D_{12}(k) & C_2(k) & D_{21}(k) & D_{22}(k) \end{bmatrix} \in \Omega. \tag{5}$$

Method of Tensor-Product transformation

The Tensor-Product (TP) model transformation is a numerical tool for reformulating quasi-LPV models into a polytopic form [Bar04]. The non-linear dynamics is described by the state-dependent convex combination of Linear Time Invariant (LTI) vertex systems.

The main algorithmic steps of the TP transformation are summarized below. The first step is the evaluation of the investigated quasi-LPV system over an arbitrary selected domain Ψ, representing the parameter variation set of the scheduling parameters. The qLPV system matrices are then sampled over an n_p dimensional predefined grid defined over Ψ. Higher-Order Singular Value Decomposition (HOSVD) is then applied for the decomposition of the resulting hyper-dimensional data matrix. The number of local LTI systems used for approximating the non-linear dynamics can be determined according to the singular values and their condition numbers. Finally, the orthonormal and discretized weighting functions of the polytopic model ensuring convexity are constructed by executing the HOSVD decomposition of the LPV data matrix.

Definition 5 (Robust controlled positive invariance) *The set $\mathscr{S} \subseteq \mathscr{X}$ is said to be robust controlled positively invariant if there exists a control such that for all $x(0) \in \mathscr{S}$ and for all allowable disturbance sequences the condition $x(t) \in \mathscr{S}$ holds for all $t \geq 0$ [BM08, Ker00].*

Definition 6 (Robust controllable set) *The t-step robust controllable set $\mathscr{K}_t(\mathscr{X}, \mathscr{T})$ is the largest set of states in $\mathscr{X}$, for which there exists an admissible control sequence such that an arbitrary terminal set $\mathscr{T} \subset \mathscr{X} \subset \mathbb{R}^{n_x}$ is reached in exactly t steps, while keeping the evolution of the state inside $\mathscr{X}$ for the first $t-1$ steps, for all allowable disturbance sequences [BM08, Ker00].*

Definition 7 (Linear Matrix Inequality) *A linear matrix inequality (LMI) is an inequality*

$$F(x) := F_0 + x_1F_1 + x_2F_2 + \ldots + x_mF_m \succ 0 \tag{6}$$

where $F(\cdot)$ is an affine function mapping a finite dimensional vector space $\mathscr{X}$ to the set of real symmetric matrices $\mathbb{S}$, i.e. $F_i = F_i^T \in \mathbb{R}^{n\times n}$. The $\succ 0$ reads as positive definite, i.e. $u^T F(x)u > 0$ for all $u \in \mathbb{R}^n$ [BGFB94, SW05].

Thesis 1
Parameter-dependent models of freeway traffic

The discretized version of the extended second-order Payne-Whitham model has been selected to represent freeway dynamics due to its ability to reproduce various traffic phenomena [Pay71, Whi74]. The model uses two dynamical equations to describe the temporal evolution of density (ρ_i) and space-mean speed (v_i) of an arbitrary Δ_i length section of freeway with n lanes. The governing equations are:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\Delta_i n}\left[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)\right], \tag{7}$$

$$s_i(k) = \beta_i \cdot q_{i-1}(k), \tag{8}$$

$$\begin{aligned} v_i(k+1) &= v_i(k) + \frac{T}{\tau}\left[V(\rho_i(k)) - v_i(k)\right] + \frac{T}{\Delta_i} v_i(k)\left[v_{i-1}(k) - v_i(k)\right] \\ &- \frac{\nu}{\tau}\frac{T}{\Delta_i}\frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + \kappa} - \frac{\delta T}{\Delta_i n}\frac{r_i(k) v_i(k)}{\rho_i(k) + \kappa}, \end{aligned} \tag{9}$$

$$V(\rho_i(k)) = v_{free} exp\left[-\frac{1}{a}\left(\frac{\rho_i(k)}{\rho_{cr}}\right)^a\right], \tag{10}$$

$$q_i(k) = \rho_i(k) \cdot v_i(k) \cdot n. \tag{11}$$

Variables $q_i(k)$, $s_i(k)$ and $r_i(k)$ denote the out-, off-ramp and on-ramp flow of segment i respectively. Furthermore, a, v_{free}, ρ_{cr}, β_i, ν, τ, δ, κ are constant model parameters. The state vector of an arbitrary long stretch is denoted by $x(k)$ and consists of density and space-mean speed variables of the interconnected segments. The on-ramp volumes $r_i(k)$ are considered as the control input $u(k)$ while stretch's boundary variables are collected into the generalized disturbance vector $d(k)$. Equations (7)-(11) are then written as a generic non-linear model:

$$x(k+1) = f(x(k), u(k), d(k)). \tag{12}$$

The following method is proposed to transform the non-linear model (12) into a LPV form (1a):

1. Determine the steady-state solutions of difference equations (12), satisfying $x(k+1) = x(k) = x^*$. The resulting algebraic equations in the generic form of $x^* = f(x^*, u^*, d^*)$ are underdetermined, therefore additional design freedoms are given, which can be capitalized according to the nature of the underlying problem.

2. Constitute shifted variables as differences from steady-state values respectively: $\tilde{x}(k) = x(k) - x^*$, $\tilde{u}(k) = u(k) - u^*$ and $\tilde{d}(k) = d(k) - d^*$.

3. Rewrite system dynamics (12) in terms of shifted variables and steady-state values. Time varying variables can be then factorized from the obtained $h(\tilde{x}(k))$ non-linearities by applying the following transformation valid for functions satisfying $h(0) = 0$:

$$h(\tilde{x}(k)) = H(\tilde{x}(k))\tilde{x}(k), \quad H(\tilde{x}(k)) = \int_0^1 \frac{\partial h(\varphi\tilde{x})}{\partial\varphi} d\varphi, \tag{13}$$

   with the use of an auxiliary variable denoted by φ. The necessary condition is fulfilled for the traffic model in question due to the construction of the shifted coordinate frame. Consequently, variables can be factorized by using (13).

4. Rearrange the factorized equations according to the involved variables (e.g. state, input or disturbance) to obtain the generic quasi-LPV structure of (7)-(11):

$$\tilde{x}(k+1) = A(p(k))\tilde{x}(k) + B(p(k))\tilde{u}(k) + E(p(k))\tilde{d}(k), \tag{14}$$

   where the scheduling parameter vector $p(k)$ is used for capturing non-linearities, therefore depends on $\tilde{x}(k)$.

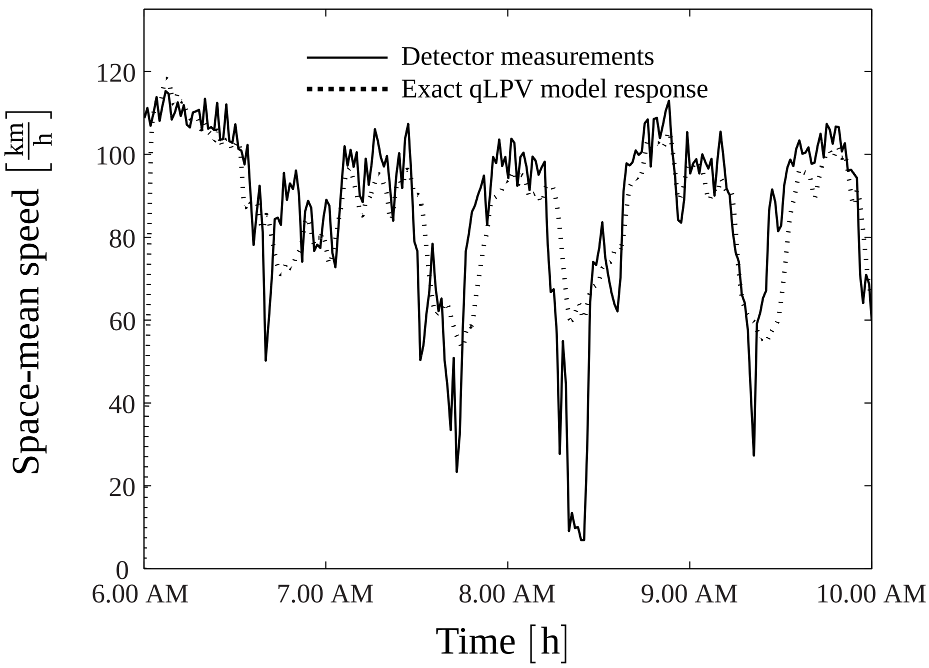
Since no approximation has been introduced through the derivation, the obtained parameter-dependent structure in eq. (14) is numerically equivalent with the non-linear representation of the process eqs. (7)-(11).

The affine parametrization of the established qLPV structure (14) has also been obtained in Chapter 3, by introducing four scheduling parameters per segments. The physical domains of the involved variables can be constructed from detector measurements. Consequently the qLPV model can be evaluated and polytopic representation is obtained by using the Tensor-Product transformation. The detailed derivation of the polytopic structure is discussed in Chapter 3.

Numerical properties of the obtained representations have been addressed in the first part of the dissertation. Computational complexity and requirements are compared in terms of the number of Linear Matrix Inequalities implied by the different structures. Reasonable approximations are proposed to relax numerical requirements, by decreasing the dimension of the scheduling parameter.

Finally, the validation of the developed models is given by using real traffic measurements collected at the A12 freeway in the Netherlands. Fig. 1 shows the comparison of the space-mean speed and flow responses of the exact parameter-dependent model with data provided by detectors. As one can depict, the established model

(a) Comparison of space-mean speed responses



(b) Comparison of traffic flow responses

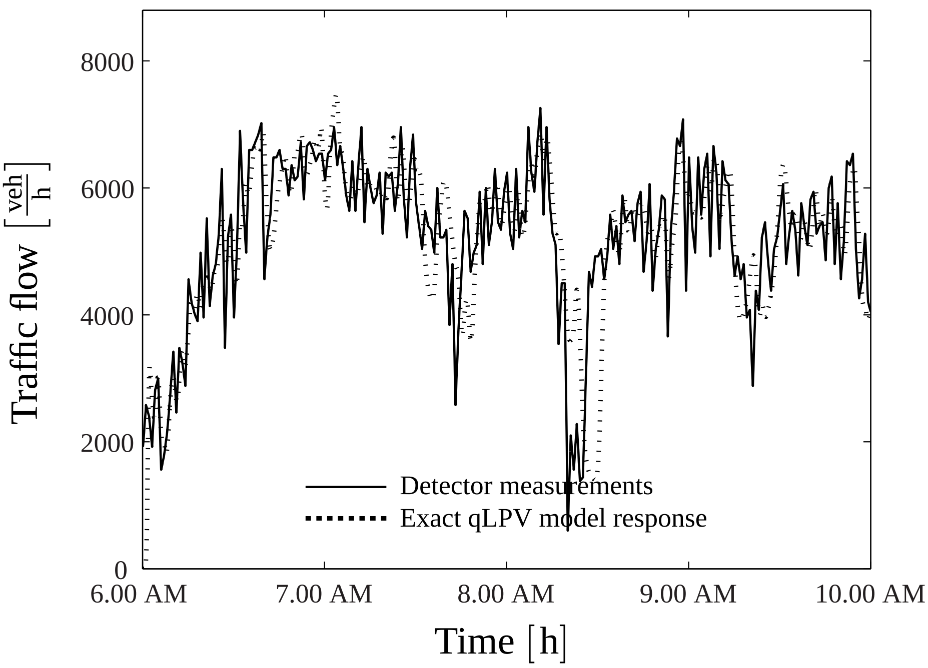


Figure 1: Comparison of exact qLPV speed and flow responses with detector measurements

preserves the numerical accuracy of the second-order macroscopic model and is able to reproduce various traffic phenomena.

Thesis 1 is discussed in details under Chapter 3 of the dissertation and can be summarized as follows:

Thesis 1
The parameter-dependent reformulation of the extended Payne-Whitham type non-linear, second-order, macroscopic freeway model has been established. A generic Linear Parameter Varying structure has been obtained, as well as an affine parameterized one, by using non-conventional transformation techniques. Furthermore, parameter-dependent polytopic formalism is also introduced for traffic modeling purposes. Numerical issues are addressed and a comparative numerical analysis is given to demonstrate the capabilities of the newly developed model structures, using a simple case study. Reasonable approximations have been introduced to relax complex computational requirements. [LKVB10, LKvW^{+}11]

Thesis 2
Analysis of freeway traffic models

A set-theoretic framework is established for analyzing the ramp metering problem, in Chapter 4. Due to its numerically favorable properties, the analysis is based on the polytopic description.

The generic derivation proposed in Chapter 3 (Thesis 1) has been adjusted to the underlying control problem by capitalizing design freedoms offered by steady-state conditions. It is known from traffic flow theory, that the minimization of the network wide control objective has a strong relationship with the maximization of the outflow. Furthermore, the capacity outflow is related to a distinct density value, called critical density. Consequently, the non-linear dynamics have been shifted into an operation point representing the theoretically achievable maximal throughput. This way the origin of the centered coordinate frame coincides with the objective of traffic control. In addition, disturbances acting on the system are categorized as measured and unmeasured ones [BM08]. While the information of measured disturbances can be incorporated through the determination of the control action, control decision should be taken by preparing for the worst possible unmeasured disturbance value.

Accordingly, two algorithms are developed in Chapter 4 to investigate the effect of physically constrained on-ramp algorithms. No preliminary assumptions have been made on either the structure or the computation method of the control sequence, therefore the proposed framework is independent from these issues. Firstly, the *maximal robust controlled positive invariant set* algorithm is adopted for the underlying control problem [Bla99]. Traffic situations which can be maintained with constrained on-ramp inputs are identified by the result of the established iterative algorithm. In these situations additional traffic throughput degradation can be prevented by appropriate control of on-ramp volume. Secondly, an algorithm has been established for determining the *t-step robust controllable sets*. These controllable sets reflect the traffic control objective: those congested traffic situations can be characterized, where congestion can be dissolved by appropriate control of on-ramp volume. Set-theoretic algorithm has been established to compute the t-step robust controllable sets of the local ramp metering in Chapter 4.

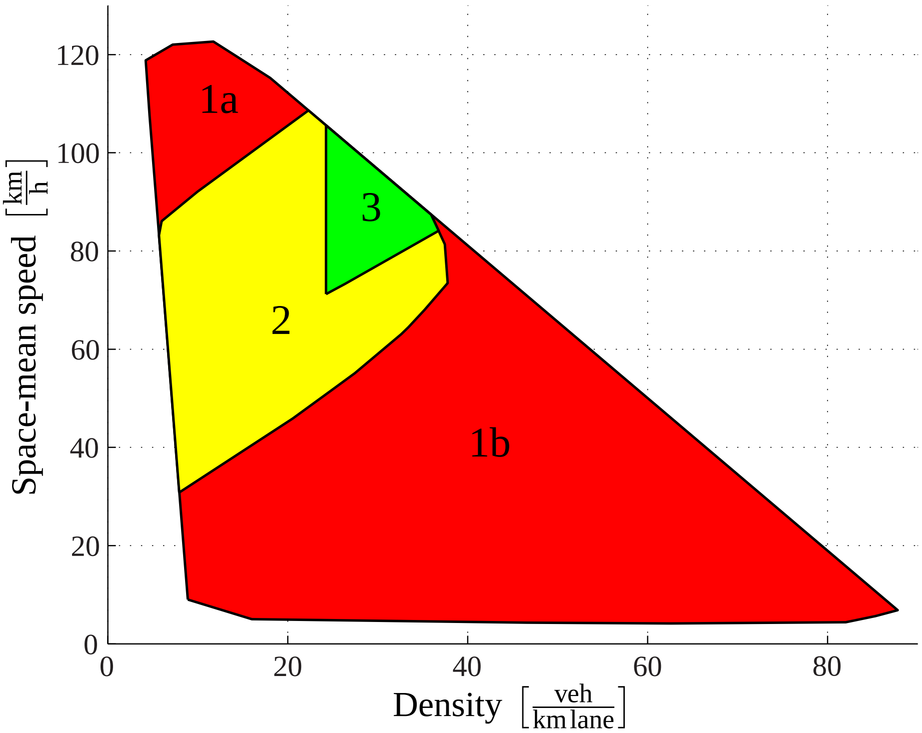


Figure 2: Characterization of traffic situations

For better viability the results are merged together and illustrated in Figure 2. The maximal robust controlled invariant set is labelled by 2, the controllable set by 3. The region outside of the invariant set labelled by 1 and can be further subdivided into two parts (1a and 1b respectively in Figure 2). The region 1a outside of the invariant set, with $\rho < \rho_{cr}$ cannot be sustained due to the high incoming volume. Obviously, low density regions disappear because of the incoming vehicles to the segment. On the other hand the region 1b is not sustainable neither because of the following reasons. If a shock wave reaches an initially congested segment, while the inflow still delivers vehicles to the segment, further degradation of the throughput cannot be avoided by any ramp metering algorithm. In view of these argumentations, the physical interpretation of the maximal robust controlled invariant region (region 2 in Figure 2) becomes clear. Notice that region 2 also contains congested situations, however by appropriate selection of the on-ramp volume the further degradation of throughput can be avoided. This property clearly justifies the importance of any ramp metering algorithm. The investigation of the controllable sets (region 3 in Figure 2) provides additional information on the effectiveness of ramp metering. These controllable sets characterize traffic situations, where ramp metering is able to subsequently dissolve congestion and steer the segment's conditions into the stable (uncongested) region. The set-theoretic methods are discussed more into the details in Chapter 4 of the dissertation. Consequently, the contributions of the chapter can be summarized as:

Thesis 2
Set-theoretic algorithms are established for the analysis of the local ramp metering problem. Traffic situations where traffic breakdown can be avoided by using physically limited on-ramp strategies are identified by the maximal robust controlled invariant set. Furthermore traffic situations where the ramp metering can subsequently steer the system into the stable region are characterized by the t-step robust controllable sets [LKPV, LPKV].

Thesis 3
Control of freeway traffic

A novel ramp metering design method is proposed and established in Chapter 5, where the ramp metering problem is reformulated by using Linear Parameter Varying concepts.

Throughout the transformation proposed in Chapter 3 two key ideas are used for adjusting the structure according to the control task. Firstly, the origin of the centered coordinate frame is set to represent maximal section throughput (see Chapter 4-5 for more details). Secondly, the steady-state on-ramp volume is determined as the algebraic mean of its allowable domain. This way, physical constraints are take a 0-symmetric form in the shifted coordinate frame. The centered input constraints are then modelled by a saturation function:

$$\sigma(u(k)) = \begin{cases} u(k) & |u(k)| \leq \bar{u} \\ \text{sign}(u(k))\bar{u} & |u(k)| \geq \bar{u} \end{cases}, \tag{15}$$

and Linear Parameter Varying concepts are applied for input saturated systems. Namely, we introduce the saturation parameter as [WGP00]:

$$\theta(u(k)) = \frac{\sigma(u(k))}{u(k)}, \tag{16}$$

and define $\theta(0) = 1$ in addition. LPV paradigms can be in a straightforward way extended to address the problem, by considering the saturation parameter $\theta(u(k))$ as an additional scheduling parameter of the system.

The TP model transformation of the system is then performed over the extended scheduling domain containing the saturation parameter in addition. Consequently, a polytopic model is obtained implicitly containing hard physical input constraints. The unmeasured disturbance vector $(w(k))$ of the system contains undesired effects while the performance output $z(k)$ is defined to represent theoretically achievable maximal throughput. The problem is then formulated as a formal induced $\mathscr{L}_2$ norm minimization problem.

Solution of the proposed control problem is solved by defining a dynamic controller. The closed-loop interconnection of the system and the controller is required to satisfy the dissipativity inequality in the form of [SW05]:

$$\mathscr{V}(\xi(k+1)) - \mathscr{V}(\xi(k)) \leq s(w(k), z(k)). \tag{17}$$

For the underlying problem a quadratic storage function $\mathscr{V}(\xi(k)) = \xi^T(k)P\xi(k)$ is applied, while the storage function coincides with the induced $\mathscr{L}_2$ control objective, i.e.: $s(w(k), z(k)) = \gamma^2 \|w(k)\|^2 - \|z(k)\|^2$. A non-linear matrix inequality representation of the dissipative inequality (17) is then derived using algebraic manipulations. In order to obtain a numerically solvable problem a congruent transformation is applied. Consequently, the non-linear matrix inequality is transformed into a Linear Matrix Inequality form, i.e. the control synthesis problem is derived as a convex optimization problem subject to LMI constraints. In order to investigate the numerical properties and effectiveness of the proposed method, two case studies are considered in Chapter 5.

Firstly, a simple benchmark problem is formulated and implemented in MATLAB/SIMULINK environment. A traffic scenario have been artificially constructed, including free-, congested- and peak-hour conditions. The designed controller is compared with the uncontrolled case and the well-known and widely used ramp metering algorithm ALINEA [PBHS91, PHSM98]. Fig. 3 shows the density evolution of the three cases. As one can depict the newly proposed algorithm keeps the segment density around the desired critical one, consequently improves the section's

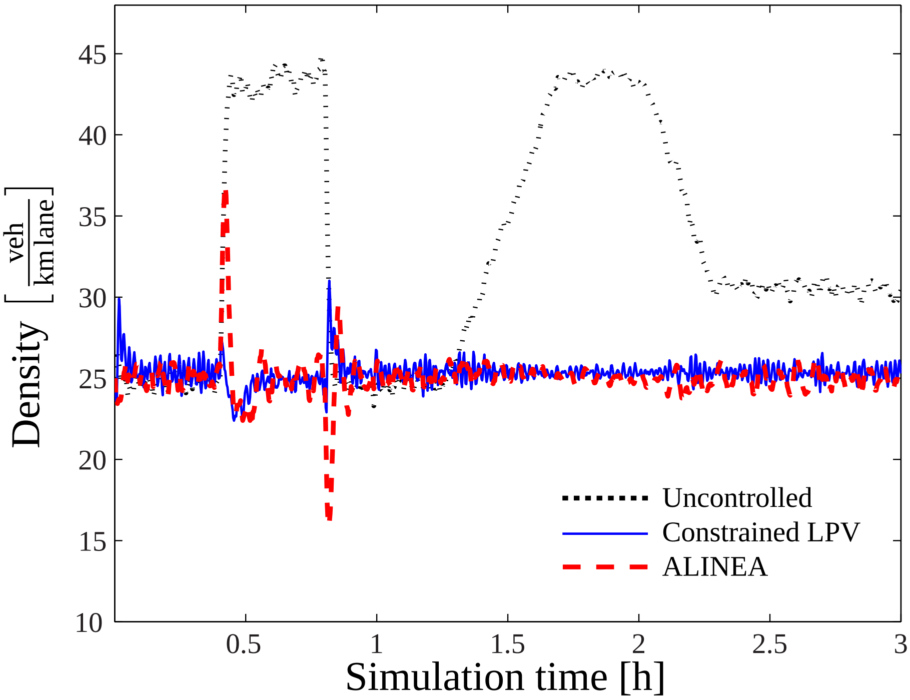


Figure 3: Comparison of the density evolution for the uncontrolled, polytopic controlled and the ALINEA controlled cases

throughput. The effects of unmeasured disturbance is rejected on the performance output according to the design objective. Furthermore, input constraint violation is also avoided by using the proposed implicit constraint handling method.

Secondly, the test of the constrained LPV controller in a commercial traffic simulation software environment has also been carried out in Chapter 5. Through the closed-loop structure with a realistic, microscopic simulation tool the constrained LPV controller showed its transferability and effectiveness by suppressing shock waves.

Thesis 3 is discussed in details under Chapter 5 of the dissertation and reads as follows:

Thesis 3

A novel ramp metering algorithm is proposed by using the parameter-dependent formulation of the second-order macroscopic model. Physical limits on the ramp volume are handled through an implicit structure. The ramp metering problem is formulated as a formal induced $\mathscr{L}_2$ norm minimization problem where the throughput of the section is maximized by suppressing shock waves and other disturbance effects. Comparative numerical examples of the isolated ramp problem are given to validate the developed methodology and investigate its efficiency [LPKa, LPKb].

Future works

According to the parameter-dependent modeling formalisms, the Linear Fractional Transformation (LFT) description has not yet been established for freeway systems. The LFT framework is widely used for representing parameter-dependent models, and has some computationally favorable advantage. It would be worth to introduce such representation for the underlying traffic system.

The developed parameter-dependent models involve a high number of scheduling parameters. Consequently, LPV design techniques became numerically challenging. In order to ease these computational requirements, model reduction techniques can be introduced in the near future. In a similar manner, the results of the controller design can be extended for higher dimensional freeway control problems by applying decentralized and distributed control schemes. Both topics (i.e. LPV model reduction and distributed LPV design) are active and challenging research topics.

The set-theoretic analysis framework can be further extended in several directions. One possibility is the consideration of queue dynamics and the analysis of the problem according to the extended model description. Consequently, the t-step robust controllable sets could form the basis of a novel control design. Predictive control techniques based on controllable and invariant sets are known in the literature, therefore it seems a straightforward application of the analytical results.

Nevertheless, some extensions and applications of the developed LPV models have already appeared in the traffic community. In [ZSHB10] the established parameter-dependent description has been further developed to model vehicular emissions. Also, it was found that the optimum of traffic emissions is related to a higher density value than the critical one. If one unites the classical traffic objective with the vehicular emission objective, the results of the set-theoretical analysis may be useful to characterize the trade-off.

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