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Booklet of PhD Theses

DYNAMICS OF TOWED WHEELS
– Nonlinear Theory and Experiments –

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Introduction

The wheel is one of the oldest and most important inventions of mankind. It has been developing during thousands of years reaching its actual form we can see today. The most relevant innovation was carried out by John Boyd Dunlop in 1887, who made the first pneumatic tyre to avoid the headache caused by the noise of his son's bicycle on the paved road surface.[1] *At that time, Dunlop did not suspected how much "headache" will be generated by his invention in the future for engineers...*

Almost all road vehicles roll on pneumatic tyres and their motion is primarily determined by the forces transferred to the vehicle from the ground by the wheels. Since the properties of the tyre strongly influences the manoeuvrability and stability of vehicles, the investigation of tyre and tyre-ground contact models is an essential task in vehicle dynamics. Precise mechanical models of tyres and tyre-ground contacts are required for numerical simulations in vehicle dynamics in order to explain experienced motions or to predict possible vehicle behaviour. The mechanical models of tyre also play a key role in the development of vehicle control systems such as the ABS, ESP, and so on.

Some of the vibrations in vehicles are related to tyre dynamics. In this study, the lateral vibration of towed wheels – also called shimmy – is under investigation, which is one of the most fascinating dynamic phenomena. Anybody can experience shimmy on supermarket trolleys or on two-wheeled suitcases. Shimmy on vehicles usually presents a safety hazard and its ultimate elimination is a problem for engineers at design stage. It may appear on airplane landing gears, on front wheels of motorcycles and cars, on caravans, rear wheels of semi-trailers and articulated buses.

The shimmy of towed wheels may be caused by the elasticity of the towing bar suspension and the attached vehicle structure, by the elasticity of the tyre on the wheel or by the combination of the two cases. On one hand, the analyses of shimmy is difficult due to the fact that the vehicle itself is a complex dynamical system serving several low-frequency vibration modes which may all be important components of the dynamical behaviour at different running speeds and conditions. On the other hand, difficulties in the analyses are also originated in the modelling of the wheel-ground contact.

[1] Despite of the fact that Dunlop is often said to be the inventor of the pneumatic tyre, note that Robert William Thomson patented his 'Aerial Wheels' in 1846 more than forty years before Dunlop.

Aim of the work

The analytical handling of complex multi degree-of-freedom (DoF) models are usually impossible. Moreover, such complex systems can show very different behaviours for slightly different parameter set-ups, which causes difficulties in the experimental validation of the chosen mechanical models. This problem can be even greater if the non-linearities have relevant effects on the vehicle behaviour. However, a complex system can be simplified if the relevant vibration modes are properly selected. Namely, the system can be modelled by a simple, usually analytically manageable low DoF mechanical model, which can also capture and explain the practically observed relevant vibrations.

The goal of this study is the construction and the analysis of low DoF mechanical models of towed wheels that describe the well-known properties of shimmy with small number of parameters. Thus, we investigate the linear stability of the stationary rolling in wide ranges of the towing speed and the caster length in order to detect linearly unstable islands of the stability charts. We also manage to prove the existences of subcritical Hopf bifurcations and quasi-periodic vibrations in our properly constructed models by means of theoretical and experimental methods.

Mechanical models

Two essentially different low DoF mechanical models are considered and analysed by analytical, numerical and experimental methods. The first model consists of a rigid wheel with elastic suspension (see Figure 1(a)). This model is analysed by analytical and numerical methods with special attention to the nonlinear dynamics of the system. The corresponding mathematical model is a strongly nonlinear three dimensional system of ordinary differential equations (ODEs) that may produce several stable and unstable limit cycles leading to bistability and isola in certain realistic parameter regions.

The second model is constructed from a rigid suspension and a towed wheel of elastic tyre (see Figure 1(b)). The lateral deformations of the tyre are described by the *exact stretched string model*. With the appropriate choice of the boundary conditions, the relaxation length of the tyre is taken into account among other conventional tyre parameters like the specific stiffness and damping. The Newtonian equation of motion becomes a second order integro-differential equation (IDE). The rolling condition is formulated as a kinematic constraint that leads to a partial differential equation (PDE). The travelling wave solutions of the deformation allow us to transform the PDE-IDE system into a delay-differential equation (DDE) with *distributed delay*. The linear stability investigation of the DDE shows that the stationary rolling motion may lose its stability via co-dimension one or co-dimension two Hopf

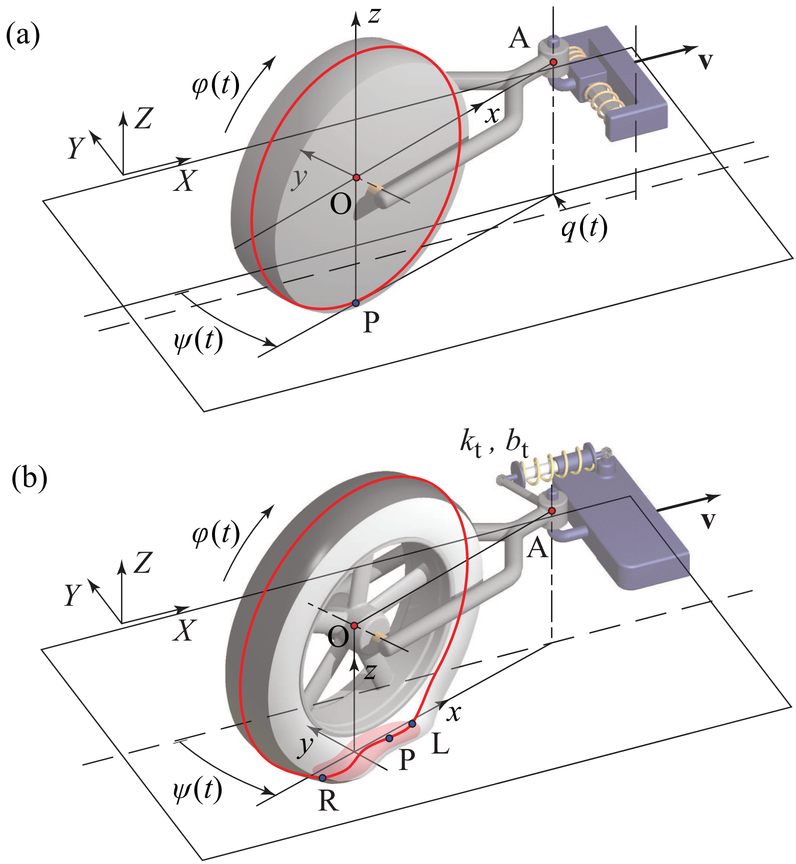

Figure 1 Mechanical models: (a) single contact point model, (b) elastic tyre model.

bifurcations as the parameters (like the towing speed and the caster length) are varied. Consequently, self-excited periodic and quasi-periodic oscillations will appear. The stability chart in the plane of the above parameters is determined analytically and checked by numerical simulations and extensive laboratory experiments in order to confirm the relevance of the memory effect of the time delayed tyre model.

Theses

Thesis 1

A low degree-of-freedom mechanical model of a towed rigid wheel with elastic suspension was constructed and analysed. It was proven that the stationary rolling of the towed rigid wheel is asymptotically stable if and only if:

$$L > L_{\mathrm{cr}}(V) = \frac{V(1 - 4\zeta^2 - 2\zeta\kappa V)}{2\zeta + \kappa V},$$

for the positive dimensionless parameter values of caster length L, towing speed V, damping ratio ζ and inertia parameter κ.

The Centre Manifold reduction of the damped system was carried out and the sense of the Hopf bifurcation related to the linear stability boundary was determined in closed form. It was shown that unstable self-excited vibrations exist around the stable stationary rolling if the damping ratio is smaller than a critical value. A useful upper estimation of the critical damping ratio was calculated, and it was shown that the loss of the stability of stationary rolling leads to small amplitude stable oscillations if

$$\zeta > \frac{\sqrt{\kappa^2 V^2 + 2} - \kappa V}{4},$$

considering positive parameters only. This estimation helps to guarantee that small amplitude oscillations will signal the danger of possible loss of stability. The analytical results were also confirmed by numerical continuation technique accomplished in the open source software AUTO97.

These results are detailed in Chapter 3 of the dissertation.
Related publications: [3, 4, 19].

Thesis 2

The low degree-of-freedom model of the towed rigid wheel was investigated globally by numerical continuation technique outside the parameter region where the local bifurcation analysis was valid. Fold bifurcation of periodic orbits linked to the theoretical Hopf bifurcation point was detected, which verified the existence of stable large amplitude oscillations even around the unstable self-excited oscillations linked to the subcritical Hopf point. The separation of a periodic branch – a so-called isola – was discovered at certain critical parameter values. A bifurcation chart was constructed in the plane of the towing speed and the caster length, in which a so-called bistable parameter range of the towed wheel was identified where the stable stationary rolling and large amplitude stable oscillations coexist.

This provides essential new information for the design of such rolling systems, since neither linear stability analysis nor local bifurcation analysis, similarly, neither extensive numerical simulation nor experimental work can predict the existence of those violent oscillations, which appear with low probability at certain large perturbations only while the stationary rolling is quite robustly stable.

These results are detailed in Chapter 3 of the dissertation.
Related publications: [3, 4, 19].

Thesis 3

A low degree-of-freedom mechanical model of a towed wheel of elastic tyre was constructed where the tyre was modelled as a stretched string of elastic support. The corresponding equations of motion were transformed into a delay-differential equation with the help of a travelling wave trial solution for the tyre-ground contact region. Linear stability charts were constructed in the plane of the towing speed and caster length parameters.

It was shown that the unstable parameter domains of the towed tyre can be reduced by increased damping of the tyre, while the increase of the tyre relaxation length magnify the unstable parameter regions.

For large towing speeds, an essential stability limit was identified for the critical caster length

$$l_{\mathrm{cr}} = a + \sigma \, ,$$

where a is the half length of the contact patch and σ is the relaxation length of the tyre.

The existence of quasi-periodic oscillations were proven, the most relevant one was found at the rightmost double Hopf bifurcation point of the stability chart. If this quasi-periodic oscillation exists, it is in the parameter region:

$$0.2a < l < 0.5a \quad \text{and} \quad a f_{\mathrm{n}} < v < 3 a f_{\mathrm{n}} \, ,$$

where f_{n} is the natural frequency of the steady caster-wheel system in Hertz.

A relevant unstable island can exist for large caster length ($l > l_{\mathrm{cr}}$) around the towing speed domain

$$2 a f_{\mathrm{n}} \leq v \leq 4 a f_{\mathrm{n}} \, .$$

These results are detailed in Chapter 4 of the dissertation.
Related publications: [3, 5, 6, 9, 17, 20].

Thesis 4

The additional effects of elastic steering mechanism and shimmy damper were investigated. It was shown that the application of a torsional spring at the king pin can eliminate the double Hopf bifurcations and quasi-periodic self-excited oscillations can not occur. It was also shown that a fixed amount of viscous damping has a better effect on stability at the tyre than it has at the king pin.

It was proven for the model in question that even a pushed wheel of elastic tyre can exhibit linearly stable stationary rolling in a wide speed range if appropriate torsional stiffness and damping are installed at the king pin. It was also shown that the distributed damping of the tyre leads to towing speed dependent static (saddle-node) stability boundary even for towed wheels.

These results are detailed in Chapter 4 of the dissertation.
Related publications: [2, 15, 16].

Thesis 5

By means of the modification of a conveyor belt, a test rig was designed and constructed to carry out a series of experiments on the shimmy of towed wheels of elastic tyres. The most relevant theoretically predicted stability boundary and the corresponding vibration frequencies of the arising self-excited vibrations were identified during the experiments in a wide range of towing speeds for caster lengths less than the sum of the half contact length and the tyre relaxation length ($l < a + \sigma$).

The memory effect of the delayed tyre model was also validated by the modal testing of the towed wheel during rolling with various towing speeds. The impact experiments clearly identified three branches of frequencies belonging to three different pairs of complex characteristic roots with near-zero real parts for short caster ($l < a + \sigma$), while a single branch of frequencies was found for long caster ($l > a + \sigma$), which perfectly followed the predicted variation of a relevant pair of characteristic roots around the imaginary axis of the complex plane.

These results are detailed in Chapter 5 of the dissertation.
Related publications: [6, 10, 11, 18, 15].

Thesis 6

The nonlinear behaviour of the system with respect to the geometric nonlinearities and Coulomb friction force was analysed by numerical simulations. The partial sliding of the contact patch was modelled by the implementation of the governing equation of the damped stretched string tyre model into the simulation code as a phenomenon of fast time scale relative to the wheel oscillation of slow time scale. The experimentally detected transient and fully developed vibration signals were compared to these simulation results, and a good match was presented.

The memory effect of the delayed tyre model was confirmed also by the measured transient small-amplitude vibrations and the fully developed large-amplitude quasi-periodic ones. Waterfall diagrams were measured and simulated which explained the evolution of the peaks in the spectra as the nonlinear vibrations developed for parameters near to the theoretically predicted double Hopf bifurcation points of the stability charts. This validated our fast and slow time scale approximation in the improved mechanical model including partial sliding within the contact patch.

These results are detailed in Chapter 5 of the dissertation.

Related publications: [6, 8, 13, 21].

Thesis 7

Experimental procedure was developed to measure the increased temperature of the tyre trailing point and the power increment of the conveyor belt due to the possible increased rolling resistance of the towed wheel at certain speeds. With the help of these measurements, we proved the physical existence of the linearly unstable islands in the stability charts for long caster ($l > a + \sigma$) where the self-excited vibration has so small amplitude due to the microslips that it cannot be distinguished from the background noise in the system. This way we also proved that the memory effect in the delayed tyre model is relevant also for long caster at certain towing speed ranges where the shimmy motion itself is hidden but its effect is relevant with respect to increased tyre temperature, rolling resistance and wear, and possible decreased ABS efficiency if implemented.

These results are detailed in Chapter 5 of the dissertation.

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