CRACK-RELATED DAMAGE ASSESSMENT OF CONCRETE BEAMS USING FREQUENCY MEASUREMENTS

PhD dissertation

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GENERAL NOTATIONS

\( M \) \hspace{1cm} \text{mass matrix of the structure}
\( K, D \) \hspace{1cm} \text{stiffness matrix of the structure}
\( G \) \hspace{1cm} \text{flexibility matrix of the structure}
\( C \) \hspace{1cm} \text{damping matrix of the structure}
\( \Phi \) \hspace{1cm} \text{mode shape matrix}
\( k_N \) \hspace{1cm} \text{stiffness matrix of } N\text{-th element of the structure}
\( \varepsilon_N \) \hspace{1cm} \text{deformation matrix of } N\text{-th element of the structure}
\{F\} \hspace{1cm} \text{load vector}
\{u\} \hspace{1cm} \text{displacement vector}
\phi \hspace{1cm} \text{mode shape vector}
\omega, \lambda, f \hspace{1cm} \text{natural frequency}
\alpha \hspace{1cm} \text{acceleration or deflection}
\omega \hspace{1cm} \text{crack width}
W \hspace{1cm} \text{internal strain energy}
t \hspace{1cm} \text{time}
\rho \hspace{1cm} \text{curvature}
\varepsilon \hspace{1cm} \text{specific strain}
M \hspace{1cm} \text{bending moment}
A \hspace{1cm} \text{area}
1

INTRODUCTION
1.1 GENERAL

Civil engineering (CE) structures are built to fulfil one or more functional demand set by the owners on the basis of service obligations, private reasons or business considerations. The design of a civil engineering structure is determined by:

- technical,
- economic and
- aesthetic

requirements. Normally, the technical requirements are given by standards and other technical specifications. The appearance of the whole structure, the type of the load bearing structure and the applied materials (concrete, steel, timber, masonry, etc.) are chosen by the designers taking into account the above requirements. In many cases the structure which will be realized is selected in a design competition.

1.1.1 Design working life

CE structures are intended to fulfil their functional demands for a given period of time (design working life). According to experience, the structures can be kept in service economically at the intended technical level with the specified structural reliability (i.e. safety and serviceability) during this period. The envisaged design working life according to the Eurocode (EN 1990, 2002) for CE structures with different functions can be seen in Table 1.1.

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1.1.2 Deteriorating effects

During their working life, CE structures are exposed to various deteriorating effects that unfavourably influence their durability.

Under usual service conditions the expected service loads cause ageing of both the whole structure and the component materials. If these loads have a cyclic character then fatigue-type damage may occur depending on the sensitivity of the structure to fatigue.
Accidental actions due to human activities (e.g. collisions) or natural disasters (e.g. earthquakes), excluding the cases when their severity are too high and, due to this, the structure collapses, generally cause local damage to the load bearing structure. This local damage may lead to a local functional disorder, which - depending mainly on ductility and the structural type - may be the basis for damage, which gradually develops due to the effects of the normal service loads.

Aggressive environmental effects cause corrosion on the exposed parts of the structure which may result degradation in the material properties or - in more severe stages - directly influence the reliability of the structure. Depending on the aggressiveness of the environmental conditions, the Eurocode (*EN 206-1, 2000; EN 1992-1-1, 2004*) defines different exposure classes for concrete structures. For structures in each class, different values of parameters connected with the durability and different serviceability requirements have to be taken into account in the design. Nowadays, considerable number of papers and research projects (*COST, 2003*) deal with the influences of unfavourable ambient conditions on the durability of CE structures, especially for prestressed concrete structures (*Mutsuyoshi et al., 2004*).

According to the experience on concrete structures, the environmental attacks have a more significant effect on the durability than the usual service loads and the accidental actions. This relates especially to concrete highway bridges built in continental regions and exposed to frequently applied de-icing materials to maintain the function of the bridge in wintertime degrade the durability of reinforced concrete very quickly (*Andrade, 2003*).

### 1.1.3 Maintenance of CE structures

After a certain period of time in service and without any preventive measures and activities against the above deteriorating effects, the structural reliability of a structure may drop below an acceptable level set by the relevant standards. At that time the existing durability deficiencies may turn into a structural problem. It means that from this time the necessary design requirements are no longer entirely fulfilled. To raise the reliability above the acceptable level again, generally the application of a cost-sensitive rehabilitation, repair or strengthening procedure is needed.

#### 1.1.3.1. Life-cycle cost analyses

A maintenance system is intended to keep the reliability (safety and serviceability) of structures between the designed and the acceptable technical level consuming the minimum possible costs throughout the full working life. This cost minimization can only
be achieved by the application of an effective resource management system that focuses straight on the life-cycle cost.

The life-cycle cost (i.e. total cost in Fig. 1.1) of a CE structure includes not only the construction cost invested during the construction period but all the other costs (such as maintenance cost, operating cost, cost of loss due to unserviceability, cost of loss due to local failures, etc.) arising during the usual service of the structure. Parts of the latter group of costs can be taken as approximately constant every year (foreseeable maintenance and operating costs) but the others highly depend on the applied risk, which can numerically be transformed into reliability of the structure (Fig. 1.1). Within the life-cycle cost management, all the above costs incurred during a given reference period, which is generally equal to the design working life, are summarized to get the total cost function of the structure. The minimum of this function in the cost-reliability diagram according to Fig. 1.1 gives the optimal reliability, which the design of the structure should be based on.

This indicates that:

- how crucial the life-cycle cost analysis is when managing highly important and expensive CE structures in long term, and
- the applied maintenance system should be in accordance with that assumed in the life-cycle cost analysis (see Sect. 1.2).

This life-cycle cost analysis has been successfully applied to road design in the USA to compare alternative concrete pavement designs (Waalkes, 2003). This was clearly used as an economic tool that determined which alternative had the best value. This analysis is much more difficult for complex CE structures due to the considerably higher number of variables that influence the performance of the structure compared to
that for pavements but an appropriately simplified, life-cycle cost analysing procedure should be an achievable goal for bridge managing companies in the near future as well.

1.1.3.2 Maintenance program and need for condition control
Following that a previous (possibly life-cycle cost) analysis declared the necessary extent of maintenance in the life-cycle resource management system, a maintenance program has to be elaborated, which contains the necessary maintenance strategies, measures, tools and activities to be applied in practice.

The amount of cost to be spent annually during the maintenance of a CE structure (constant annual maintenance cost) mainly depends on the age and – in close relation to this - on the general condition (information on the degree and severity of deterioration-induced damage, the current level of reliability compared to the acceptable level, etc.) of the structure. In order to be able to rationally plan a maintenance program and, as part of this, to minimize the maintenance cost for the future a database is needed which contains up-to-date information on the general condition of structures to be kept in service.

For this reason, in 1998, the Hungarian concrete highway bridges were classified into the following five categories depending on their level of deterioration (Farkas, 1998) (Fig. 1.2). The classification was based solely on visual inspections.

- Class 1: No visible deterioration, good general condition
- Class 2: Initial deterioration (small, local damage only)
- Class 3: Medium deterioration
- Class 4: Considerable deterioration (insufficient reliability caused by advanced corrosion)
- Class 5: Dangerous condition (structural stability problems).

This situation has been favourably changed a bit after the large extension of the national motorway network in the last few years. Since 2003 the number of motorway and highway bridges operated by the State Motorway Management Co. and the Hungarian Roads Management Co. increased by approximately 10% (Bedics et al., 2008). Yet, the proportion of bridges belonging to Classes 2-5 is higher than 50% in 2010. The structural reliability of these bridges is lower than required, therefore in these cases effective rehabilitation is expected in the near future.
A similar program is running in the USA (Chase and Laman, 2000). The Federal Highway Administration (FHA) controls 473,594 highway bridges on the territory of the USA, of which 99,912 (21%) bridges belong to the National Highway System (NHS) which carries about 60% of all traffic and about 80% of all truck traffic. In 2000, the FHA carried out a thorough survey of all the highway bridges based mainly on visual inspections. According to the results, 24% of bridges outside the NHS and 28% of bridges on the NHS were structurally deficient. There were five possible reasons for a bridge to be classified as structurally deficient (in the sequence of occurrence regarding the total number of bridges):

- structural evaluation rating of 2 or less (in a 5 grade scale, 5 is the best value)
- bad substructure
- bad deck
- bad superstructure
- waterway appraisal rating of 2 or less (in a 5 grade scale, 5 is the best value).

For structurally deficient bridges on the NHS, the most frequently found reasons were: bad deck and bad substructure. Although only 6% of structurally deficient bridges belong to the NHS, 43% of the structurally deficient deck area (over 88% of them are made of concrete) is on the NHS. As a final solution to reduce the number of structurally deficient bridges, currently the USA replaces these bridges at a rate of about 5,600 per year but the strategic goal established by the FHA is 7,000 per year.

According to the current situation and its extrapolation into the near future in Japan (Fig. 1.3), the number of highway bridges older than 50 years will reach half of all highway bridges by 2030.
All the above national bridge condition evaluation programs conclude the need to develop and to apply condition control systems, which are able to record and to evaluate structural conditions before deterioration results in visible symptoms, and to provide rapid, reliable and quantitative measurement of bridge performance in practice.

1.2 CONDITION CONTROL OF CIVIL ENGINEERING STRUCTURES

Engineers need to know the actual causes and the possible consequences of sudden damage or continuous deterioration due to long term environmental and aging processes on the structural reliability of a CE structure in order to make meaningful and cost-effective management decisions (Enright and Frangpool, 2000). Early detection and localization of structural defects allows the owners to plan conveniently programmed repair and renovation works. Therefore a condition control system, which covers:

- official measures
- scheduled investigation programs, including
  - the type and frequency of inspections/monitoring
  - the components of structures to be investigated
  - the performance criteria to be fulfilled
  - the rules for documentation of the results
- measures to be done if performance criteria are not fulfilled

and is able to quickly mobilize condition evaluation procedures, is an essential need for owners to maintain CE structures in recent days. The recently published fib Model Code for Service Life Design (fib, 2006) proposed condition control levels according to Table 1.2 in order to keep the appropriate level of reliability for CE structures during their service life.
**Table 1.2** Condition Control Levels in the fib Model Code for Service Life Design (fib, 2006)

<table>
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<tr>
<th>Condition Control Levels</th>
<th>Characteristics</th>
<th>Examples</th>
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<td>CCL3</td>
<td>Extended inspection</td>
<td>Systematic inspection and monitoring of relevant parameters for the deterioration process(es) that is (are) critical</td>
</tr>
<tr>
<td>CCL2</td>
<td>Normal inspection</td>
<td>Regular visual inspection by qualified personnel</td>
</tr>
<tr>
<td>CCL1</td>
<td>Normal inspection</td>
<td>No systematic monitoring nor inspection</td>
</tr>
<tr>
<td>CCL0</td>
<td>No inspection</td>
<td>No possible inspection, for instance due to lack of access</td>
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Sect. A1 summarizes the main aspects of the existing condition control systems applied to CE structures then provides background to their comparison.

Condition control systems are mainly intended to identify damage. A full damage identification procedure consists of four levels; all levels include the previous level, as follows (Rytter, 1993; Doebling et al., 1996):

- **Level 1:** *Detection*: Determination that damage is present somewhere in the structure
- **Level 2:** *Localization*: Determination of the geometric location of the damage in the structure
- **Level 3:** *Quantification* of the severity of the damage
- **Level 4:** *Prediction* of the remaining service life of the structure.

To perform Level 1, suitable diagnostic techniques (see Sect. A1.2) are needed as a minimum. To perform Level 2 and especially Level 3, in addition to suitable diagnostic techniques, the key issue is the experience of the working staff.

At this point, a clear distinction has to be made between the concepts of “damage identification” and “damage assessment”. In this context, damage assessment means an evaluation process (method), which provides information on the structural performance of a CE structure based on the analysis of the global response data. Therefore, damage identification should be considered as a precondition for damage assessment because it covers activities, which focus mainly on the first three of the above levels (Level 1-3).

Using these definitions it becomes obvious that the realisation of Level 4 is only possible if a damage assessment of the structure is made first in order to obtain information on the current performance of the structure. Then, in the possession of the structural performance data, the evaluation of the long-term behaviour (service life prediction) of the damaged structure can be performed on a reliability basis and, if necessary, by fatigue life analysis.
Consequently, the above levels of a full damage identification procedure should be extended by a new intermediate level, which is a precondition to perform Level 4 and called Level 3a, which aims to provide information on the performance of the whole structure by using a damage assessment method as follows:

- Level 3a: *Evaluation* of the structural performance based on damage assessment.

In this thesis, mainly the proposed Level 3a will be addressed. Sect. A1.3 discusses the important aspects of damage assessment.

### 1.3 VIBRATION-BASED DAMAGE IDENTIFICATION AND ASSESSMENT FOR CE STRUCTURES - AN OVERVIEW

This section gives an overview and deals only with general considerations, while the state-of-the-art of the vibration-based damage identification methods based on a detailed literature review can be found in Sect. 1.4.

#### 1.3.1. History

The engine of the development of vibration-based damage identification methods in civil engineering fields were, as usual in any technical area, spectacular failures resulting in loss of human lives (catastrophes), economic concerns (cost-effective maintenance) and technical innovations (computer-based measuring and evaluating tools).

The idea of dynamic-based damage identification for special CE structures arose when the oil industry started to pay considerable attention to the assessment of offshore structures (oil platforms and light stations). The first achievements were introduced at the Offshore Technology Conferences held in the USA in the 1970s. Later, from the early 1980s, these techniques and methods have been applied to aerospace structures (space shuttles and robots). Thanks to the promising first results achieved in this field and the catastrophic bridge collapses (Tacoma Narrows Bridge, 1940, Silver Bridge at Point Pleasant, 1967), the application of these methods for bridges considerably increased from the 1980s. Nowadays vibration-based damage identification is mainly used as part of a monitoring program for bridges. Additionally, vibration-based damage identification has also been applied to many types of CE structures such as simple steel, concrete, composite elements of prefabricated structures as well as frames, entire buildings, dams, tanks, vessels, pipes, etc. (*Doebling et al., 1996, 1998*). It is also frequently used for experimental purposes.
1.3.2 Vibration-based damage identification methods

The basic idea of these methods is that modal characteristics (natural frequencies, mode shapes, modal damping) are functions of the physical (mass, stiffness, internal damping) as well as structural (geometry, support conditions) properties of the structure. Therefore any change in the physical and/or structural properties induces a change in the modal characteristics, which can be detected by suitable diagnostic techniques.

The vibration-based damage identification methods can be classified by different aspects as follows:

a) Damage identification level
As introduced in Sect. 1.2, Level 1,2,3 and the proposed Level 3a are the central fields of damage identification for CE structures. However, it is impossible to define exact limits between these levels and to list methods that concentrate on only one of them. Methods dealing with damage identification generally address more of the above levels but their applicability and efficiency on each level is different. Therefore, the governing rule of the categorization of damage identification methods given in Sect. 1.4 is not the level which is addressed, but the dynamic characteristic which is measured and/or analysed.

b) Linear versus non-linear methods
In Sect. 1.4, first of all linear damage identification methods (see Sect. A1.3.1) which focuses on deducing change in the mechanical and the structural properties of a structure from the change in its modal properties will be discussed. Non-linear methods identifying and assessing non-linear structural response and non-parametric models will only be summarized in Sect. 1.4.4 and Sect.A3.4.

c) Model-based versus non-model-based methods
The damage assessment procedure may be either model-based or non-model-based. Model-based methods are supplemented by previously adjusted, robust numerical models and the identification is based on the comparison between the numerically calculated and the measured modal quantity. The practical applicability of these model-based methods is limited because previous tests are needed to properly adjust the numerical model and considerable amount of time-consuming numerical computation has to be carried out. Since either the available finite element (FE) softwares are only capable of linear-elastic processing or the use of their built-up non-linear modules is time-consuming, the FE supported model-based methods are generally linear methods.
The non-model-based methods are fully based on modal data measured on-site and their comparison. For comparative purposes, numerical damage indices (such as MAC, COMAC, GI, LI, etc., see in Sect. 1.4) are often defined. This is favourable from a practical point of view, but the efforts made on on-site measurements and signal processing as well as on the extraction of the necessary modal parameters may also be significant. In order to be able to follow the changes in the measured modal data during the full service life of the structure, a baseline is needed for the comparison. This is generally the undamaged structure or, if it is not available, the results of a previous measurement. Due to their character, the non-model-based methods are capable of handling non-linear problems and are often referred to as experimental modal analysis.

1.3.3. Benefits and challenges

From the beginning, the practical reasons for the wide and rapid spread of vibration-based damage identification in the civil engineering fields were as follows:

- **Information content.** The dynamic response of a structure contains all information on the structure. If it is fully registered and a global assessment is made, the location of damage does not need to be known in advance.

- **Possibility for structural monitoring.** The dynamic characteristics are the functions of the physical properties (geometry, mass, stiffness, material law, boundary conditions). If one or more of them change due to, for instance, a possible damage, it results in a change in the dynamic characteristics. The reversing of the procedure makes the monitoring of structures theoretically possible.

- **Practical applicability.** The applicability of many diagnostic techniques is limited due to accessibility and because the installation of the measuring equipment is difficult (see Sect. A1.2). The dynamic response of a structure can be recorded at customized (but not optional) points, which allow the positions of the measuring sensors to be chosen to suit the test situation (to maximize the sum of the magnitudes of the mode shape vectors).

However, the big challenge, which has continuously existed from the beginning, was the sensitivity of dynamic characteristics to the following effects. These apply to all vibration-based methods which use global damage assessment.

- **Sensitivity to local damage.** Especially the lower modes, which can be measured easily in-situ, proved to be insensitive to local damage. To develop effective assessment methods and to define sensitive damage indices, sometimes complicated algorithms and postprocessing procedures have to be conducted.
Sensitivity to errors. Random errors may be sourced from inevitable noises in the recorded structural response (environmental effects, electrical disturbances, background vibration noises, imperfections in the data acquisition system, etc.). Signal processing errors come from the applied calculation algorithms. Alampalli (1997) and Messina et al. (1996) investigated the sensitivity of modal parameters (natural frequencies, modal damping ratios and mode shapes) to these errors by statistical analysis and found that especially the natural frequencies associated with lower modes were much less sensitive (the maximum coefficient of variation was smaller than 1%) than other modal parameters. Similar results have been obtained by Farrar et al. (1997) and Doebling et al. (1997b).

Sensitivity to environmental effects. Often the distinction between the possible reasons for a change in the damage index, whether it is caused by real damage or one (or more) of the ambient effects, is difficult.

Another type of sensitivity analysis was undertaken by Ebert (2001) on reinforced concrete beams and plates using a non-linear stochastic finite element model. He assumed stochastic material properties spatially distributed in the structure and studied the influence of randomly occurring local damage (considered by numerically simulated cracks) on the change in the modal properties.

1.4 VIBRATION-BASED DAMAGE IDENTIFICATION METHODS (STATE-OF-THE-ART)

As a governing rule, the following categorization focuses on the dynamic characteristic which is measured and/or analysed as a damage index. This can be either one or more of the basic modal parameters (natural frequencies, mode shapes, damping ratios) or their derivatives such as mode shape curvatures, strain energy, flexibility, etc. This review makes a distinction between the model-based and the non-model-based methods.

The state-of-the-art is mainly based on previous literature reviews of this field by Doebling et al. (1996, 1998), Salawu (1997a), Sohn et al. (2004) and Wenzel et. al (2003). In this section, only the essence of methods and the conclusions of a deep literature review on vibration-based damage identification are provided. The full review can be found in Sect. A3.
1.4.1 Damage identification based on changes in the basic modal parameters

In this section, methods which use natural frequencies, mode shapes and damping ratios as damage indicators are introduced and summarized.

1.4.1.1 Methods based on changes in the natural frequencies

In literature, the terms natural frequency, modal frequency and resonant frequency are often used. However, all three refer to the same item. The applied methods can be classified as model-based or non-model-based methods.

1.4.1.1.1 Model-based methods

*Doebling et al. (1996, 1998)* categorized the essences of these methods as solutions for the forward or the inverse problem.

**The forward problem**

The methods representing the forward problem generally compare numerically calculated natural frequencies associated with modelled damage to the corresponding measured natural frequencies. These methods often lead to iterative procedures and are usually used for damage detection (Level 1) and sometimes for approximate damage localization (Level 2).

**The inverse problem**

The inverse problem addresses Level 2 and partly Level 3. These methods calculate the damage parameters (mainly the damage location and sometimes its severity) from the measured natural frequency shifts by using so called “prediction” functions describing how local damage influence the natural frequencies (or their shifts) of the whole structure. The essence of the difference between the methods solving the forward and the inverse problem is the application of these prediction functions by the latter methods.

1.4.1.1.2 Non-model-based methods

Some authors investigated the influence of structural damage on the natural frequencies only experimentally and tried to establish quantitative relationships between the severity of damage and the change in the natural frequencies.

1.4.1.2 Methods based on changes in the mode shapes and in their derivatives

Mode shapes are generally measured to provide spatial information on the dynamic behaviour of the structure, therefore most often they are used together with natural frequency measurements. Their parallel use allows more effective damage localization (Level 2) compared to methods based only on natural frequency measurements.
One part of the methods which analyse mode shape changes use direct displacement amplitude measurements (displacement mode shapes) while others focus on one or more derivatives of the mode shapes (slope, curvature, strain energy), which are either indirectly calculated from the measured displacement mode shapes or directly (experimentally) determined from e.g. strain measurements.

1.4.1.2.1 Methods using displacement mode shapes

These methods can also be categorized as model-based and non-model based methods.

Model-based methods

Model-based methods investigate the effects of simulated damage on an analytical or a numerical model. The damage identification (detection, localization and sometimes quantification) is based on the comparison of the derived model parameters (e.g. amplitudes, slopes) with the corresponding measured values.

Non-model-based methods

The non-model-based methods are generally based on modal data correlation by focusing on the graphical or numerical, direct comparison of the corresponding mode shapes, which have been experimentally measured in different deterioration states of the investigated structure (one of them is often the undamaged state used as a baseline).

The most commonly applied procedures to compare two sets of vibration mode shapes are the calculation of the Modal Assurance Criterion (MAC) and the Coordinate Modal Assurance Criterion (COMAC). MAC and COMAC are widely used as global damage indicators (damage indices).

Naturally, MAC and COMAC values are able to compare any set of mode shape data such as measured/measured, calculated/measured or calculated/calculated. However, the biggest advantage of MAC and COMAC is their effective applicability as a non-model based damage indicator to measured/measured data sets without any prior analytical or numerical baseline model. Many authors applied MAC and COMAC to compare corresponding mode shapes measured before and after an assumed (in real situations) or an artificially produced (under laboratory conditions) damage event.

1.4.1.2.2 Methods based on mode shape curvature

For realistic damage in CE structures subjected mainly to flexure, the mode shape displacements change in a very small degree compared to the initial, undamaged state which makes the application of methods based simply on displacement mode shape measurements difficult. To improve their sensitivity to damage, a derivative such as the...
curvature of a mode shape (mode shape curvature) may alternatively be used instead of displacement mode shapes for structures subjected mainly to flexure. The expected improved sensitivity and the ability of these methods for more refined damage identification is based on the following assumptions:

- The damage in a structure under bending \( (M) \) is tightly accompanied by the reduction in the bending stiffness \( (EI) \), which is in direct relation to the curvature \( (M = \rho EI) \). The curvature can be deduced from displacement mode shapes or directly measured on the structure (Level 1).
- Under constant bending moment, the curvature of a cross-section is fully determined by the bending stiffness of this cross section \( (\rho = M/(EI)) \) but there is no direct relationship between the curvature and the displacement of the same cross section. Therefore the curvature is suitable for damage localization (Level 2).
- The size of damage is related to the magnitude of reduction in the bending stiffness which gives the possibility for approximate damage quantification (Level 3).

The curvature \( (\rho) \) at each point of the mode shape can be either calculated by an appropriate formula from the displacement mode shape or directly determined from strain measurements using the following relationship between strain and curvature for beams under flexure:

\[
\varepsilon = \rho y 
\]

(1.1)

where \( \varepsilon \) is the directly measured strain (mostly on the surface of the structure) and \( y \) is the distance between the strain measurement point and the neutral axis.

1.4.1.2.3 Methods based on strain energy

These methods are very similar to, and so, can be considered as an improvement of methods using mode shape curvatures. The main difference is the quantity which the applied damage index is based on. For this purpose, the strain energy defined on the basis of the curvature of the investigated mode shape is applied by these methods.

1.4.1.3 Methods based on changes in damping parameters

The most commonly observed damping behaviour of vibrating civil engineering structures mainly originates from the internal friction of structural materials, which results in continuous energy dissipation during the motion of the structure. This energy dissipation is even more intensive in the presence of cracks that open and close during vibration. This either consumes the input energy given by the exciting effect for structures under forced vibration or gradually decreases the amplitudes of the free-vibrating structures. In experimental works, the degree of damping is generally measured by the mo-
dal damping ratio or by the logarithmic decrement of damping. In literature, the damping models are not exactly elaborated at this stage and, due to this, many modelling issues are unanswered and many results are contradictory. Notwithstanding, significant attempts have been made to use the damping characteristics, mainly as an addition to other damage indices, for damage assessment purposes of civil engineering structures (Stubbs and Osegueda, 1990a, 1990b; Brincker et al., 1995b).

1.4.2 Damage identification methods based on dynamically measured flexibility

Flexibility-based damage identification methods use the flexibility matrix for the characterization of the changes in the structural behaviour. The flexibility matrix \( [G] \) defined as the inverse of the stiffness matrix relates the applied static force \( \{F\} \) to the resulting displacement \( \{u\} \) of the structure as follows:

\[
\{u\} = [G] \{F\}
\]  
(1.2)

It means that the \( i \)-th column of the flexibility matrix represents the displacement pattern of the structure subjected to a unit force applied at the \( i \)-th of the degrees of freedom (DOF). Hence, the flexibility matrix can be estimated from a modal test using only on-site data such as the mass-normalized, measured mode shapes \( \{\psi_i\} \) and the associated natural frequencies \( \lambda_i \) as follows:

\[
[G] = [\Phi] \begin{bmatrix} \lambda_1 & \cdots & \lambda_n \end{bmatrix}^{-1} [\Phi]^T = \sum_{i=1}^{n} \frac{1}{\lambda_i^2} \{\psi_i\} \{\psi_i\}^T
\]  
(1.3)

where \([\Phi]\) is the mode shape matrix, \( [\Lambda] = \text{diag}(\lambda_i^2) \) is a diagonal matrix containing the natural frequency squares and \( n \) is the total number of DOF. Although the in-situ measured mode shapes are not scaled, there are postprocessing techniques to unit mass-normalize them \( ([\Phi]^T [M] [\Phi] = [I], \text{ where } [M] \text{ and } [I] \text{ are the mass and the unity matrices, respectively}) \) computationally.

For practical applications, damage is generally identified by the comparison of flexibility matrices synthesized from mode shapes measured on the damaged structure with that either measured on the undamaged (or a previous baseline) structure or calculated from a FE model. The synthesis of the complete flexibility matrix from on-site measured mode shapes is unrealistic due to the fact that only the first few (lowest-frequency) modes are measured, hence, the \([G]\) matrix computed according to \( Eq. (1.3) \) remains only an estimation. However, because of the inverse relationship to the square of natural frequencies, the flexibility matrix is most sensitive to changes in these lowest-frequency modes, which ensures quick convergence when computing it from a limited
number of (low-frequency) modes. This consequently allows handling the flexibility matrix as a reliable damage index for such non-model-based methods.

1.4.3 Matrix update methods
Matrix update methods for damage identification are based on fully numerical, mostly finite element (FE) models. The damaged state of a structure is determined by modifying (updating) the typical structural properties, such as stiffness, mass, and/or damping properties (matrices), in the FE model to minimize the differences in the modal behaviour between the FE model and the damaged structure. If these methods are used for damage identification, the process starts from the FE model of the undamaged (or previously damaged and appropriately identified) structure, which perfectly fits its modal behaviour. Then, its update has to be based on the measured modal data of the damaged structure. The presence, the location and the severity of damage are indicated by the place and the magnitude of changes made in the FE model parameters during the update process.

1.4.3.1. Mathematical background
The update process mathematically is an optimization problem of the structural equation of motion: starting from the nominal FE model of the undamaged structure and iteratively transforming it into the FE model of the damaged structure represented by the measured modal data.

1.4.3.2. Differentiation of methods
The elaborated matrix update methods differ in the mathematical procedures. The optimal matrix update methods, the sensitivity-based update methods, the eigenstructure assignment methods and hybrid methods were applied for identification purposes.

1.4.4 Non-linear damage identification methods
The appearance of the non-linear modal behaviour for civil engineering structures was attempted to be identified in structures with opening and closing cracks. The presence of such cracks is typical and expected for operating concrete structures but generally unwelcome and may be the result of local damage for structures made of steel and other usual materials. The reasons for non-linearities are detailed in Sect. A1.3.1. Two main issues are analysed in literature. The first is the intensity of the modal test. Many authors observed that the best way to identify non-linearities in a structure is to
carry out modal tests in different response levels and to evaluate the responses appropriately.

The second issue is the applied evaluation procedure with the main focus on the signal-processing technique. Many techniques are investigated and their sensitivity on different non-linearities is pointed out.

1.4.5 Conclusions

Regarding the considerations listed in Sect. 1.3.3, the achievements of vibration-based damage identification methods published in literature can be concluded as follows:

A) Information content

[A1] Methods based on changes in the basic modal parameters

a) Despite their global nature, the natural frequencies themselves do not provide exact spatial information about local damage with the exception of high natural frequencies, which are associated with local, damage-induced responses.

b) If extending the natural frequency measurements by parallel mode shape recording, theoretically the full spatial information about local damage can be gained. The higher the number of recorded modes the more complete the information content of the dynamic response of the structure is.

[A2] Methods based on dynamically measured flexibility, matrix update methods, non-linear methods

Each of these methods uses the on-site measured natural frequency and associated mode shape data as input parameters, therefore, the information content of these methods and their improvement is consequently the same as for methods referred to above in [A1]/b).

B) Practical applicability

[B1] Methods based on changes in the basic modal parameters

a) The lower natural frequencies can be measured with high confidence (with low coefficient of variation), therefore they are best suited for field tests. The use of established relationships between damage location and natural frequency shifts theoretically allows limited damage localization in easy circumstances (Level 1 and limited Level 2).

b) The technical difficulties in installing measurement points and/or the narrow frequency range of the applied excitation often limit the possibility for measuring a "sufficient" number of high modes (natural frequencies together with associated mode shapes). For limited number of measured modes, and especially in the ab-
sence of high modes, the limited information content generally does not allow exact damage localization and quantification even in the possession of measured mode shapes.

c) The *model-based methods* require analytically or numerically computed sensitivity values obtained from an appropriately adjusted model. The considerable amount of computation needed to model all possible damage events may be performed for simple laboratory tests but is unrealistic for real structures. Moreover, the accuracy of these methods highly depends on the quality of the numerical model. Therefore, the practical applicability of these methods is highly limited. This applies to methods focusing simply on natural frequency shifts as well as to those analysing natural frequency and associated mode shape changes simultaneously.

However, some *non-model-based methods* do not require analytical or numerical models but use only test data from the measured modes and are based on simple assumptions about the structural behaviour. This is favourable from a practical point of view but the effort made on the on-site measurements and the on-site-recorded signal processing as well as on the extraction of the necessary modal parameters may also be significant. For these methods a baseline is needed for the comparison-based evaluation of results, which can be either the undamaged structure or the results of a previous investigation. The defined numerical damage indices (*MAC*, *COMAC*, *GI*, *LI* and *MSECR*) proved to be efficient in practical cases and made possible damage localization (Level 2).

d) The *damping* models are not exactly elaborated at the present stage and, due to this, many modelling issues are unanswered and many results are contradictory, therefore, these models are not commonly used in practice.

[B2] Methods based on dynamically measured flexibility, matrix update methods, non-linear methods

The *flexibility methods* can be taken as fully non-model-based methods and are practical for intermittently conducted routine surveys on structures without any known, present damage and with an appropriately documented previous stage as a baseline, because quick convergence can be reached when compiling the flexibility matrix from on-site measured low-frequency modes.

The *matrix update methods* answer the forward problem (*Sect. 1.4.1.1.1*) by using a robust mathematical background (matrix iteration), therefore, they require considerable computational capacity, which greatly limits their practical applicability.
The non-linear methods use similar mathematical means as the matrix update methods, therefore their applicability in practice is also limited.

C) Sensitivity

[C1] Methods based on changes in the basic modal parameters

Each mode has different sensitivity to damage because a frequency shift or a mode shape change depends on the nature, the location and the severity of the damage. The insensitivity of lower modes is generally due to their global nature. For damage located in low stress regions (close to nodes) of the considered mode, the associated frequency shift or mode shape change will also be insensitive. However, due to the increased information content on the dynamic response, the sensitivity of methods focusing on simultaneous natural frequency and associated mode shape measurements is generally higher than that for methods based simply on natural frequency measurements.

[C2] Methods based on dynamically measured flexibility, matrix update methods, non-linear methods

Because of the inverse relationship to the square of natural frequencies, the flexibility matrix, which is directly used by the flexibility methods, is most sensitive to changes in these lowest-frequency modes.

In addition to the amount and the quality of input data, which is tightly determined by the number and the accuracy of measured modes of the investigated structure, the sensitivity of matrix update methods and non-linear methods is highly affected by the applied mathematical procedure.
FOCUS OF THE THESIS
2.1 CONCEPTION OF THE RESEARCH

The primary motivation of the research covered by this thesis was already addressed in Sect. 1.2. A literature review of vibration-based damage identification methods (Sect. A3) pointed out that the existing methods do not really address such issues, which are particularly important from the owner’s, the maintainer’s and the designer’s point of view. These issues are related to structural safety/reliability and durability and are well-founded input for further decisions regarding the necessity of possible structural strengthening or the estimation of the remaining service life. The implemented methods focus only on the first three levels of damage identification (detection, localization and maybe quantification) set by Rytter (1993), but Level 4 as well as the related safety and durability concerns remain untouched. The need for assessment methods, which are able not only to identify damage by dynamic methods but to provide information on the structural performance, led to the proposal of Level 3a as a new interim level after Level 3 in the above damage identification system defined in Sect. 1.2. The global purpose of this research was to fulfil Level 3a of crack-related damage assessment and to carry it out on concrete beams under experimental conditions.

Before defining what tasks are necessary to achieve experimental damage identification and assessment, some issues, which are closely touched by this research, have to be clarified.

2.1.1 Differentiation of model-based and non-model-based methods

Many of the existing procedures are supplemented by previously adjusted, robust numerical models. Damage identification is often based on the comparison of numerically calculated and measured values of the investigated modal quantity. The practical applicability of these “model-based methods” is limited because previous tests are needed to properly adjust the numerical model (i.e. to eliminate the model uncertainties) and considerable amount of time-consuming numerical computation has to be carried out.

The “non-model-based methods” are fully based on on-site-measured modal data and their comparison. This is carried out by defining numerical damage indices (MAC, CO-MAC, GI and LI, flexibility matrix, etc.) directly on the basis of the measured data. This is favourable from a practical point of view but the effort made on on-site measurements and on on-site-recorded signal processing as well as on the extraction of the necessary modal parameters may also be significant. For monitoring purposes (i.e. to
follow the changes in the measured modal data during the service life of the structure) a baseline, which is generally the undamaged structure, or, if it is not available, the results of a previous measurement is needed for the comparison.

### 2.1.2 Model uncertainties in testing concrete structures

*Fig. 2.1* shows the scheme of static and dynamic tests regularly made on concrete bridges recently. On-site investigations are usually complemented by analytical or numerical analyses. They are fully based on *assumptions* regarding the load, the geometry, the material properties, the material law and the structural behaviour, which result in *model uncertainties*. Consequently, the results of these analytical and numerical models include the effects of model uncertainties.

![Scheme of static and dynamic tests for concrete bridges](image)

The outcome of a typical static test is generally a comparison of the calculated (model-based) and the measured (non-model-based) values of the investigated quantity, which, due to the often limited accessibility of the structure, is generally a deformation or a cracking parameter. If the difference does not exceed an acceptable level, which is intended to consider model uncertainties, the test is declared successful.
When carrying out dynamic modelling (i.e. modal analysis), an additional structural behaviour model, which appropriately characterizes the structure under vibration, and therefore, is not included in the static model, has to be adopted. Typically for concrete bridges, this model describes the behaviour of active (opening and closing) cracks under vibration. For long-term investigations, the degradation of structural parameters due to cyclic effects may also be necessary to consider in the additional structural behaviour model. When executing on-site dynamic tests (signal recording + processing) and trying to compare the calculated (model-based) dynamic parameters with the measured (non-model-based) ones, the uncertainties in the additional structural behaviour model also work against their exact coincidence.

### 2.1.3 Reducing or eliminating the effects of model uncertainties

Due to the above model uncertainties, it proved to be difficult to find a close coincidence between numerically calculated and corresponding on-site measured dynamic properties for real structures (Rizos et al. (1990), Salawu and Williams (1994), Farrar and Jauregui (1998), etc.). The effectiveness of such tests can be improved considerably if:

1. The effect of model uncertainties is reduced or eliminated for the model-based damage identification procedures;
2. Non-model-based damage identification procedures are developed and used.

The model uncertainties are fully eliminated if relationships between measured (non-model-based) static characteristics and measured (non-model-based) dynamic parameters are quantitatively described (link a1 in Fig. 2.1). The description of relationships between measured (non-model-based) dynamic parameters and calculated (model-based) static characteristics is helpful in reducing the number, and consequently, the effect of model uncertainties. Calculated static characteristics are obtained from static models, therefore their applicability has to be verified by comparison to their corresponding measured (non-model-based) values through conformity checks (link a2 in Fig. 2.1).

### 2.1.4 Explaining the omission of dynamic models

The omission of dynamic models from damage identification was due to the intention to ignore model uncertainties, which come when modelling the dynamic behaviour of damaged concrete beams. Sect. A1.3 introduces how difficult it is to take into account all parameters which influence the dynamic behaviour of a concrete structure in the
dynamic model. Sect. A1.3.1 discusses that especially the crack-related behaviour of concrete beams often leads to non-linearity, thus, such behaviour can only be described by non-linear (numerical, FEM) models. The applicability of analytical models in these cases is practically impossible. Sect. A3 introduces the extent of computational effort necessary to develop and run non-linear dynamic models. Their adjustment to the actual structural response is complicated. Sect. 3.4 verifies how difficult it is to read out the structural behaviour-related effects from the electronically recorded vibration signals.

2.1.5 Applied method of work to fulfil Level 3a damage assessment

Due to reasons given in Sect. 2.1.4, no dynamic model was developed and, consequently, the structural behaviour of test beams was described by static characteristics calculated from a static model. Referring to Sect. 2.1.3, static-based damage indices were defined for identification purposes and structural performance-related static characteristics were defined to reflect on the structural performance of the tested specimens. Hence, the establishment of any relationship between the measured dynamic properties and the structural performance-related characteristics fulfilled Level 3a damage assessment according to Sect. 2.1.

2.2 GOALS OF THE RESEARCH

To address Level 3a damage assessment under experimental conditions, the following were the goals of this research:

a) Modelling crack-related damage under experimental conditions on various types of concrete beams;

b) Elaborating signal processing tools to process numerically-recorded vibration data and then to provide a statistical basis for the determined dynamic parameters (here the first two natural frequencies). Development of appropriate frequency evaluation techniques to reflect on different damage cases.

c) Definition of damage indices, which are able to identify crack-related damage as well as to measure its extent (identification).

d) Establishing relationships between the measured dynamic parameters and the related structural performance-characterizing properties (assessment).

e) Application of the method (or one of its part) on a real structure.
Although the intended research is not material related, the work in this thesis fully focused on structures made of concrete and does not aim to deal with any other issues related to other materials.

2.3 DESCRIPTION OF THE RESEARCH

Laboratory tests were conducted to thoroughly describe the crack-related degradation of concrete model beams by structural performance-related parameters as well as by basic modal data. For basic modal data, the first two natural frequencies were determined. Crack-related damage of beams was identified by appropriately defined, static-based damage indices. Some of these indices (non-model-based indices) were based simply on experimentally registered (directly-measured) parameters (e.g. crack pattern) and the others (model-based indices) were obtained by calculations. The assessment of the structural performance of the test beams included the establishment of relationships between the basic modal data and the structural performance-related parameters, their quantitative description and their conclusion. The tests have been carried out at the Budapest University of Technology and Economics, Dept. of Structural Engineering. The practical applicability of frequency measurements was verified by a field test executed on an existing concrete highway bridge.

2.3.1 Test variables

Deterioration of concrete beams due to either aging, environmental attack or mechanical effects generally results in cracking. Crack-related damage identification and assessment carried out on such beams for various reasons often focuses on the determination of the extent of cracking. Cracking is not only a visible sign of deterioration but a systematic pattern of individual cracks and, consequently, a structural characteristic tightly influenced by many performance-related structural parameters. For concrete beams under flexure, the most important of these parameters are the properties of bond between reinforcement and concrete, the amount of bending reinforcement as well as the presence, the bond-related behaviour (bonded or unbonded) and the intensity of prestressing.

Therefore, while using cracking to model the deterioration process, the introduced test was aimed at investigating the influence of the above test variables on the extent of cracking and, consequently, on the relationship between basic modal data and the structural performance-related parameters.
2.3.2 Focuses of the chapters

Chapter 3 focuses on tasks a) and b) of Sect. 2.2 by introducing the conducted laboratory tests. Detailed descriptions of test beams, the execution of the artificial deterioration process, the measured and calculated structural performance-related parameters, the applied excitation techniques, the dynamic measurements, the signal processing as well as the procedure of signal evaluation will be given.

Chapter 4 deals with task c) of Sect. 2.2 by defining, determining and classifying static-based damage indices used for crack-related damage identification purpose.

Chapter 5 focuses on tasks d) of Sect. 2.2 by establishing relationships between the first two natural frequencies and the structural performance-related parameters for assessment purpose.

Chapter 6 demonstrates the practical applicability of frequency measurements with a simple dynamic test carried out on an existing concrete highway bridge.

Chapter 7 summarizes the new scientific results of this work.
EXPERIMENTAL DETERIORATION PROCESS
3.1 INTRODUCTION
For experimental purposes, simply supported, reinforced, prestressed and post-tensioned reinforced concrete beams were tested under flexure. No primarily shear-related behaviour was considered and investigated, therefore all test set-ups were chosen accordingly.

3.2 GENERAL PROCEDURE
All of the test beams first underwent an artificially-produced deterioration process in several steps. This process was chosen depending on the beam type and, in order to investigate minor aspects, was slightly modified for a few specimens within the beam family of the same type. Then for a few of the reinforced (reinforced concrete) beams, external post-tensioning with gradually increasing intensity as a simulated strengthening effect was applied in three steps. For the prestressed beams, no post-tensioning was applied after the deterioration process. In the following, each of the previous deterioration and strengthening steps will be mentioned simply as state and marked by natural numbers (and by their variations when the general process was slightly modified) starting from 1 to 5 (7 for P2/1 & P2/2, see Table 3.6) for the deterioration period and from 6p to 8p (see Table 3.4) for the subsequent strengthening period.

The detailed introduction of the test set-up and the description of the process can be found in Sect. 3.3.2. The deduction of damage indices on the basis of parameters characterizing the cracking state of the beam and the computation of modal data from the registered signals will be detailed in Sect. 3.4.3.

3.2.1 Reinforced concrete (non-prestressed) beams
For the reinforced concrete (non-prestressed) beams (R1, R2, P1, P2 in Fig. 3.1) each state in the deterioration period (from 1 to 5 (7 for P2/1 & P2/2)) included a loading phase, during which the intended cracking state of the beam was first produced then the loading was completely removed, and a following measuring phase, during which dynamic measurements resulting in the investigated modal data were carried out. In the strengthening period (from 6p to 8p) the system of the applied post-tensioning was identical; only the intensity of the post-tensioning force increased gradually state by state but was constant in each state. No removal or decreasing of the post-tensioning force took place and no external load was applied to the beams between the consecutive states in the strengthening period.
3.2.2 Prestressed beams
For the prestressed beams (P2p in Fig. 3.1) no strengthening state was applied, furthermore, the loading phases in the deterioration period were significantly modified in comparison to that for the reinforced concrete beams. Similarly to prestressed beams used in practice, the cracking and decompression moments of the tested beams were significantly increased by the applied prestressing therefore very small degree of cracking and consequently very small crack-induced shifts in modal parameters could be expected at practical load levels (before the design moment capacity is reached). In fact, the primary deterioration effect for this kind of beam in practice used to be the breaking of prestressing tendons, which leads to decrease in the prestressing action and consequently allows more intensive cracking and external load-induced deformations. For this reason the intended deterioration process for these beams in the test mainly focused on the investigation of shifts in modal parameters due to gradual tendon breaks. For this test, only bonded tendons (in form of wires) were applied, therefore the tendon breaks were simulated by simply, mechanically cutting wires through the concrete cover at many sections along the beam length state by state. After the prestressing effect had been decreased to some extent, loading phases as part of the deterioration period and similar to that for the reinforced concrete beams were also applied.

3.3 DESCRIPTION OF THE TEST
The properties of the test beams, the applied prestressing and post-tensioning system, the test set-up and the procedure of the test will be detailed.

3.3.1 Test beams
The prototypes of the test beams are widely used in practice as precast, prestressed concrete floor beams in buildings with clear spans ranging between 2.4 m and 6.6 m (type P2p in Fig. 3.1 can be considered as an original product). However, in accordance with the aim of the present test, the test beams were manufactured with the same geometry but with different type (reinforcing bar or prestressing wire) and amount of reinforcement compared to the original product. Other investigated parameters were the effect of prestressing as well as the application of external post-tensioning to reinforced concrete (non-prestressed) specimens as a simulation of a possible strengthening effect.
3.3.1.1 Geometric, material and reinforcement properties of the test beams

All the test beams had the same concrete cross section (Fig. 3.1) whose nominal geometric data can be seen in Fig. 3.2.

The total length was 4.4 m for the R1, R2, P2, P2p marked specimens and 3.4 m for the P1 marked specimen (Table 3.1). All beams were prismatic, both the concrete section and the embedded longitudinal reinforcement were unchanged along the full length of each beam.

![Cross sections of test beams](Fig. 3.1)

![Geometric dimensions of the cross section (notations)](Fig. 3.2)

The beams had different types of reinforcement and different steel ratios according to Fig. 3.1 and Table 3.1.

| Table 3.1 Geometrical and reinforcement data of test beams |
|---|---|---|---|---|
| Beam type | No. of beams | Length Total length $L_b$ [m] | Span $L$ [m] | Strength $f_{pk}/f_{p0.1k}$ or $f_{tk}/f_{yk}$ [N/mm²] | Type (surf.) | Reinforcement | No. & $\phi$ [mm] of bars | Steel ratio, $\rho$ [%] | Initial prestress |
| P1 | 1 | 3.4 | 3.2 | 1770/1520 | prestressing steel (tr.) | $2\phi5.34$ | 0.527 | 0.284 | unstressed |
| R1 | 4 | 4.4 | 3.8 | 600/500 | reinforcing steel (sp.) | $2\phi8$ | 1.183 | 0.636 | - |
| P2 | 3 | 4.4 | 3.8 | 1770/1520 | prestressing steel (tr.) | $(7+1)\phi4.7$ | 1.436 | 0.769 | unstressed |
| R2 | 3 | 4.4 | 3.8 | 600/500 | reinforcing steel (sp.) | $3\phi8$ | 1.774 | 0.955 | - |
| P2p | 3 | 4.4 | 3.8 | 1770/1520 | prestressing steel (tr.) | $(7+1)(5+1)(3+1)(2+1)\phi4.7$ | 1.436 | 0.769 | 0.61$f_{pk}$ | 0.549 | 0.329 | 0.220 | 0.61$f_{pk}$ |
The steel ratio ($\rho$) was calculated on the basis of the effective cross sectional area ($b_w d$) as well as the total concrete section ($A_c$). The R1 and R2 specimens contained reinforcing steel, the P1 and P2 specimens contained unstressed prestressing wires and the P2p marked specimens contained the same reinforcement as for P2 specimens but the wires in the P2p specimens were prestressed (with an initial prestress of 1080 N/mm² ($0.61 f_{pk}$)). The outer surface was trochoidally ribbed for the prestressing wires and spirally ribbed for the reinforcing bars.

Table 3.2 Material properties of test beams

<table>
<thead>
<tr>
<th></th>
<th>Design value</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concrete: C35/45 ($f_{ck}=35$ N/mm²)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive strength</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{cd}=f_{ck}/1.5; f_{cm}=f_{ck}+8$ [N/mm²]</td>
<td>23.3 ($f_{cd}$)</td>
<td>43.0 ($f_{cm}$)</td>
</tr>
<tr>
<td>Mean value of flexural tensile strength</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{ctm, fl}=(1.6-h[m])\times0.3 f_{ck}^{2/3}$ [N/mm²]</td>
<td>4.53</td>
<td></td>
</tr>
<tr>
<td>Short-term modulus of elasticity, $E_{cm}=22(f_{cm}/10)^{0.3}$ [N/mm²]</td>
<td></td>
<td>34077</td>
</tr>
<tr>
<td><strong>Reinforcing steel: S500B ($f_{yd}=600$ N/mm²)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter, $\phi$ [mm]</td>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>Tensile strength</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{cd}=f_{yd}/1.15$, and $f_{cm}=f_{yd}/(1-1.645\times0.025)$ [N/mm²]</td>
<td>435 ($f_{cd}$)</td>
<td>626 ($f_{cm}$)</td>
</tr>
<tr>
<td>Modulus of elasticity, $E_s$ [N/mm²]</td>
<td></td>
<td>200000</td>
</tr>
<tr>
<td><strong>Prestressing steel: 1770 ST ($f_{pd}=1770$ N/mm²)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter, $\phi$ [mm]</td>
<td>5.34 or 4.70</td>
<td></td>
</tr>
<tr>
<td>Tensile strength</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{pd}=f_{p0,1k}/1.15$, and $f_{pm}=f_{pd}/(1-1.645\times0.025)$ [N/mm²]</td>
<td>1322 ($f_{pd}$)</td>
<td>1846 ($f_{pm}$)</td>
</tr>
<tr>
<td>Modulus of elasticity, $E_p$ [N/mm²]</td>
<td></td>
<td>205000</td>
</tr>
</tbody>
</table>

The applied materials, their nominal strengths and the deduced properties can be found in Table 3.2. The design values for strength ($f_{cd}$, $f_{cm}$, $f_{yd}$, $f_{pd}$) and the moduli of elasticity ($E_{cm}$, $E_s$, $E_p$) were taken or determined for calculation purposes on the basis of the Eurocode (EC, EN 1992-1-1:2004).

In case of bending failure for lightly and normally reinforced beams, the bending capacity is influenced first of all by the steel strength while the compressive strength of concrete does not have a significant effect on it. Therefore, the steel strengths were checked by standardized breaking tests for a few of the test beams (Table 3.3). After the test, the specimens were cut out of beam sections which were situated in uncracked zones during the loading phases. Taking the individual measured strengths as mean values ($f_m$), having the characteristic values (defined as 5% quantile values) of the applied steel classes ($f_k$) and assuming normal distribution for the steel strengths as usual, the coefficient of variation (COV) for the measured strength ($\nu_m$) for both the yield and the tensile strengths of steels were calculated as follows:

$$\nu_m = \frac{1-f_k/f_m}{1.645}. \quad (3.1)$$
When determining the mean tensile strength of steels \((f_{tm}, f_{pm})\) in Table 3.2 for cross-sectional data calculations, the applied COV values equal to 0.025 were taken on the basis of the average COV obtained in Table 3.3.

### 3.3.1.2 Reduction of prestress in the P2p specimens

For the prestressed P2p type specimens, the deterioration process was partly modelled by artificial tendon breaks made by sawing the intended number of wires in the bottom flanges of the beams through the concrete cover at selected cross-sections (cut points) along the beam length as shown in Fig. 3.3. The numbers below the cut points show the number of cut wires at that particular section in the corresponding state.

![Fig. 3.3 Wire cuts for the P2p specimens](image)

For the P2p/1 specimen, first gradual wire cuts were made only at two sections symmetric to the midspan (states 41-44). Then the cuts were expanded uniformly for the full span (states 5-55) producing a relatively smooth transition in the prestressing force.
For the P2p/2 and P2p/3 specimens, this transition was quicker because five wire cuts at each cut point were completed through only two states. Note that for the P2p/2 and P2p/3 specimens the cut points are not uniformly distributed along the span but rather positioned toward the midspan. In state 3 for the P2p/1 beam and state 2 for the P2p/2&3 beams, only the concrete cover was sawed without any wire cuts. Because of full bond around the tendons, individual wire cuts result in only a local decrease of the prestressing action along the span. This decrease was considered as linear along the transmission length \( l_{pt} \) on both sides of the cut, which was calculated on the basis of the mean value of tensile strength \( f_{ctm} \) of concrete according to the EC as follows:

\[
l_{pt} = \alpha_1 \alpha_2 \phi \frac{\sigma_{p,eff}}{f_{bpt,m}} = 165 \text{ mm.} \quad (3.2)
\]

Here \( f_{bpt,m} = \eta_{p1} \eta_1 f_{ctm} \) is the mean value of bond stress and \( \phi \) is the diameter of indented wire (\( \eta_{p1} = 2.7 \)) with circular cross section (\( \alpha_2 = 0.25 \)). Sudden stress release (\( \alpha_1 = 1.25 \)) and “good” bond conditions (\( \eta_1 = 1.0 \)) were assumed. As a result of this, all prestressing related data suddenly changed at the cut points and were consequently considered as functions of distance along the beam length as shown in related figures of Sect. 3.3.1.4 and Sect. 3.3.2.1.

### 3.3.1.3 Details of post-tensioning

To simulate a possible strengthening effect, a few, reinforced concrete beams (P1, R1/4, R2/1, P2/1 and P2/2) were equipped with an external post-tensioning system after the intended deterioration process (according to Sect. 3.3.2) had been completed.

![Post-tensioning equipment](image)

**Fig. 3.4** Post-tensioning equipment

This system consisted of two unbonded, ST 1860 type, wedge-anchored, nearly centrically-positioned, straight-line monostrands with a cross-section of 150 mm\(^2\) each and 40 mm thick anchorage plates at both ends. The tensioning force was applied simulta-
neously in both strands in three steps (6p, 7p and 8p according to Table 3.4) by a 608 kN (650 bar) capacity hydraulic jack, whose self-weight was measured as 0.3 kN, at one end of beams (Fig. 3.4). The tensioning process was monotonic, thus the tensioning force was not removed or decreased between the consecutive tensioning steps. The general arrangement of the post-tensioning system for the P1 marked specimen is shown in Fig. 3.5.

![Fig. 3.5 Arrangement and geometry of post-tensioning for the P1 specimen](image)

The tensioning force was intentionally centric, but owing to the different reinforcements slightly varying eccentricity occurred in the vertical plane for the different beam types. However, considering its extent, the resulting flexural effect was quite low and the centric compression effect remained dominant.

<table>
<thead>
<tr>
<th>Post-tensioning step</th>
<th>Respective pressure in jack [bar]</th>
<th>Measured slips [mm]</th>
<th>Calculated (with $E_p=195000$ N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>force per strand [kN]</td>
<td>anchored end, $\Delta_1$</td>
<td>active end $\Delta_{2,1}$ $\Delta_{2,2}$</td>
</tr>
<tr>
<td>6p</td>
<td>21</td>
<td>10</td>
<td>8.6</td>
</tr>
<tr>
<td>7p</td>
<td>203</td>
<td>95</td>
<td>6.5</td>
</tr>
<tr>
<td>8p</td>
<td>321</td>
<td>150</td>
<td>5.8</td>
</tr>
</tbody>
</table>

The applied, pressure-adjusted tensioning force was controlled by measuring the wedge slips at both ends ($\Delta_1$ and $\Delta_{2,2}$) as well as the push-out length of the jack ($\Delta_{2,1}$) after each stressing step for the P1 marked specimen. Because of the good coincidence between the intended and the deduced force (<5%) no further control was found to be necessary (Table 3.4). In the following the intended post-tensioning forces (10, 95 and 150 kN/strand for states 6p, 7p and 8p respectively) will be taken into account for calculation purposes.

### 3.3.1.4 Cross-sectional data

Cross-sectional data of the test beams (Table 3.5) were calculated on one hand as input for the deduction of damage indices and on the other hand to provide background information to the applied loading phases.
### Table 3.5 Cross-sectional data of test beams

<table>
<thead>
<tr>
<th>Beam type</th>
<th>Bending capacity [kNm]</th>
<th>Cracking moment [kNm]</th>
<th>Decompression moment [kNm]</th>
<th>M(\text{Rm})</th>
<th>M(\text{Rd})</th>
<th>M(\text{dec})</th>
<th>M(\text{dec}/M\text{Rd})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean value (M_{\text{Rm}})</td>
<td>Design value (M_{\text{Rd}})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1 without post-tens.</td>
<td>13.06</td>
<td>9.13</td>
<td>3.19</td>
<td>1.13</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>post-tensioned</td>
<td>14.92</td>
<td>10.78</td>
<td></td>
<td>0.10</td>
<td>0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6p</td>
<td>21.33</td>
<td>12.60</td>
<td></td>
<td>10.76</td>
<td>1.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7p</td>
<td>23.20</td>
<td>9.09</td>
<td></td>
<td>16.99</td>
<td>1.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8p</td>
<td>10.12</td>
<td>6.92</td>
<td>3.28</td>
<td>1.12</td>
<td>0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>post-tensioned</td>
<td>12.09</td>
<td>8.74</td>
<td></td>
<td>10.68</td>
<td>0.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6p</td>
<td>23.66</td>
<td>12.70</td>
<td></td>
<td>16.86</td>
<td>1.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7p</td>
<td>23.74</td>
<td>9.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R1 without post-tens.</td>
<td>24.26</td>
<td>16.27</td>
<td>3.40</td>
<td>1.14</td>
<td>0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>post-tensioned</td>
<td>24.27</td>
<td>16.11</td>
<td></td>
<td>10.88</td>
<td>0.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6p</td>
<td>25.06</td>
<td>13.88</td>
<td></td>
<td>17.17</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7p</td>
<td>24.87</td>
<td>10.51</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8p</td>
<td>16.54</td>
<td>11.58</td>
<td>1.12</td>
<td>1.12</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>post-tensioned</td>
<td>24.92</td>
<td>12.76</td>
<td></td>
<td>10.61</td>
<td>0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6p</td>
<td>24.10</td>
<td>9.01</td>
<td></td>
<td>16.75</td>
<td>1.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7p</td>
<td>14.75</td>
<td>9.99</td>
<td>3.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2 without post-tens.</td>
<td>22.74</td>
<td>15.22</td>
<td>11.53</td>
<td>8.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>post-tensioned</td>
<td>14.12</td>
<td>9.67</td>
<td>7.66</td>
<td>4.42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6p</td>
<td>9.41</td>
<td>6.51</td>
<td>5.72</td>
<td>2.52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2p wire cuts made</td>
<td>29.68</td>
<td>19.27</td>
<td>15.37</td>
<td>11.96</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 cut wires</td>
<td>22.74</td>
<td>15.22</td>
<td>11.53</td>
<td>8.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 cut wires</td>
<td>14.12</td>
<td>9.67</td>
<td>7.66</td>
<td>4.42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 cut wires</td>
<td>9.41</td>
<td>6.51</td>
<td>5.72</td>
<td>2.52</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When calculating the bending capacity of cross sections, a rectangular idealized stress-strain diagram (with \(\varepsilon_{\text{cu3}}=0.35\%\), \(\alpha=1.0\) and \(\gamma_{c}=1.5\)) for concrete and a linear elastic part with horizontal top branch for the idealized stress-strain diagrams (with \(\gamma_{s}=1.15\)) for both reinforcing and prestressing steel was taken into account according to the EC. Using the material properties given in Table 3.2 and the nominal geometric dimensions according to Fig. 3.2 as well as the external post-tensioning for the related beams according to Sect. 3.3.1.3, the mean and the design values of ultimate bending capacity \(M_{\text{Rm}}\) and \(M_{\text{Rd}}\) as well as the cracking or decompression moments \(M_{\text{r}}\) and \(M_{\text{dec}}\) were calculated for each related beam type. The \(M_{\text{dec}}\) values were simply computed as bending moments resulting in zero stress in the extreme bottom fibre of post-tensioned beams without any limitation in the intensity of the compression stress in the extreme top fibre, therefore \(M_{\text{dec}}\) values higher than \(M_{\text{Rm}}\) or \(M_{\text{Rd}}\) had only theoretical significance. The eccentricity-change of the external post-tensioning force due to beam deformation was neglected. The P2p type specimens remained uncracked under full prestressing and no external load (compressive stress exists at the extreme top fibre) and the same can be stated for beams under post-tensioning in states 6p-8p. Only flexure-related data were calculated because the applied loading phases precluded shear failure (see Sect. 3.1).
As indicated in Sect. 3.3.1.2 for the P2p type specimens, the prestressing-related data of cross-sections along half of the beam length are shown in Fig. 3.6 (where \( I \) is the moment of inertia of the uncracked section, \( M_p \) is the bending moment due to prestressing, \( M_c \) and \( M_{dec} \) are the cracking and the decompression moments respectively and the numbers in the indices related to the state number according to Fig. 3.3).

### 3.3.2 Test set-up and programme

As indicated in Sect. 3.2, modelled, subsequent deterioration and strengthening periods were carried out on all test beams except for the P2p type specimens, for which no “strengthening” period was applied. Each state in these two periods included a loading and a following measuring phase. In this context wire cuttings for the P2p type specimens as well as the application of post-tensioning for the R1, R2, P1, P2 type specimens were considered as special loading phases.

According to the general process of the test, in the \( i \)-th state first the loading phase \( i \) was applied to the beam to produce the intended degree of cracking. Under this loading the deflection at the midspan, the maximum crack width and, in a few cases, the
crack pattern was registered. After the loads were removed, the residual deflection was also registered at midspan. Then the dynamic measuring phase $i$ took place, during which the first two natural frequencies were determined by using special excitation effects. Then the $(i+1)$-th state with the next loading and measuring phases followed.

### 3.3.2.1 Loading phases

The cracking states were produced by force-controlled four-point bending according to Fig. 3.7 and Table 3.6. The calculated data of Table 3.6 are based on Sect. 3.3.1.4.

![Arrangement of beams under four-point bending](image)

The acting forces ($F$) were equal to each other and symmetric to the midspan in each load position. Sliding of the bottom beams on supports during loading was possible at both ends. The midspan-deflection was measured in each loading phase under loading and after load removal. A periodic exciter and an acceleration detector were fixed to the beams as necessary parts of the dynamic measuring equipment (see Fig. 3.11). Their weights were negligible compared to that of the beams and thus they insignificantly influenced the dynamic behaviour of the beams. Self-weight ($g_0=0.39 \text{ kN/m}$) was the only load on the specimens in states 0 and after the load removal in all other states. The general concept of choosing the position and the intensity of acting forces in the subsequent states was to produce gradually increasing flexural cracking in the beams. According to this concept, flexural cracking is:

- present in a place where the bending moment is higher than the decompression moment (prestressed beams) and has previously exceeded the cracking moment;
- intensified if the length of sections containing cracks and/or the crack sizes are increased.
<table>
<thead>
<tr>
<th>Beam</th>
<th>Mark of the deterioration state</th>
<th>Load</th>
<th>Length of the cracked zone</th>
<th>Bending moment ratios at midspan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.16</td>
</tr>
<tr>
<td>P1</td>
<td>1</td>
<td>5.13</td>
<td>0.5</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.63</td>
<td>1.0</td>
<td>2.45</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.13</td>
<td>1.5</td>
<td>2.87</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9.63</td>
<td>1.0</td>
<td>3.48</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>8.13</td>
<td>0.5</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td>0 (for all P2 beams)</td>
<td>0</td>
<td>0</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>1/23 (for P2/2 &amp; P2/3)</td>
<td>2.13</td>
<td>0.5</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>1/3 (for P2/3)</td>
<td>4.38</td>
<td>0.5</td>
<td>2.43</td>
</tr>
<tr>
<td></td>
<td>1/12 (for P2/1 &amp; P2/2)</td>
<td>6.13</td>
<td>0.5</td>
<td>2.79</td>
</tr>
<tr>
<td></td>
<td>2/12 (for P2/1 &amp; P2/2)</td>
<td>7.13</td>
<td>1.0</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>3/3 (for P2/3)</td>
<td>8.13</td>
<td>1.5</td>
<td>3.02</td>
</tr>
<tr>
<td></td>
<td>3/12 (for P2/1 &amp; P2/2)</td>
<td>8.63</td>
<td>1.5</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td>4/12 (for P2/1 &amp; P2/2)</td>
<td>11.13</td>
<td>2.0</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>4/3 (for P2/3)</td>
<td>12.13</td>
<td>2.5</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>5/12 (for P2/1 &amp; P2/2)</td>
<td>11.13</td>
<td>1.5</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>6/12 (for P2/1 &amp; P2/2)</td>
<td>11.13</td>
<td>1.0</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>7/12 (for P2/1 &amp; P2/2)</td>
<td>11.13</td>
<td>0.5</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>0 (for all R1 beams)</td>
<td>0</td>
<td>0</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>1/4 (for R1/4)</td>
<td>10.63</td>
<td>2.5</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>3/4/2 (for R1/1 &amp; R1/3)</td>
<td>10.88</td>
<td>2.5</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>3/2/3 (for R1/1 &amp; R1/3)</td>
<td>11.13</td>
<td>2.5</td>
<td>3.24</td>
</tr>
<tr>
<td></td>
<td>4/1/2 (for R1/1 &amp; R1/3)</td>
<td>12.13</td>
<td>2.5</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>4/3/2 (for R1/1 &amp; R1/3)</td>
<td>14.13</td>
<td>2.5</td>
<td>3.35</td>
</tr>
<tr>
<td></td>
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### Table 3.6 Data of loading phases

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<th>Beam</th>
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<th>Bending moment ratios at midspan</th>
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<td>17.13</td>
<td>2.5</td>
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</table>
Fig. 3.8 shows the associated moment lines of each loading phase as well as levels of cracking ($M_r$) and decompression ($M_{dec}$) moments for the P1 and each of the P2p specimens along one half of the beam lengths. As mentioned in Sect. 3.3.1.2, the sudden changes in the $M_r$ and $M_{dec}$ lines for the P2p type specimens represent the effects of wire cuts. For the P1, P2, R1 and R2 beams first the length of the cracked zone ($L_r$ in Table 3.6) had been gradually increased (in the first deterioration states) then the crack sizes were significantly opened (last states). Loading phases with $M_{max}/M_{Rd}$ values smaller than 1.0 represent usual load cases in practice, but phases when $M_{Rd}<M_{max}<M_{Rm}$ are not relevant from a practical point of view rather are able to model structural behaviour close to bending failure. The same loading phases were applied for each specimen within the P1 and R2 beam families.
For a few specimens within the P2 and R1 beam families, several load phases were modified in order to investigate minor aspects. For example, for the P2/3 beam the cracked length extension and the crack size increase were simultaneously produced in each subsequent state. For the R1/1-3 beams, no significant damage by crack size increase was produced after the maximum cracked length was reached while this was not the case for the R1/4 specimen.

A relatively small number of loading phases was introduced to the P2p type (prestressed) beams. In these cases the main focus was the investigation of tendon breaks, which was modelled by wire cuts according to Sect. 3.3.1.2. Note that states in which wire cuts were made are missing from Table 3.6 (such as 3, 41-44 and 5-55 for the P2p/1 beam and states 2, 3 and 4 for the P2p/2&3 beams). Although all cracks should have theoretically closed due to prestress during the load removal after state 1, the basic strategy was still to produce very small cracking before the wire cuts for the P2p/1 beam and, just the opposite of that, large cracking for the P2p/2&3 beams (as shown by the $M_1$ lines in Fig. 3.8). The number of cut wires as well as the gradation in cutting was different (fewer cuts and less rapid cutting rate for the P2p/1 specimen).

Another aspect was to apply states with a loading phase (state 31 and 41) immediately after states with wire cuts (state 3 and 4) for the P2p/2&3 beams and no states with loading phases between states with wire cuts (states 41-55) for the P2p/1 beam.

Regarding the strengthening states (6p-8p), the strategy was first to eliminate cracking from the significantly deteriorated beams by external post-tensioning (state 6p) and then to increase the tensioning intensity (states 7p & 8p). The specimens concerned and the process itself is detailed in Sect. 3.3.1.3 and Table 3.6. Fig. 3.9 shows the bending moment ($M$) (under self-weight and post-tensioning) and the associated decompression moment ($M_{dec}$) lines in states 6p-8p for the half length of the P1 beam (a similar tendency in the $M/M_{dec}$ ratio was also observed in the corresponding states for the P2/1&2, R1/4 and the R2/1 specimens). To have reference data with regard to the intensity of applied post-tensioning, the relevant $M_{dec}/M_{Rtd}$ ratios have been calculated and included in Table 3.5.
3.3.2.2 Dynamic measuring phases

Each state, independently of the preceding loading phase (either a four-point loading or a wire cut or a post-tensioning), ended in a measuring phase with the aim of determining the first two natural frequencies. The numerical values of natural frequencies were obtained statistically from the analysis of signals, which were recorded, transformed and stored in situ by the appropriate measuring equipment. The vibration signals were recorded in two ways: in the first case under the free vibration of beams and in the second case under a nearly harmonically excited vibration of beams being very close to resonance.

Measuring equipment and device settings

The measuring equipment included an accelerometer (type Hottinger B3-5) fixed to the beam at about L/5 distance from one of the supports according to Fig. 3.7. Its position was chosen by considering on one hand to avoid coincidence with acting forces under four-point loading as well as the close vicinity of nodes of expected mode shapes during vibration and on the other hand to obtain as large amplitudes as possible in order to better measurability. During the measuring process, the vibration signals produced by the accelerometer went to an analyser (Tektronix 500), which first recorded the acceleration-time (\(a-t\)) function then immediately made a fast Fourier transform (FFT) and directly produced its frequency spectrum.

According to the procedure of the fast Fourier transform, the recorded \(a-t\) function is approximated in discrete (sample) points by the linear combination of “basic” trigonometric time functions with unit amplitudes and different frequencies \((f_b)\). The frequency spectrum is defined as a relationship between the frequencies of basic functions and their associated relative weights in the linear combination at the best fit. The smallest basic frequency \((f_{b1})\) is equal to the reciprocal of the total length of the time function to be approximated \((T\ [s])\) and the others are multiples of \(f_{b1}\), therefore the resolution of the frequency spectrum \((\Delta f)\) is equal to \(f_{b1}\). The total length of the recorded time function depends on the sampling frequency \((f_s)\) and the number of discrete sampling points \((N)\). Regarding the sampling frequency, \(f_s \geq 2,0f_{\text{max}}\) is recommended according to Shannon’s rule in order to avoid deformed (false) spectrums where \(f_{\text{max}}\) is the highest frequency to be measured. (In the case of the P1 specimen, the first natural frequency \((f_1)\) was expected below 40 Hz and the second one \((f_2)\) below 140 Hz. For the rest of longer specimens, lower corresponding natural frequencies were expected.) If fixing \(f_s\) then \(N\) determines the resolution of the frequency spectrum. In this test, most often \(f_s=512\) Hz was used for the measurements of both \(f_1\) and \(f_2\), which was much higher.
than the recommended limit \( f_s > 2.0f_2 \). Because the FFT requires \( 2^n \) number of samples to compute \( 2^n-1 \) number of basic functions, furthermore, frequency spectrums with an appropriately high resolution frequency scale were demanded, \( N \) was set as \( N=4096(=2^{12}) \) for each sample. Due to lower accuracy in fitting basic functions with high frequencies, the frequency spectrum often becomes deformed (false) in the highest frequency domains. For that reason a default setting of the applied analyser was to calculate only the first 1600 spectrum ordinates, which resulted in a frequency domain ranging from 0 Hz to \( f_{\text{lim}}=1600f_{b1} \). The measurable natural frequency had to fall into this domain thus the device settings had to be chosen accordingly. Finally, the above set parameters resulted in a total length of \( a-t \) functions of \( T=N/f_s=4096/512=8 \) s, a frequency spectrum resolution of \( \Delta f=1/T=1/8=0.125 \) Hz and a frequency range limit of \( f_{\text{lim}}=1600\Delta f=1600\times0.125=200 \) Hz. Considering \( f_1 \) as being about 1/4-th of \( f_2 \), sometimes, in order to improve the resolution of spectrums, \( f_s=256 \) Hz sampling frequency was used only for the \( f_1 \) measurements, which resulted in \( T=16 \) s, \( \Delta f=1/T=1/16=0.0625 \) Hz and \( f_{\text{lim}}=1600\Delta f=1600\times0.0625=100 \) Hz.

**Excitation techniques**

The differences between excitation effects used both in practice and for testing purposes were discussed in Sect. 1.3.4. It was also detailed that a simple mechanical impulse, which is often used in practice for excitation purposes, results in more intensive excitation forces in the lower frequency domain compared to the higher ones (see Fig. A2.2).

In the test two types of excitation were investigated. The first was a single mechanical impulse, which is widely used for experimental purposes.

\[ F \approx \sin(k\pi \tau / T)/(k\pi \tau / T) \]

\[ F \approx \text{constant} \]

![Fig. 3.10 Schematic comparison of excitation forces for the applied techniques](image)

The second was a nearly harmonic effect, by which only the close vicinity of natural frequencies were excited with about constant excitation force. A scheme on the differences in magnitudes of the excitation forces for the two applied techniques can be seen in Fig. 3.10 where \( k \) is an integer, \( \tau \) measures the duration of the impulse within the \( T \) interval of the excitation effect. The reason for the application of these two excita-
tion techniques was to analyse their effects on the shape of the resulting frequency spectrums.

The mechanical impulse was made by a rubber covered mallet to the beam at the section symmetric with the fixed accelerometer to the midspan. This impulse was considered as an approximation of the Dirac-$\delta$ effect with $A \neq \infty$ and $\Delta t = \tau \neq 0$ (see Fig. A2.2), thus more intensive excitation was expected for $f_1$ compared to $f_2$.

In the case of the second technique, an exciter was fixed to the beam about symmetrically with the accelerometer to the midspan as shown in Fig. 3.11. The magnitude of the excitation force could be varied by either driving cap screws with different lengths perpendicularly into the rotating axle or by changing the revolution of the exciter. Due to the pure circular movement of the eccentric mass, the exciter with constant revolution provided a harmonic, sinusoidal excitation force. During the measuring process, the revolution of the exciter was slightly altered in such a way that the resulting excitation frequencies varied in the close vicinity of the expected natural frequencies (see Fig. 3.10). The variation in intensity of the excitation force over this very narrow excitation frequency range (max. 1 Hz) could be taken as negligible. However, considering the existing degree of structural damping of the beam as well as the revolution needed to match the excitation frequency to the natural frequency, the magnitude of the excitation force could be controlled by the appropriate selection of the rotational mass together with its eccentricity. Depending on the natural frequency to be measured, different cap screws were applied to keep the amplitudes in the $a$-$t$ function within acceptable limits. When the excitation frequency coincided with the natural frequency of the beam, resonance effects occurred, which were clearly visible as sudden amplitude increases in the plotted $a$-$t$ function of vibration. At those moments, the magnitude of amplitudes in the $a$-$t$ function belonging unambiguously to resonance effects could be identified and then, during the measurement, only parts of $a$-$t$ function containing amplitudes close to that measure were recorded for further analysis.
3.4 SIGNAL PROCESSING

The evaluation of the recorded $a$-$t$ functions took place in two steps. First the frequency spectrums were produced still by the analyser then a statistical analysis of them was carried out manually. In the following, the main aspects of this evaluation process as well as the numerical values of the obtained natural frequencies will be introduced.

3.4.1 Computing individual frequency spectrums

Getting the beam into vibration with impulse excitation and thereafter letting it move under free vibration, the signs of excitation in the structural response quickly (after a few periods) diminish down to an ignorable level and the signs of free vibration become dominant. Because of this dominance, the relative weight of especially the natural frequency (associated with the mode, on which the damped free vibration runs) and, due to signal imperfections, of frequencies close to the natural frequency will be significantly higher in the frequency spectrum compared to others which are poorly represented in the free vibration. The applied length of the recorded $a$-$t$ functions ($T$) included many impulses related, subsequent free vibrating time periods. However, note that irregularities are present in the $a$-$t$ functions at moments when the impulses are activated and that curve fitting to the $a$-$t$ function becomes difficult and unreliable at ranges with very small amplitudes because disturbing effects of external noises and the influence of structural damping become more intensive. One of the recorded $a$-$t$ functions belonging to state 0 of the P1 specimen and part of its associated frequency spectrum containing $f_1$ is shown in Fig. 3.12 ($f_s$=256 Hz, $\Delta f$=0.0625 Hz) in the case of impulse excitation.

![Fig. 3.12 Time function due to impulse excitation and its associated frequency spectrum](image_url)

If using the harmonic excitation technique, the opposite takes place regarding the structural response. Because in this case the excitation is continuous in time, the excitation frequencies are dominant in the structural response and the natural frequencies
disappear from it very quickly (after the first few periods). If intending to measure a natural frequency, as many coincidences as possible have to be induced between the excitation and the natural frequency by altering the excitation frequency accordingly during the recorded time period. Coincidences result in resonance effects. Having many resonance effects in the $a-t$ function, the relative weight of the associated natural frequency will be increased in the frequency spectrum. If the recorded $a-t$ function includes no real coincidence (resonance effect) then the related frequency spectrum will be deceiving because the frequency with the highest ordinate will be the most dominant excitation frequency instead of the natural frequency. One of the recorded $a-t$ functions belonging to the same state as for Fig. 3.12 and part of its associated frequency spectrum containing $f_1$ is shown in Fig. 3.13 ($f_s=256$ Hz, $\Delta f=0.0625$ Hz) in the case of harmonic excitation.

![Time function due to harmonic excitation and its associated frequency spectrum](image)

**Fig. 3.13** Time function due to harmonic excitation and its associated frequency spectrum

Note that the same frequency value at the peak ordinates on Fig. 3.12 and Fig. 3.13 is not by chance but, of course, also not a necessity for spectrums representing individual measurements. Observe for Fig. 3.13 on one hand that the ratio of the highest amplitude (presumably belonging exactly to resonance effects) to each of the other ordinates remained below 2.5 throughout the full $a-t$ function, which indicated a structural response really close to resonance. On the other hand, the magnitude of the maximum amplitude in the $a-t$ function was adjusted to about the same degree as for the impulse excitation by appropriately controlling the excitation force. Also note that when using the same device settings for the frequency spectrum computations for both types of excitations, a more reliable (much higher relative weight of the peak ordinate and no secondary local max. ordinates next to the peak) and a bit narrower (especially in the close vicinity of to the peak ordinate) spectrum was obtained from the $a-t$ function of the harmonically excited vibration.
3.4.2 Computing average frequency spectrums

It was seen in Sect. 3.4.1 and Sect. 3.3.2.2 that the “correctness” of natural frequencies obtained from individual frequency spectrums significantly depends on the excitation input for both types of excitation. For the impulse excitation, the reasons are the lower excitation force in the higher frequency domains, the irregularities in the $a$-$t$ function at impulse activation points and the unreliability of free vibration at ranges with very low amplitudes. For the nearly harmonic excitation input, the main reason is the imperfect coincidence between the excitation and the natural frequency during the recorded $a$-$t$ function. Therefore, a statistical analysis, during which the individual frequency spectrums have been averaged manually, was carried out for both types of excitation which resulted in the average frequency spectrums. For each state of the beams, a minimum of 30 individual frequency spectrums were included in this process. Because averaging decreases the relative weight of non-regular frequencies to a much higher degree than that of frequencies close to the natural frequencies, the reliability of natural frequencies given by these average frequency spectrums is expected to improve significantly in comparison with that given by the individual spectrums.

Fig. 3.14 shows the resulting average frequency spectrums belonging to state 0 of the P1 beam computed to determine $f_1$ and for both types of excitation.

The average spectrum belonging to the impulse excitation (Fig. 3.14a) shows only slight changes (in shape far from $f_1$ and in the value of $f_1$) compared to the corresponding individual spectrum shown in Fig. 12. The standard deviation (SD) of frequencies belonging to the maximum ordinates in the individual spectrums is 0.144 Hz. Additionally, the general shape of the spectrum line, the number and positions of local peaks next to $f_1$, the relative weight of both local peaks and $f_1$ itself as well as the width of the peak region in the spectrum line around $f_1$ (~0.5 Hz) are almost unchanged. Based on this, in the case of impulse excitation, it can be concluded that:
• the recorded individual $a$-$t$ functions regularly contain frequencies which differ from the natural frequency, represent the imperfectness of structural response under free vibration and are identified as secondary peaks with unchanged position and about constant relative weight in the frequency spectrum.

• the individual frequency spectrums are able to provide a relatively good estimation of $f_1$ with relative weights significantly higher than that of secondary peaks.

• consequently, the average frequency spectrum does not improve the reliability of $f_1$ significantly in comparison with the individual spectrums.

However, the average spectrum belonging to the harmonic excitation (Fig. 3.14b) shows visible changes compared to the individual spectrum shown in Fig. 13. The SD of frequencies belonging to the maximum ordinates in the individual spectrums is 0.152 Hz. The relative weight of $f_1$ decreased, the position of $f_1$ shifted to a greater extent than experienced for the impulse excitation and the width of the peak region in the spectrum line around $f_1$ became wider. The conclusions for the harmonic excitation are that:

• the resulting spectrums (both individual and average) are effectively able to filter out frequencies which differ from the natural frequency.

• the individual frequency spectrums give a bit less reliable estimation of $f_1$ than experienced for the impulse excitation.

• the average frequency spectrum improves the reliability of $f_1$ in comparison with the individual spectrums.

However, the picture changes considerably when analysing average frequency spectrums related to $f_2$ and belonging to state 0 of the P1 beam as shown in Fig. 3.15. The same excitation techniques, the same measuring equipment and procedure as well as the same computational process were applied as for the determination of $f_1$.

![Average frequency spectrums for the second natural frequency ($f_2$)](image)
Using impulse excitation (Fig. 3.15a) it can be seen that the peak region in the spectrum line around $f_2$ became much wider than that around $f_1$. Additionally, regular frequencies similar to that in Fig. 3.14a existed and were represented as secondary peaks with high relative weights in the average spectrum close to the natural frequency. The differences between relative weights belonging to the natural frequency and to secondary peaks were relatively small. However, the SD of frequencies at maximum ordinates in the individual spectrums resulted in 0.234 Hz, which, considering the magnitude, was quite acceptable. These facts indicated a bit lower reliability in the value of $f_2$ compared to $f_1$ if using impulse excitation input.

In using harmonic excitation, an average spectrum similar to that for $f_1$ was obtained, although the peak region around $f_2$ became wider (~3.0 Hz) than that for $f_1$ (~0.5 Hz). However, a narrow interval of frequencies before $f_2$ with non-negligible relative weights may be observed as a secondary peak. Its existence is supported by the fact that the SD of frequencies at maximum ordinates in the individual spectrums resulted in 0.650 Hz, which was more than four times the corresponding SD for $f_1$.

If further analysing the average frequency spectrums related to $f_2$ as shown in Fig. 3.15, two main issues should be discussed regarding the aspects set previously for average spectrums related to $f_1$. The first is the significant widening of the spectrum line around $f_2$ (Fig. 3.15a) compared to that for $f_1$ (Fig. 3.14a) when using impulse excitation. This, as an addition to the related discussion on the individual spectrums in Sect. 3.4.1, may be explained by the following two reasons:

- The free vibration, following that the excitation impulse has been activated, runs at the lowest energy mode. Generally, as is the case for these simply-supported beams, the first mode stores the lowest energy, therefore $f_1$ is expected to be over-represented in the recorded a-t functions. The $f_2$ may be excited to an acceptable level only if the impulse effect is activated at places around the highest amplitudes of the associated 2nd mode. After the first few periods, the vibration returns to the lowest energy (1st) mode while the highest modes (including the 2nd mode) gradually diminish. This makes the measurement of modal data associated with higher modes difficult and less reliable. In contrast with this, the excitation force of the harmonic excitation can, between certain limits, be adjusted freely, thus $f_2$ may be excited with as high an excitation force as needed to get “smooth” frequency spectrums.

- As discussed in Sect. 1.3.4 and Sect. 3.3.2.2, the excitation force of the impulse excitation decreases with the frequency according to the $\sin(x)/x$ function. Thus $f_2$ is
less intensively excited and, consequently, less represented in the subsequent free vibration than \( f_1 \).

The second issue is the observed secondary peaks in the average spectrums related to \( f_2 \) for both types of excitation. The mode shape associated with \( f_2 \) is asymmetric with one internal node at the midspan, thus the two beam halves vibrate in opposite phases. If the distribution of bending rigidity along the beam length is not exactly symmetric about the midspan, which may occur e.g. due to non-visible cracks, then the natural frequency, which is associated with the strictly symmetric mode shape, slightly shifts and an additional (virtual) frequency appears and represents itself in the frequency spectrum as a secondary peak with significant relative weight close to the shifted natural frequency (Huszár, 2009). This effect remains marginal if the whole beam vibrates in the same phase as in the 1st mode. The presence of this multiple peak is clearly visible in Fig. 3.15b for the harmonic excitation but is partly masked by the other above-mentioned secondary frequencies in Fig. 3.15a for the impulse excitation. However, using harmonic excitation, the presence and the intensity of the less dominant peak may also be deceiving in cases when incorrectly coinciding the excitation frequency with the frequency associated with the dominant peak. The high SD and the intensive secondary peak before \( f_2 \) in Fig. 3.15b support these suppositions.

3.4.3 Computing the average of natural frequencies based on individual spectrums

Sect. 3.4.2 drew attention to the presence of multiple peaks around \( f_2 \) in the computed average frequency spectrums and introduced the reasons behind them. Also being aware of the fact that if a multiple peak occurs, none of the local sub-peaks in the multiple peak corresponds to the exact natural frequency, which is associated with a strictly symmetric mode shape and represented as a single, narrow peak in the average frequency spectrum, the question remains: what to consider as a natural frequency during the evaluation process when multiple peaks exist and, moreover, when the local sub-peaks fall relatively far from each other. The question is even more stressed because asymmetry in the crack pattern is relevant, likely and actually not controllable during the experimental deterioration process (and also in practice). A typical example is shown in Fig. 3.16 which introduces average frequency spectrums belonging to state 4 of the P1 beam for both types of excitation. It can be seen that the frequency difference between local sub-peaks in Fig. 3.16a is \( \sim 7\% \). For deterioration states with significant cracking, similar average frequency spectrums were obtained.
In order to numerically eliminate the multiple peaks and be able to set $f_2$ for further evaluation and for comparability reasons, it was decided to consider $f_2$ as the average of frequencies associated with the maximum ordinates in the individual frequency spectrums. The reasons behind this decision were as follows:

- for average frequency spectrums without multiple peaks (typical around $f_1$) the difference between the average of frequencies associated with the maximum ordinates in the individual frequency spectrums (henceforth in this section shortened as average frequency) and the frequency associated with the maximum ordinate in the average frequency spectrum remains marginal even if the “hill” around the peak in the average spectrum is wide.
- due to the mentioned frequency shifts in the vicinity of multiple peaks, the average frequency value falls certainly closer to the exact value of the natural frequency, which is associated with strictly symmetric mode shape and represented as a single, narrow peak in the average frequency spectrum, than either of the duplicated peaks.
- for harmonic excitation, the number of false individual spectrums (which derived from incorrect coincidence of the excitation frequency with one of the sub-peaks in the average spectrum) relative to the total number of individual spectrums can be considered as a stochastic parameter that depends first of all on the abilities of the operator, who adjust the excitation frequency to cause resonance effects. Presumably this stochastic parameter nearly identically influences the positions of sub-peaks in the average spectrum as well as the above average frequency because the individual spectrums are common input for both. Therefore no additional uncertainty is imported when dealing with the average frequency calculated simply on the basis of individual spectrums instead of sub-peaks in the average spectrum.
In order not to lose information on the presence of multiple peaks when using the average frequency values, the associated SD values will also be calculated. If frequencies associated with sub-peaks in a duplicated peak significantly differ from the average frequency, then the magnitude of SD values clearly reflects it (the higher the SD the more dominant the presence of the duplicated peak in the average spectrum). The SD associated with \( f_2 \) after state 4 of the P1 beam (the related frequency spectrums are shown in Fig. 3.16) resulted in 4.125 Hz (!) for impulse excitation and 1.546 Hz for harmonic excitation, which were much higher than the previously (in Sect. 3.4.2) calculated corresponding SD for state 0 of the same beam (the frequency spectrums belonging to state 0 of the P1 beam without significant duplicated peaks are shown in Fig. 3.15).

3.5 RESULTS OF FREQUENCY MEASUREMENTS

Following the procedure detailed in Sect. 3.3.2 and applying the signal evaluation technique discussed before in Sect. 3.4, the measured values of the first (\( f_1 \)) and the second (\( f_2 \)) natural frequencies of beams belonging to states in the deterioration period listed in Fig. 3.3, Table 3.4 and Table 3.6 are included in Table 3.7a-e. The frequency results belonging to states in the strengthening period (states 6p, 7p and 8p, in which post-tensioning was applied) will be analysed separately in Chapter 5.

3.5.1 Natural frequencies for the deterioration states

In the case of the P1 beam, the recorded vibration signals derived from both impulse and harmonic excitations according to Sect. 3.3.2.2 were evaluated according to Sect. 3.4.1-3.4.3. The \( f_1 \) and \( f_2 \) values in Table 3.7a correspond to averages of 30 frequencies each associated with the maximum ordinates in the related individual frequency spectrums (average frequencies based on 30 individual frequency spectrums). The associated SD values (\( s_1 \) and \( s_2 \) respectively) are also given in Table 3.7a; their highest values are bold-faced.

<table>
<thead>
<tr>
<th>Beam</th>
<th>State</th>
<th>Natural frequencies and their SD [Hz] determined from</th>
<th>Relative differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Impulse excitation</td>
<td>harmonic excitation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( f_1 )</td>
<td>( s_1 )</td>
</tr>
<tr>
<td>P1</td>
<td>0</td>
<td>56.37</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>52.35</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>51.94</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>3.1</td>
<td>51.10</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td>51.06</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>30.67</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>30.52</td>
<td>0.15</td>
</tr>
</tbody>
</table>
The last two columns of Table 3.7a contain the relative difference between the corresponding frequencies computed from signals, which resulted from different types of excitation. As shown, it remains below 0.6% for $f_1$ and 3% for $f_2$ in each state. This relative difference for $f_1$ is quite acceptable and demonstrates that both types of excitation can reliably be used for experimental purposes if a suitable statistical evaluation process is applied. In states when the highest relative differences occurred (mostly for $f_2$) the corresponding SD values were also high. This clearly indicates the presence of multiple peaks in the frequency spectrums as discussed in Sect. 3.4.

In light of the good coincidence between the determined natural frequencies obtained from signals with different excitation inputs for the P1 beam, it was generally decided to use signals derived only from the impulse excitation for further investigation purposes. When significant multiple peaks in the individual frequency spectrums occurred (in the case of high SD values), further evaluations were carried out to find the probable reasons behind them by the use of signals derived from harmonic excitation (see further comments to this in Sect. 3.5.2). In order to spare the available storage capacity and to reduce the necessary computational effort, the number of individual spectrums, on the basis of which the average frequencies were calculated, was reduced to 10 (from 30, which was taken into account in the case of the P1 beam). Therefore Table 3.7b–e contain the first two natural frequency values ($f_1$ and $f_2$) computed (similarly to that in Table 3.7a for the P1 beam) as the averages of frequencies associated with the maximum ordinates in the related 10 individual frequency spectrums as well as the related SD values (see further comments in Sect. 3.5.2). High SD values are bold-faced.

Table 3.7b Measured values of natural frequencies and the associated SD for the P2 beams

<table>
<thead>
<tr>
<th>State</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.37</td>
<td>3.12</td>
<td>0.35</td>
<td>0.19</td>
</tr>
<tr>
<td>1</td>
<td>19.89</td>
<td>0.45</td>
<td>2.93</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>20.45</td>
<td>0.03</td>
<td>7.11</td>
<td>0.26</td>
</tr>
<tr>
<td>3</td>
<td>20.10</td>
<td>0.12</td>
<td>7.78</td>
<td>7.42</td>
</tr>
<tr>
<td>4</td>
<td>19.70</td>
<td>0.19</td>
<td>7.62</td>
<td>6.88</td>
</tr>
<tr>
<td>5</td>
<td>19.70</td>
<td>0.06</td>
<td>7.41</td>
<td>6.78</td>
</tr>
<tr>
<td>6</td>
<td>19.32</td>
<td>0.16</td>
<td>7.46</td>
<td>3.81</td>
</tr>
</tbody>
</table>

Table 3.7c Measured values of natural frequencies and the associated SD for the R1 beams

<table>
<thead>
<tr>
<th>State</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.10</td>
<td>0.00</td>
<td>72.43</td>
<td>0.23</td>
</tr>
<tr>
<td>1</td>
<td>18.64</td>
<td>0.08</td>
<td>74.97</td>
<td>0.52</td>
</tr>
<tr>
<td>2</td>
<td>18.25</td>
<td>0.00</td>
<td>72.86</td>
<td>0.27</td>
</tr>
<tr>
<td>3</td>
<td>18.09</td>
<td>0.06</td>
<td>71.76</td>
<td>0.26</td>
</tr>
<tr>
<td>4</td>
<td>18.13</td>
<td>0.00</td>
<td>71.22</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>17.50</td>
<td>0.00</td>
<td>69.62</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Table 3.7d Measured values of natural frequencies and the associated SD for the R2 beams

<table>
<thead>
<tr>
<th>State</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.37</td>
<td>0.06</td>
<td>99.86</td>
<td>0.35</td>
</tr>
<tr>
<td>1</td>
<td>19.89</td>
<td>0.45</td>
<td>2.93</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>20.45</td>
<td>0.03</td>
<td>7.11</td>
<td>0.26</td>
</tr>
<tr>
<td>3</td>
<td>20.10</td>
<td>0.12</td>
<td>7.78</td>
<td>7.42</td>
</tr>
<tr>
<td>4</td>
<td>19.70</td>
<td>0.19</td>
<td>7.62</td>
<td>6.88</td>
</tr>
<tr>
<td>5</td>
<td>19.70</td>
<td>0.06</td>
<td>7.41</td>
<td>6.78</td>
</tr>
</tbody>
</table>

Table 3.7e Measured values of natural frequencies and the associated SD for the R3 beams

<table>
<thead>
<tr>
<th>State</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>20.10</td>
<td>0.00</td>
<td>72.43</td>
<td>0.23</td>
</tr>
<tr>
<td>1</td>
<td>18.64</td>
<td>0.08</td>
<td>74.97</td>
<td>0.52</td>
</tr>
<tr>
<td>2</td>
<td>18.25</td>
<td>0.00</td>
<td>72.86</td>
<td>0.27</td>
</tr>
<tr>
<td>3</td>
<td>18.09</td>
<td>0.06</td>
<td>71.76</td>
<td>0.26</td>
</tr>
<tr>
<td>4</td>
<td>18.13</td>
<td>0.00</td>
<td>71.22</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>17.50</td>
<td>0.00</td>
<td>69.62</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Chapter 3  EXPERIMENTAL DETERIORATION PROCESS

Table 3.7d Measured values of natural frequencies and the associated SD for the R2 beams

<table>
<thead>
<tr>
<th>State</th>
<th>R2/1 $f_i$</th>
<th>R2/1 $a_i$</th>
<th>R2/1 $f_j$</th>
<th>R2/1 $a_j$</th>
<th>R2/2 $f_i$</th>
<th>R2/2 $a_i$</th>
<th>R2/2 $f_j$</th>
<th>R2/2 $a_j$</th>
<th>R2/3 $f_i$</th>
<th>R2/3 $a_i$</th>
<th>R2/3 $f_j$</th>
<th>R2/3 $a_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.89</td>
<td>0.00</td>
<td>77.22</td>
<td>1.06</td>
<td>0</td>
<td>10.13</td>
<td>0.02</td>
<td>78.36</td>
<td>0.11</td>
<td>0</td>
<td>10.13</td>
<td>0.02</td>
</tr>
<tr>
<td>1</td>
<td>11.1</td>
<td>0.00</td>
<td>91.58</td>
<td>0.33</td>
<td>1</td>
<td>11.01</td>
<td>0.01</td>
<td>84.87</td>
<td>0.33</td>
<td>1</td>
<td>11.01</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>10.83</td>
<td>0.07</td>
<td>70.64</td>
<td>0.71</td>
<td>2</td>
<td>10.13</td>
<td>0.02</td>
<td>72.24</td>
<td>0.13</td>
<td>2</td>
<td>10.13</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>10.19</td>
<td>0.06</td>
<td>70.17</td>
<td>0.22</td>
<td>3, 4</td>
<td>10.16</td>
<td>0.00</td>
<td>72.08</td>
<td>0.13</td>
<td>3, 4</td>
<td>10.16</td>
<td>0.00</td>
</tr>
<tr>
<td>3_1</td>
<td>10.19</td>
<td>0.06</td>
<td>70.17</td>
<td>0.22</td>
<td>3, 4</td>
<td>10.16</td>
<td>0.00</td>
<td>72.08</td>
<td>0.13</td>
<td>3, 4</td>
<td>10.16</td>
<td>0.00</td>
</tr>
<tr>
<td>3_2</td>
<td>18.10</td>
<td>0.05</td>
<td>69.80</td>
<td>0.18</td>
<td>5, 2</td>
<td>18.13</td>
<td>0.03</td>
<td>71.87</td>
<td>0.22</td>
<td>5, 2</td>
<td>18.13</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>18.57</td>
<td>0.04</td>
<td>66.53</td>
<td>0.06</td>
<td>4</td>
<td>17.90</td>
<td>0.04</td>
<td>69.22</td>
<td>0.16</td>
<td>4</td>
<td>17.90</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>15.60</td>
<td>0.07</td>
<td>65.23</td>
<td>0.05</td>
<td>5</td>
<td>16.30</td>
<td>0.07</td>
<td>69.21</td>
<td>0.06</td>
<td>5</td>
<td>16.25</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 3.7e Measured values of natural frequencies and the associated SD for the P2p beams

<table>
<thead>
<tr>
<th>State</th>
<th>P2p/1 $f_i$</th>
<th>P2p/1 $a_i$</th>
<th>P2p/1 $f_j$</th>
<th>P2p/1 $a_j$</th>
<th>P2p/2 $f_i$</th>
<th>P2p/2 $a_i$</th>
<th>P2p/2 $f_j$</th>
<th>P2p/2 $a_j$</th>
<th>P2p/3 $f_i$</th>
<th>P2p/3 $a_i$</th>
<th>P2p/3 $f_j$</th>
<th>P2p/3 $a_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25.32</td>
<td>0.07</td>
<td>97.56</td>
<td>0.07</td>
<td>0</td>
<td>24.64</td>
<td>0.04</td>
<td>94.09</td>
<td>1.58</td>
<td>0</td>
<td>25.56</td>
<td>0.02</td>
</tr>
<tr>
<td>1</td>
<td>25.25</td>
<td>0.00</td>
<td>96.59</td>
<td>0.06</td>
<td>1</td>
<td>25.65</td>
<td>0.05</td>
<td>92.47</td>
<td>0.39</td>
<td>1</td>
<td>24.67</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>25.18</td>
<td>0.06</td>
<td>95.70</td>
<td>0.06</td>
<td>2</td>
<td>24.13</td>
<td>0.03</td>
<td>91.03</td>
<td>0.15</td>
<td>2</td>
<td>24.51</td>
<td>0.04</td>
</tr>
<tr>
<td>3c</td>
<td>25.19</td>
<td>0.06</td>
<td>95.13</td>
<td>0.10</td>
<td>3c</td>
<td>23.64</td>
<td>0.04</td>
<td>89.39</td>
<td>0.21</td>
<td>3c</td>
<td>24.03</td>
<td>0.05</td>
</tr>
<tr>
<td>41c</td>
<td>25.17</td>
<td>0.00</td>
<td>94.60</td>
<td>0.12</td>
<td>4</td>
<td>22.36</td>
<td>0.03</td>
<td>85.23</td>
<td>1.46</td>
<td>4</td>
<td>22.53</td>
<td>0.00</td>
</tr>
<tr>
<td>42c</td>
<td>25.13</td>
<td>0.00</td>
<td>94.70</td>
<td>0.12</td>
<td>5</td>
<td>21.88</td>
<td>0.03</td>
<td>80.10</td>
<td>0.64</td>
<td>5</td>
<td>19.71</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Unfortunately some electronic data have been lost due to unexpected events such as temporary power loss or disk failure during the test. This was the case between states 0 and 2 for the R2/3 beam as well as in states 41c and 42c for the P2p/1 beam; their data are missing from Table 3.7.

The analysis of the above frequency results in relation to test variables and the defined damage indices can be found in Chapter 5.

3.5.2 Evaluation technique applied for multiple peaks

In calculating the average of frequencies associated with the maximum amplitudes in the individual frequency spectrums (average frequencies) together with their SD, the sudden increase of SD value from a previous acceptable level (< 1 Hz) was found to be a good indication of mode deformation, and consequently, of multiple (mostly duplicated) peaks in the related frequency spectrums. In states within the deterioration period this mode deformation was clearly caused by a non-symmetric crack pattern and, consequently, non-symmetric bending rigidity distribution along the beam length. Although all beams were geometrically prismatic and the loading was symmetric in each state (except when tendon cuts were made in state 43c for the P2p/1 beam), uneven tensile strength of concrete may result in a non-symmetric crack pattern. Sect. 3.4.2 and Sect. 3.4.3 discussed how non-symmetric bending stiffness distribution resulted in
duplicated peaks in the relevant frequency spectrums especially for modes with asymmetric shapes (here the second mode was affected). Sect. 3.4.3 also discussed that if these sub-peaks fell relatively close to each other and their relative weights were nearly identical in the average spectrum (this was the case if their relative weight were either nearly identical or alternately became maximum in the subsequent individual spectrums) then the calculation of the average frequency (average of frequencies associated with the maximum amplitudes in the individual spectrums) together with its SD was able to reflect on these kinds of duplicated peaks (see Fig. 3.16a) and to be a good estimation of the real natural frequency (associated with no mode deformation). However, when the relative weights of the sub-peaks are significantly different in each individual spectrum and one of them is dominant (this is presumably the case when the difference in bending stiffness between the two beam halves becomes significant due to a non-symmetric crack pattern) then the average frequency reflects only the dominant peak with possibly low SD (in cases when the sub-peaks fall relatively far from each other). This was definitely the situation through states 4, 5, 6 and 7 for the P2/1 beam; Fig. 3.17 shows the average frequency spectrums (based on 10 individual spectrums) belonging to state 5.

In calculating the average frequency for \( f_2 \) and the associated SD based only on signals derived from impulse excitation according to Sect. 3.5.1, the frequencies resulted in 85.61 Hz, 83.40 Hz, 83.65 Hz and 76.48 Hz (associated with SD of 0.30, 0.54, 0.29 and 0.60) for states 4, 5, 6 and 7 respectively, which did not fit into the usual decreasing tendency of \( f_2 \) observed both through previous states of this P2/1 beam and for all other beams together. The calculated SD values were acceptably low. However, the average spectrum showed another local peak around 70 Hz with much lower (but the same order of) relative weight compared to the calculated frequencies around 80 Hz (Fig. 3.17a). To check this observation, recorded signals derived from harmonic excita-
tion were also analysed. Clear resonance effects could be detected around both 70 Hz and 80 Hz. The computed average spectrum based on ten individual spectrums, five of which belonged to natural frequencies around 70 Hz and another five, to that around 80 Hz, gave a similar result (Fig. 3.17b). Based on this, it was decided to include individual spectrums with frequencies having a maximum relative weight around 70 Hz into the statistical analysis. The average frequency in Table 3.7b for these states were calculated from 30 data, 15 of which fell into a relatively narrow frequency interval around 80 Hz and another 15 of which fell into a similar interval around 70 Hz. Consequently, the associated SD values became considerably high but, in accordance with the original intention, unambiguously indicated the presence of duplicated peaks. Similar but less intensive duplications around $f_2$ could be observed in states where the SD values are bold-faced in Table 3.7a-e.
DEFINITION OF DAMAGE INDICES
4.1 GENERAL

Damage indices are used to express information on the structural behaviour in numerical form. This chapter deals with the definition, classification, calculation and comparison of damage indices related to the investigated test beams. In this research, only static-based indices are used in accordance with Sect. 2.3. The applicability of model-based indices is confirmed by the corresponding non-model-based indices through conformity checks. The defined indices will be used in Chapter 5 to identify crack-related damage in test beams.

4.1.1 Interpretation of crack-related damage

At the moment when a reinforced concrete beam section cracks under flexure, the strain energy stored previously in the concrete in the tension zone of the section discharges and is immediately transformed first into kinetic energy, which results in a sudden deflection increment of the beam, then again into strain energy, which will be stored by mostly the tension reinforcement. While this strain and the corresponding tension force develops in the tension reinforcement, a parallel force transmission from steel to concrete also takes place along the transmission length of the reinforcing bars. During this latter force transmission, irreversible micro-cracks develop in the concrete around the bars, which "softens" the concrete zones beyond the crack faces.

If a reinforcing bar at a crack yields then residual strain remains in the bar after load removal. If this crack often opens and closes afterwards due e.g. to further loading, the crack gradually deteriorates (grains fall off the crack faces). For the test beams, all the above events (micro-cracking, plastic strain in bars, crack surface deterioration):

a) dissipate small parts from the energy transmitted to the beam by the acting forces in the consecutive loading phases. This dissipated energy is directed to degrade (damage) zones, which are beyond the linear elastic stage, primarily in the vicinity of cracks (energy-based approach);

b) are mostly irreversible and result in preventing the main crack to perfectly close after load removal. Open cracks result in local bending stiffness decrease along the beam length (crack-amount-based approach).

Instead of a theoretical approach, we suppose that:

a) the amount of energy dissipated in the vicinity of cracks is proportional to that transmitted to the beam by the acting forces in the current loading phase;

b) the extent of bending stiffness decrease is proportional to the extent of cracking, which is measured by the number and the width of individual cracks.
The following damage indices are defined in accordance with either of the above approaches.

### 4.1.2 Practical aspects of damage index definition

From a practical point of view, if neglecting accidental events and specially designed (e.g. over-reinforced) elements, cracking is an obvious, reliable and most frequently appearing indication of damage for concrete structures being in service. The reasons behind cracking may be different (see Sect. 1.1.2) such as mechanical forces, restrained deformations, long-term or cyclic loads, chemical attacks, environmental effects, etc; furthermore, the extent of cracking is tightly influenced by many structural parameters such as intensity and type of subjecting forces, reinforcement ratio, presence of prestressing, etc. However, cracking is definitely unfavourable from the point of view of serviceability and durability. Therefore, instead of the reasons behind cracking, which are of low interest from the point of view of damage identification; the presence (detection and localization) as well as the extent (quantification) of cracking are more important information on the deterioration of the structure and, consequently, are aimed to be addressed.

In this test, cracking was caused and further intensified by simple mechanical effects (forces and wire cuts). The consecutive loading phases were planned to gradually intensify cracking either by extending the cracked length of the beam or by widening the already existing cracks. The resulting cracks seriously influenced the structural response of the beams. These crack-influenced parts of the structural response or its consequences (e.g. residual effects after the removal of crack-inducing mechanical effects) were intended to be indicated by damage indices. Therefore, all damage indices were such defined to be influenced by cracking.

According to the interpretation of damage indices in Sect. 2.3, the non-model-based indices were not based on any previous calculation model; they were defined as parameters, which were directly measured on the beams or deduced from the measured parameters without any additional model. The model-based indices were based on a previous static model and obtained from it by calculations.

### 4.1.3 Local versus global indices

A damage index is considered to be local if the value of the index depends on features belonging to a local zone of the structure. Thus, these indices do not reflect closely on the structural behaviour of the structure as a whole (their information content on the
Consequently, from the point of view of damage detection (Level 1) and localization (Level 2), they are extremely effective when damage happens within the related zone, but unusable when damage happens outside this zone. Global indices are generally obtained from structural parameters gathered from many (preferably all) zones of the structure. Consequently, features of a local zone do not influence the value of a global index considerably. Global indices may be more suited for damage detection if the location of damage is unknown, but local indices are more practical if the potential damage can be localized to a particular zone of the structure.

When using global indices, the effectiveness of damage localization (and maybe quantification) may be increased if not only one but more structural parameters, which are primarily linked to different zones of the structure, are investigated (and numerically determined). That is why the first two natural frequencies are simultaneously investigated in this test. Natural frequency shifts are influenced mostly by zones of beams where the highest amplitudes of the associated mode shapes are located. Therefore, damage happened around the midspan zone is expected to result in a higher shift in $f_1$ than in $f_2$ and, taking the opposite of this, a higher shift in $f_2$ suggests damage occurred around the quarters of the beam.

### 4.1.4 Cumulative versus incrementative indices

Some damage indices are definitely intended to measure damage accumulated in the structure due to previous deteriorating effects. Theoretically, they cannot decrease during a deterioration process unless the healing of damage is possible. Healing of cracks for some civil engineering structures under special service conditions is possible, even without any repair work, but in this test this scenario is neglected. Therefore, if no new damage develops in an intermediate stage within a deterioration process then the applied damage index should be defined so that its value corresponding to this stage is at least equal to that corresponding to the previous stage. These cumulative indices are capable in the first place of damage quantification (Level 3).

If a damage index is intentionally used instead for detection purposes (Level 1) then its increment between the subsequent states has to be deduced. In this case, the increment becomes equal to zero for states without new damage.

For this test, the applied damage indices are defined according to their incrementative form but their cumulative form is also discussed.
### 4.2 Definition of Damage Indices

The following tables contain the numerical values of all non-model-based and model-based damage indices defined below in Sect. 4.2 and Sect. 4.3. Further comments to explain the data included in the tables are also given in the relevant sections.

#### Table 4.1a Damage indices for the P1 beam

<table>
<thead>
<tr>
<th>Beam</th>
<th>Non-model-based indices</th>
<th>Model-based indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l_1$, $l_2$, $l_3$, $l_4$</td>
<td>$A_1$, $A_2$, $W$, $\Delta W$</td>
</tr>
<tr>
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<td>under</td>
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<td>2</td>
<td>1881</td>
<td>333</td>
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<tr>
<td>3</td>
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</tr>
<tr>
<td>5</td>
<td>3431</td>
<td>65</td>
</tr>
</tbody>
</table>

#### Table 4.1b Damage indices for the P2 beams

<table>
<thead>
<tr>
<th>Beam</th>
<th>Non-model-based indices</th>
<th>Model-based indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l_1$, $l_2$, $l_3$, $l_4$</td>
<td>$A_1$, $A_2$, $W$, $\Delta W$</td>
</tr>
<tr>
<td></td>
<td>under</td>
<td>after</td>
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<td>0.0</td>
<td>0.0</td>
</tr>
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</tr>
<tr>
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<td>2307</td>
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</tr>
<tr>
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<td>3982</td>
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<tr>
<td>7</td>
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</table>

#### Table 4.1c Damage indices for the R1 beams

<table>
<thead>
<tr>
<th>Beam</th>
<th>Non-model-based indices</th>
<th>Model-based indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l_1$, $l_2$, $l_3$, $l_4$</td>
<td>$A_1$, $A_2$, $W$, $\Delta W$</td>
</tr>
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<td></td>
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#### Table 4.1d Damage indices for the R2 beams

<table>
<thead>
<tr>
<th>Beam</th>
<th>Non-model-based indices</th>
<th>Model-based indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l_1$, $l_2$, $l_3$, $l_4$</td>
<td>$A_1$, $A_2$, $W$, $\Delta W$</td>
</tr>
<tr>
<td></td>
<td>under</td>
<td>after</td>
</tr>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td>1/3</td>
<td>6.7</td>
<td>6.7</td>
</tr>
<tr>
<td>1/2</td>
<td>15.0</td>
<td>17.3</td>
</tr>
<tr>
<td>1</td>
<td>23.8</td>
<td>27.7</td>
</tr>
<tr>
<td>4/3</td>
<td>20.2</td>
<td>25.0</td>
</tr>
<tr>
<td>4/3</td>
<td>29.1</td>
<td>33.5</td>
</tr>
</tbody>
</table>
### Definition of Damage Indices

#### Table 4.1d Damage Indices for the R2 Beams

<table>
<thead>
<tr>
<th>R1/2</th>
<th>R1/3</th>
<th>R1/4</th>
<th>R2/1</th>
<th>R2/2</th>
<th>R2/3</th>
</tr>
</thead>
</table>

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Table 4.1d Damage indices for the R2 beams
4.2.1 Non-model-based indices

For this test, defined in their incrementative forms, the growth of “total length of cracks” ($\Delta l_r$), the growth of deflection measured under loading at midspan ($\Delta a_{\text{tot}}$), the growth of residual deflection at midspan ($\Delta a_{\text{res}}$) as well as the growth of crack width measured at midspan ($\Delta w$) were used as non-model-based indices.

4.2.1.1 Growth of “total length of cracks” ($\Delta l_r$)

The length of an individual crack is interpreted as the length of the irregular curve (or curves for branching cracks) which is outlined on the side face of the beam and stretches from the extreme tension fibre up to the tip of the crack. The total length of cracks ($l_r$) summarizes the above lengths of individual cracks over the full cracked length ($L_{\text{cr}}$) of the beam.

Defining in that way, the total length of cracks may be considered as a damage index strictly following the crack-amount-based approach according to Sect. 4.1.1 and containing information on the whole structure (global index). Because it is based only on in-situ measurements, it can be classified as a non-model-based index.

To determine the total length of cracks, the lengths of individual cracks had to be read directly from the crack pattern. This process consumed considerable time, therefore it
was gathered only for the P1, P2/1 and P2/2 beams. As an example, Fig. 4.1 shows the crack patterns recorded on the side faces of the P1 beam.

![Crack patterns recorded on the side faces of the P1 beam](image)

The length of the newly-appearing individual cracks as well as the length increment of existing cracks were recorded under loading phases and cumulated into $l_r$ for each deterioration state.

Allowing for Sect. 4.1.4, the incrementative form of $l_r$ was defined as $\Delta l_r^i = l_r^i - \max(l_r^{<i})$ where $l_r^i$ was the value of $l_r$ in the $i$-th deterioration state and $\max(l_r^{<i})$ was the maximum of $l_r$ values belonging to previous states. The values of $l_r$ and of its incrementative form ($\Delta l_r$) can be found in Table 4.1 for the related beams.

### 4.2.1.2 Growth of total ($\Delta a_{tot}$) and residual ($\Delta a_{res}$) deflection at midspan

These damage indices, defined simply as a deflection value or combination of deflection values, do not strictly follow either approach of Sect. 4.1.1. However, being the curvature along a section of a cracked beam mostly concentrated in open cracks, the curvature is very closely linked to both the number of cracks along this beam section and their widths. Fig. 4.2 schematically shows bending moment ($M$) versus curvature ($\rho = 1/r$) relationships for a reinforced (non-prestressed) and a prestressed beam cross-section. The circled zones symbolize the typical crack-related damage effects of the cross-sections, which may correspond to a crack for a reinforced cross-section and to tendon breaks for a prestressed cross-section. Both of them result in sudden curvature increment on the cross-section level and, furthermore, in deflection increment on the beam level. These latter deflection increments are intentionally reflected by these indices. Thus, they may rather be considered as following the crack-amount based approach according to Sect. 4.1.1.

Since deflection at any (e.g. midspan) cross-section of a beam depends on the magnitude and the distribution of curvature along the whole beam, damage indices based purely on deflection values may be considered as global rather than local indices. However, the local character of $\Delta a_{tot}$ and $\Delta a_{res}$, which are linked to the midspan cross-section, is also obvious if considering load configurations, which cause greater extent
of damage in the whole beam and result in smaller deflection at midspan compared to other load configurations. Because deflection values are fully in-situ-measured parameters and the deduction of $\Delta a_{\text{tot}}$ and $\Delta a_{\text{res}}$ do not add any model-based information to them, the $\Delta a_{\text{tot}}$ and $\Delta a_{\text{res}}$ indices fall within the group of non-model-based indices.

According to Sect. 3.3.2, the deflection at midspan ($a_{\text{measured}}$) was registered under loading and after load removal for each loading phase. The deflection of beams under state 0 (self-weight only) was not measured therefore all measured and deduced deflection values were related to beam shape already deformed by the self-weight. Because the deflection indicator was removed before each dynamic measuring phase, it had to be re-installed before the subsequent loading phase. Therefore, actual deflection values developed under loading ($a_{\text{tot}}$) and after load removal ($a_{\text{res}}$) had to be obtained from the measured ($a_{\text{measured}}$) values for each loading phase. Accordingly, $a_{\text{tot}}$ and $a_{\text{res}}$ associated with the investigated state were obtained as the $a_{\text{res}}$ value of the previous state (baseline) plus the $a_{\text{measured}}$ value of the investigated state. This was not the case for the P2p beams because deflection measurements during wire cuttings were impossible due to technical reasons. Thus, deflection increments in states belonging to individual wire cuts (such states are shown in Fig. 3.3 and marked by “c” in the state name in Table 3.7) were not registered and, consequently, no baselines were available for the loading phases of the subsequent states. Therefore, no $a_{\text{tot}}$ and $a_{\text{res}}$ values were calculated for these subsequent states.

The incrementative form of $a_{\text{tot}}$ was defined as $\Delta a_{\text{tot}}^i = \max\{a_{\text{tot}}^i - \max(a_{\text{tot}}^{c_i})\} \forall c_i \in [0, \text{max}])$ where $a_{\text{tot}}^i$ was the value of $a_{\text{tot}}$ in the $i$-th deterioration state and $\max(a_{\text{tot}}^{c_i})$ was the maximum of $a_{\text{tot}}$ values belonging to previous states. $\Delta a_{\text{res}}$ was analogously defined. The results can be found in Table 4.1 for all beams.
4.2.1.3 Growth of crack width at midspan ($\Delta w$)

In practice, the measurement of crack width is difficult and its outcome is unreliable. Many cracks often become not active under increasing load, thus crack width does not reflect strictly on the load intensity. Furthermore, widths of neighbouring cracks do not necessarily fall close to each other.

The crack width ($w$) in a beam cross-section is fully determined by the internal force and by the geometric, material and reinforcement properties of the cross-section, therefore this crack width itself is considered as a typical local index. Actual crack width measured directly on the beam is categorized as a non-model-based index. It clearly follows the crack-amount-based approach according to Sect. 4.1.1. In fact, the residual width of an active crack would be the ideal index to reflect fully on accumulated damage, however, its measurement with acceptable accuracy is practically unrealistic.

For this test, crack width at midspan ($w$) was measured by microscope with 0.025 mm accuracy under loading in every state for many (P2/3, all R1 and R2) but not all beams. The measured $w$ values and the incrementative form of the index ($\Delta w$), defined analogously as introduced in Sect. 4.2.1.2 for $\Delta a_{loa}$, can be found in Table 4.1 for the related beams.

4.2.2 Model-based indices

In calculating model-based indices in accordance with Sect. 4.1.1 and Sect. 2.3 for this test, assumptions were made regarding the loads, the geometry, the materials, the material law and the structural behaviour. Concerning the first three, nominal values of acting forces, nominal sizes, mean values of material properties, were used according to Sect. 3.3. With regard to material law, a linear-elastic stress-specific strain relationship was taken into account without limits in stress or strain, therefore plastic strains were not addressed by the applied calculation model.

Concerning the structural behaviour, three models were applied. The first was the well-known Bernoulli-Navier assumption (principle of plane sections) in accordance with the general beam theory. The further two models were adopted from the Eurocode 2. One of them was the calculation model of deformations of structural elements, which was applied in this test to calculate the $\rho(x)$ curvature of cross-sections along the beam length as follows:

$$\rho(x) = \zeta(x) \frac{M(x)}{E_{cm} I(x)} + \left[1 - \zeta(x)\right] \frac{M(x)}{E_{cm} l(x)}$$  (4.1)
where \( x \) measured the distance along the specimen from one of the supports; \( I_1(x) \) and \( I_{II}(x) \) were the moments of inertia of the cross-section at \( x \) in linear elastic uncracked and cracked stage, respectively; \( M(x) \) was the subjecting bending moment at \( x \); \( E_{cm} \) was the mean value of the modulus of elasticity of concrete. \( \zeta(x) \) was the distribution coefficient allowing for tension stiffening and calculated as follows:

\[
\zeta(x) = 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s(x)} \right)^2
\]  

(4.2)

where \( \beta = 1 \) took into account short-term loading; \( \sigma_{sr} \) and \( \sigma_s(x) \) were the stress in the tension reinforcement calculated on the basis of cracked cross-section due to cracking moment (\( M_{cr} \), see Table 3.5) and \( M(x) \), respectively.

---

For cross-sections without normal force (reinforced P1, P2, R1 and R2 beams), \( I_1(x) = I_1 = \text{constant} \) and \( I_{II}(x) = I_{II} = \text{constant} \) were considered. The \( \zeta(x) \) was set equal to zero for cross-sections uncracked earlier or (only for prestressed beams) being under decompression. For previously cracked reinforced cross-sections (without normal force) subjected to \( M(x) < M_{cr} \), \( \zeta(x) \) was taken to be at least equal to its value in the previous state. For prestressed (P2p type) cross-sections with no wire cuts and subjected to \( M(x) > M_{cr} \), first \( \rho \) was calculated at particular cross-sections \( \rho(x_i) \) along the cracked length of the beam and then linearly interpolated between these \( \rho(x_i) \) values for the remaining cross-sections (state 1). In the vicinity of wire cuts, \( \rho \) was...
remaining cross-sections (state 1). In the vicinity of wire cuts, \( \rho \) was calculated, on one hand, exactly at the cut (allowing for only the remaining number of wires) and, on the other hand, at the end of the transmission length on both sides of the cut. For cross-sections located between these calculation points, both inside and outside the (vicinity of) cuts, linear interpolation was applied. 

Fig. 4.3 shows the accordingly calculated curvature functions along the half length of the reinforced P1 and the prestressed P2p/3 beam. 

The third structural model was related to crack width calculation. According to this, the width of a crack (\( w_{\text{calc}} \)) in a cross-section was calculated as follows:

\[
 w_{\text{calc}} = s_{r,\text{max}} (\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}}) 
\]

(4.3)

where \( s_{r,\text{max}} \) estimates the maximum crack spacing according to Eq. (4.4):

\[
 s_{r,\text{max}} = 3.4 c + 0.425 k_1 k_2 \frac{\phi}{\rho_{p,\text{eff}}} 
\]

(4.4)

where \( c \) is the concrete cover; \( \phi \) is the bar diameter; \( k_1 \) allows for the bond properties of the reinforcement and \( k_2 \) takes into account the strain distribution along the cross-section. \( k_1 = 0.8 \) (high bond) was used for reinforcing bars (R1 and R2 beams) and \( k_1 = (0.8+1.6)/2 = 1.2 \) (ribbed wires) for prestressing wires (P1, P2 and P2p beams). Due to bending, \( k_2 \) was taken as \( k_2 = 0.5 \). \( \rho_{p,\text{eff}} \) is the effective steel ratio as follows:

\[
 \rho_{p,\text{eff}} = \frac{A_s}{A_{c,\text{eff}}} 
\]

(4.5)

where \( A_{c,\text{eff}} \) is the effective tension area of concrete, whose depth is the minimum of \( 2.5(h-d) \), \( (h-x)/3 \) and \( h/2 \) (\( h \) and \( d \) are the height and the effective depth of the cross-section, respectively; \( x \) is the depth of the compression zone). \( A_s \) represents the area of longitudinal reinforcement falling within \( A_{c,\text{eff}} \). \( \varepsilon_{\text{sm}} \) and \( \varepsilon_{\text{cm}} \) are the main strain in the reinforcement within the crack and in the (uncracked) concrete between cracks, respectively; the \( (\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}}) \) strain difference was calculated as follows:

\[
 (\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}}) = \frac{\sigma_s - k_1 f_{\text{cm}}}{\rho_{p,\text{eff}} E_s} \left( 1 + \alpha_s \rho_{p,\text{eff}} \right) \geq 0.6 \frac{\sigma_s}{E_s} 
\]

(4.6)

where \( \sigma_s \) is the stress in the outermost bar assuming cracked cross-section. Only additional stress \( (\Delta \sigma_s) \) above the stress associated with zero strain in concrete at the same level was considered for the prestressed reinforcement (for P2p beams). Due to short-term loading \( k_t = 0.6 \) was taken. Furthermore, \( \alpha_e = E_s/E_{\text{cm}} \) with \( E_s \) representing the modulus of elasticity for steel.
For this test, defined according to their incrementative form, the growth of deflection \((\Delta a_{\text{calc}})\) and the growth of crack width \((\Delta w_{\text{calc}})\), both of them calculated at midspan under loading, as well as the growth of “total of crack sections” \((\Delta A_r)\) and the growth of internal strain energy of deformation \((\Delta W)\) under loading were defined as model-based indices for each deterioration state on the basis of the above models.

### 4.2.2.1 Growth of “total of crack sections” \((\Delta A_r)\)

This index, on one hand, is similar to \(\Delta l_r\) defined in Sect. 4.2.1.1 but based on a previous calculation model. However, on the other hand, it addresses not only the length of individual cracks but their widths as well. Therefore, due to its greater information content on cracking, it is more capable of identifying the growth of crack-related damage.

The section of an individual crack \((A_{r,i})\) is interpreted as shown in Fig. 4.4 (hatched area). The tip of the crack is assumed at that level of the cracked cross-section at which the flexural tensile strength of concrete \((f_{\text{ctm,fl}})\) occurs due to the subjecting loads.

Based on this, the “total of crack sections”, which summarized \(A_{r,i}\) over the cracked length of the beam, was calculated as follows:

\[
A_r = \sum_{i \in \mathcal{L}} \frac{S_{r,i}}{S_{r,max}} A_{r,i}
\]  

(4.7)

where \(L_{r,i}\) measured the cracked length of beams in the \(i\)-th state, \(S_{r,i}\) was the distance within \(L_{r,i}\), along which \(w_{\text{calc}}\) and \(h_r\) (both included in \(A_{r,i}\)) were averaged.

This index fully complies with the crack-amount-based approach according to Sect. 4.1.1 and is able to reflect crack-related damage over the whole beam. Consequently, it can be categorized as a global index.

The results can be found in Table 4.1 for all beams. For the prestressed P2p beams, this index was determined only for states before wire cuttings. For states during and after wire cuttings, the application of the above-introduced calculation procedure became unrealistic. The incrementative form of the index \((\Delta A_r)\) was defined analogously as introduced in Sect. 4.2.1.1 for \(\Delta l_r\).

### 4.2.2.2 Growth of calculated midspan deflection \((\Delta a_{\text{calc}})\)

Similarly to \(\Delta a_{\text{tot}}\) defined in Sect. 4.2.1.2, \(\Delta a_{\text{calc}}\) was intended to address the consequences of crack-related damage, which occurred somewhere in the beam, in the midspan deflection, however, on the basis of calculation models detailed above in Sect. 4.2.2. Thus, this can also be categorized as a global rather than a local index.
In determining this index, first the total deflection at midspan ($a_{\text{calc}}$) was calculated as follows:

$$a_{\text{calc}} = \varphi \frac{L}{2} - \int_0^{L/2} \rho(x) \left( \frac{L}{2} - x \right) dx$$  \hspace{1cm} (4.8)

where $\rho(x)$ was the curvature function according to Eq. (4.1); $\varphi$ was the rotation of cross-section above one of the supports, from which $x$ was measured, and $L$ meant the beam span. All the $a_{\text{calc}}$ values included in Table 4.1 were reduced by $a_{\text{calc}}^0$, which was the calculated midspan deflection in state 0 (self-weight only) and was italicized in Table 4.1, in order to express deflection due only to acting forces in each state. Calculating this manner, $a_{\text{calc}}$ became comparable to $a_{\text{tot}}$ (see Sect. 4.2.1.2).

The results can be found in Table 4.1 for all beams. The incrementative form of the index ($\Delta a_{\text{calc}}$) was defined analogously as introduced in Sect. 4.2.1.2 for $\Delta a_{\text{tot}}$.

### 4.2.2.3 Growth of calculated crack width at midspan ($\Delta w_{\text{calc}}$)

The growth of the calculated crack width at midspan ($\Delta w_{\text{calc}}$) is the model-based analogue of $\Delta w$ defined in Sect. 4.2.1.3, therefore it can be categorized accordingly. However, a close coincidence between $\Delta w_{\text{calc}}$ and $\Delta w$ was not necessarily expected because of the unreliability of crack width measurement as discussed in Sect. 4.2.1.3.

The results can be found in Table 4.1 for all beams. The incrementative form of the index ($\Delta w_{\text{calc}}$) was defined analogously as introduced in Sect. 4.2.1.3 for $\Delta w$.

### 4.2.2.4 Growth of internal strain energy ($\Delta W$)

The aim of the definition of this index is, on one hand, to provide a full global index, in which structural behaviour of no local beam parts as well as no specific crack parameters is preferred, and, on the other hand, to strictly follow the energy-based approach according to Sect. 4.1.1. In fact, this index, similarly to the above model-based indices, is based on the curvature model introduced in Sect. 4.2.2, however, calculated as a quantity irrespective of any directly visible or measurable structural characteristic. This improves its global character but makes controllability (by non-model-based indices) difficult and, consequently, decreases its robustness.

The idea behind this index is based on the assumption that the resulting damage, as interpreted in Sect. 4.1.1, in a reinforced concrete beam cross-section, which cracked earlier under flexure and then again subjected to and deformed by flexure, is proportional to that part of the internal strain energy of the deformation which is incremental to that occurred due to previous bending deformations. According to this, no newly occurring crack-related damage is assumed if the cross-section is subjected to a bending
deformation whose internal strain energy is less than that of a previous bending deforma-
tion. Also no damage is assumed for uncracked sections which are subjected to a
bending moment less than the cracking moment, because they are assumed to re-
spond fully elastic without any non-linear effect. As shown, this approach always sup-
poses non-linearity for reinforced concrete cross-sections already cracked under flex-
ure when subjected to bending deformation, which was not achieved earlier. In fact,
this non-linearity is aimed to be addressed by this index.

Note that only bending deformation is considered by the above approach. However, it
can be extended to prestressed beams, even if they work under the combination of
axial and bending deformation, if prestressing is considered as an internal action,
which shifts the limit between the uncracked and the cracked behaviour of sections as
well as if the bending deformation due to prestressing is excluded from the subjecting,
external deforming effect. Before the first crack of the prestressed cross-section, no
damage is assumed. After the first crack, when a bending deformation occurs which
was not achieved earlier, the idea supposes crack-related damage even if cracks
(theoretically) close due to prestressing after the deforming effect disappears. In this
test, this concept was applied to the P2p beams.

![Curvature function for the P1 and the P2p/3 beams](image)

**Fig. 4.5** Curvature function for the P1 and the P2p/3 beams

The internal strain energy ($W$) of a bending deformation was defined as follows:

$$ W = \int_{L} M(x)\psi(x)dx $$

(4.9)
where \( M(x) \) and \( L_r \) are in accordance with Sect. 4.2.2 and Sect. 4.2.1, respectively, and \( \rho(x) \) is according to Eq. (4.1). Fig. 4.5 shows the calculated \( M(x)\rho(x) \) function product for the half of the reinforced P1 and the prestressed P2p/3 beam (see also Fig. 3.8 and Fig. 4.3 and note that the integral in Eq. (4.9) works only over \( L_r \)).

The results can be found in Table 4.1 for all beams. The incremental form of the index (\( \Delta W \)) was defined analogously as introduced in Sect. 4.2.1.2 for \( \Delta a_{tot} \).

### 4.2.3 Comparison of non-model-based and model-based indices

The use of model-based static indices is justified if their coincidence with reality is confirmed. It is essential for practical cases when no in-situ static measurements can be made to check the calculated indices due to limited accessibility or other technical reasons. In these cases, the applied model-based indices have to be previously verified by a baseline obtained from previous in-situ measurements. If parallel in-situ measurements may be executed, then parallel non-model based indices can be defined and compared to the model-based ones to check their conformity.

For this test, two possibilities were available to conduct these checks because parallel non-model-based and model-based indices were used with regard to deflection and crack width. Within this framework, the total deflection under loading was both measured (\( a_{tot} \)) and calculated (\( a_{calc} \)). Furthermore, similarly to this, the crack width at midspan was also measured (\( w \)) and calculated (\( w_{calc} \)). These corresponding non-model-based and model-based data could directly be compared and their coincidence could be analysed. These data are shown in Fig. 4.6a-d for a few beams, by which the typical stages of their structural behaviours can be reflected.

Owing to the unreliability of crack width measurements, the comparison of \( a_{tot} \) and \( a_{calc} \) was more appropriate for the coincidence checks. Generally, with the exception of P2p beams with cut wires, good coincidence was found while the beams remained fairly elastic (see both P1 and P2 beams plus the R1/2 beam up to state 3_2), which verified the suitability of the applied calculation (curvature) model. It was generally observed that if a higher \( a_{tot} \) value (compared to the corresponding \( a_{calc} \)) was measured immediately after the first cracks appeared, then generally the \( a_{tot} \) values slightly exceeded the corresponding \( a_{calc} \) values through all the following deterioration states (typical case for this is the R1/2 beam), otherwise the \( a_{calc} \) values remained higher (see the P1 beam).

Where measured crack width data (\( w \)) were available, their acceptable coincidence with the corresponding \( w_{calc} \) was also observed (e.g. for the P2/3 beam), however, their ratio provided small useful information. In states of the P2p beams after wire cuttings
(state 56 and 57 for P2p/1 and state 31 and 41 for P2p/2&3), significantly higher $a_{\text{tot}}$ values were measured than the corresponding $a_{\text{calc}}$ values, which was in contrast to that experienced in states before wire cuts (state 1, see Table 4.1e). This suggested that wire cuts had much more influence on curvature than estimated by the applied curvature model (in Fig. 4.3).

**Fig. 4.6** Conformity check of non-model-based and model-based indices

An interesting situation was observed for the P2/1 beam (Fig. 4.6b). Through states 3 and 4, when the acting forces were located far from the midspan, a virtual hardening in the measured midspan deflection was experienced in contrast to the calculated behaviour. This was the highest difference between $a_{\text{tot}}$ and $a_{\text{calc}}$ experienced among all beams and was also reflected by the measured natural frequencies (see Sect. 5.3.4 for the explanation).

Plastic deformation, which was expected in the last states for the R1, R2 and P2p2&3 beams (see $M_{\text{max}}/M_{\text{rm}}$ ratios higher than 1.0 in Table 3.6) but was not reflected by the applied material law, was clearly experienced for all of the related beams (visible in Fig. 4.6d for the R1/2 beam) by the significant increase of the measured $a_{\text{tot}}$ and $w$ values in the relevant deterioration states and by their detachment from the corresponding calculated values.
5

DAMAGE IDENTIFICATION
AND ASSESSMENT OF
MODEL BEAMS
5.1 INTRODUCTION

The goal of this chapter is to present crack-related damage identification and assessment of the investigated beams on the basis of their first two natural frequencies and their associated shifts measured in accordance with Chapter 3 and given in Table 3.7. Damage identification is governed by the damage indices defined in Chapter 4 and given in Table 4.1. Each level of damage identification will be discussed. The usefulness as well as the limits of applicability of static-based damage indices will be demonstrated.

Damage assessment focuses mainly on crack-related damage. For the originally non-prestressed (reinforced) beams, the gradual degradation of the beams due to intensifying cracking is assessed by the measured natural frequency shifts in typical deterioration states. Effects of test variables (see Sect. 2.3.1) on the natural frequency changes will also be discussed. For the originally prestressed as well as for the post-tensioned beams, the influence of change in prestress on the natural frequencies of the beams will be analysed for different extents of previous crack-related damage.

5.1.1 Application of damage indices in experimental damage identification and assessment

Damage indices are usually defined to express information on the structural condition in numerical form. Structural damage changes the structural information given by the index and, consequently, indicated as a change in the value of the damage index. Because they do not require an additional structural model, non-model-based indices are determined from measured structural properties. Therefore, they do not include any model uncertainty and, for this reason, are more capable of characterizing the behaviour of existing structures than model-based indices. However, for existing structures, the execution of on-site measurements is often difficult due e.g. to accessibility reasons. At that point, model-based indices calculated on the basis of previous static models are applied to help damage identification. Model-based indices contain information on the behaviour of the model of the structure, which is affected by modelled damage in the form of assumed damage cases (e.g. newly formulated cracks, intensifying existing cracking, changes in support conditions, variation in ratio of internal forces to capacity). For the identification of existing structures, the use of assumed damage cases (damage models) being very close to service conditions is essential.

When executing dynamic tests (see Fig. 2.1) on existing structures, dynamic parameters (e.g. \( f_1 \) and \( f_2 \)) are measured and, if practical, their derivatives (e.g. \( \Delta f_1 \) and \( \Delta f_2 \)) are
Dynamic-based damage assessment is carried out if the structural condition (or any change in it) is deduced from the measured dynamic parameters. To achieve this,

- first the damage has to be identified by (either non-model-based or model-based) damage indices and then related to the measured dynamic parameters (or to their derivatives);
- then the measured dynamic parameters (or their derivatives) have to be related to structural parameters which characterize the performance of the (damaged) structure.

In this test, both non-model-based and model-based indices were defined for experimental purposes. The applicability of model-based indices was verified in Sect. 4.2.3 by checking their coincidence with non-model-based indices. During the identification phase (Sect. 5.3) mainly the incrementative form of damage indices are applied and related to shifts ($\Delta f_1$ and $\Delta f_2$) of the measured natural frequencies. Each level of identification (detection, localization and quantification) is thoroughly discussed. Within the assessment phase (Sect. 5.4), the structural condition is reflected

- on one hand directly by the value of the defined model-based index ($\Delta w_{calc}$, $\Delta A_r$)
- or,
- on the other hand by performance-based properties defined for each beam in Sect. 3.3.

Concluding remarks are made in relation to the evaluation of test variables (Sect. 2.3.1) in the assessment phase with regard to aspects of practical application.

### 5.1.2 Introduction of test results

When conducting damage identification Level 1 (detection) and 2 (localization) are in focus. The way how the results are introduced should support this concept.

Detection is focused if the incrementative form of damage indices (Sect. 4.1.4) is used, because any value other than zero of the index indicates a change in the investigated parameter (here in the natural frequency). If the magnitude of the index is proportional to the magnitude of the change in the investigated parameter, then this form is very helpful even in quantifying damage (Level 3). Owing to this, the frequency changes measured in the $i$-th state will be drawn as frequency shifts ($\Delta f_i$) relative to the frequency belonging to the previous state ($\Delta f_i = f_i - f_{i-1}$).

As discussed in Sect. 4.1.3, damage within zones located around the highest amplitudes of the associated mode shape influence the frequency shifts to the highest de-
This means that single damage somewhere along the beam is expected to influence $\Delta f_1$ and $\Delta f_2$ to a different extent. Therefore, the location is focused if $f_1$ and $f_2$ as well as $\Delta f_1$ and $\Delta f_2$ are simultaneously drawn against damage indices.

Fig. 5.1a Natural frequency shifts versus damage indices for the P1 beam

Fig. 5.1b Natural frequency shifts versus damage indices for the P2 beams
Fig. 5.1c Natural frequency shifts versus damage indices for the R1 beams
Fig. 5.1d Natural frequency shifts versus damage indices for the R2 beams

The $f_1$ and $f_2$ natural frequencies (see Table 3.7) and their deduced shifts ($\Delta f_1$ and $\Delta f_2$) as well as the defined damage indices (see Table 4.1) in their incrementative form are drawn in Fig. 5.1 for the related deterioration states of the P1, P2, R1 and R2 beams. For technical reasons, all quantities in the diagrams of Fig. 5.1 were converted into percentage. The natural frequencies ($f_{1i}$ and $f_{2i}$) as well as their shifts ($\Delta f_{1i}$ and $\Delta f_{2i}$) belonging to the $i$-th state are relative to $f_0$ (the value of $f$ in state 0) for all beams. The zero axis belonging to $\Delta f_{1i}$ and $\Delta f_{2i}$ is fixed at 100% (on the left side vertical axes) and, furthermore, the positive values of $\Delta f_{1i}$ and $\Delta f_{2i}$ are drawn downwards from this zero axis. Similarly, the percentage values of incrementative damage indices are relative to the highest value of the corresponding index (given in Table 4.1), which, in fact, corresponds to the 100% value of the cumulative index.
The influence of change in prestress (i.e. the effects of wire cuts) on the frequency shifts for the prestressed P2p beams as well as the influence of change in post-tensioning through states 6p, 7p and 8p on the natural frequencies of the related beams are discussed separately in Sect. 5.4.2 and Sect. 5.4.3.

5.2 ANALYTICAL AND NUMERICAL CALCULATIONS

Despite the fact that the objective of this research is to assess damage without using dynamic models (see Sect. 2.1.4), this section provides rough estimations of the first two natural frequencies of model beams on the basis of linear elastic analyses.

Since their determination needs short time and small computation effort, rough estimations may be practical for calibrating the measuring equipment and to select the appropriate device settings. For structures with complex shape or with multiple static indeterminacies, a priori estimations based on very simple assumptions may help better understanding of the structural behaviour (e.g. during mode shape identifications).

5.2.1 Analytical estimations

Assuming beams uncracked in full length, ignoring side cantilevers beyond the supports (marked by $k$ in Fig. 3.7) and neglecting structural damping, as simplifications, the first two natural frequencies may analytically be calculated as follows:

$$f_{1,\text{calc}} = \frac{1}{2\pi} \sqrt{\frac{E_{\text{cm}} l_i}{g_0 L^4}}$$  \hspace{0.5cm} (5.1a)

$$f_{2,\text{calc}} = \frac{1}{2\pi} \frac{4\pi^2 E_{\text{cm}} l_i}{g_0 L^4}$$  \hspace{0.5cm} (5.1b)

where $g=9.81 \text{ m/s}^2$; $g_0=0.39 \text{ kN/m}$ is the self-weight; $L$ is the span; $l_i$ is the moment of inertia based on uncracked cross-section and assumed as constant over $L$; $E_{\text{cm}}$ is the short-term modulus of elasticity (see Table 3.2). Eq. (5.1a) and (5.1b) resulted in $f_{1,\text{calc}}$ and $f_{2,\text{calc}}$ given in Table 5.1. The maximum difference of measured frequencies belonging to states 0 (no cracks) in Table 3.7 compared to $f_{1,\text{calc}}$ and $f_{2,\text{calc}}$ was obtained as $+7\%$ and $+1\%$ for the P1 beam as well as $-26\% - -12\%$ and $-25\% - -8\%$ for the P2, R1 and R2 beams, furthermore $-1\% - +4\%$ and $-5\% - -2\%$ for the P2p beams, respectively. These deviations from $f_{1,\text{calc}}$ and $f_{2,\text{calc}}$ indicated that the dynamic behaviour of a real concrete beam could be significantly different from that of an improperly adjusted calculation model. However, if assuming perfect geometry ($L$), these frequency deviations were necessary consequences of deviations in either $EI$ or $g_0$ and were clear indi-
cations of either insufficient density or existing but invisible micro-cracks in the concrete material or any hidden defect in the beam.

Table 5.1 Analytically calculated natural frequencies

<table>
<thead>
<tr>
<th>Beam</th>
<th>Based on uncracked beam</th>
<th>Based on fully cracked beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1^{\text{, calc}}$ [Hz]</td>
<td>$f_2^{\text{, calc}}$ [Hz]</td>
</tr>
<tr>
<td>P1</td>
<td>34.05</td>
<td>136.20</td>
</tr>
<tr>
<td>P2 &amp; P2p</td>
<td>24.79</td>
<td>99.17</td>
</tr>
<tr>
<td>R1</td>
<td>24.36</td>
<td>97.43</td>
</tr>
<tr>
<td>R2</td>
<td>24.55</td>
<td>98.18</td>
</tr>
</tbody>
</table>

When substituting $I_{\text{II}}$ in Eq. (5.1a) and (5.1b) with $I_{\text{II}}$ (fully cracked beam), which is the moment of inertia on the basis of cracked cross-section, then Eq. (5.1a) and (5.1b) resulted in $f_1^{\text{, II, calc}}$ and $f_2^{\text{, II, calc}}$ (see Table 5.1). These values were much smaller than those belonging to the last deterioration states of beams (even for states when $M_{\text{max}}/M_{Rm} > 1.0$) given in Table 3.7.

It was concluded that the corresponding $f_1^{\text{, calc}}$ and $f_2^{\text{, calc}}$ values may only be used as rough upper and lower estimations of natural frequencies for beams damaged by cracks. However, the magnitude of difference between the measured natural frequencies of a visibly undamaged beam and the corresponding $f_1^{\text{, calc}}$ values may be informative regarding the presence of invisible hidden defects in the beam.

5.2.2 Numerical estimations

Analytical calculations become difficult if the structural system becomes complicated. Due to the application of the introduced post-tensioning (see Sect. 3.3.1.3) to the related, originally non-prestressed beams, the previous simply-supported system was transformed into a complex, non-symmetric system consisting of the beam itself, the external strands (without any deviators), the anchorage plates and the jack. This modification in the structural system resulted in a considerable change of the natural frequencies and made second order numerical analysis necessary when calculating the natural frequencies at varying post-tensioning force in the strands.

![Mode shapes of the system including the dominant first and second beam modes](image-url)
Similarly to the above analytical approach (Sect. 5.2.1), linear elastic material law, main values of the material properties according to Table 3.2 and no embedded reinforcement were considered in the model.

Even the identification of modes became difficult. Due to the big difference in bending stiffness between beam and strands, the vibration of strands was dominant together with hardly visible amplitudes of the beam for most of the modes. Therefore, those modes and the associated frequencies of the post-tensioned system were selected, in which the vibration of the beam according to its first or second mode dominated in comparison with that of the strands (see Fig. 5.2a for the first and Fig. 5.2b for the second beam mode). Fig. 5.2c shows another (non-selected) mode, which is very close in frequency to that in Fig. 5.2b and in which the second beam mode also exists but dominates much less compared to that in Fig. 5.2b. This mode selection was carried out for each level of post-tensioning force in the strands (states 6p, 7p and 8p) according to Sect. 3.3.1.3.

Table 5.2

<table>
<thead>
<tr>
<th>State</th>
<th>P1 beam (L=3.2 m)</th>
<th></th>
<th>P2, R1, R2 &amp; P2p beams (L=3.8 m)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_{1,\text{num}} ) [Hz]</td>
<td>( f_{2,\text{num}} ) [Hz]</td>
<td>( f_{1,\text{num}} ) [Hz]</td>
<td>( f_{2,\text{num}} ) [Hz]</td>
</tr>
<tr>
<td>0 (no post-tens.)</td>
<td>34.82 (1\text{st} mode)</td>
<td>139.18 (2\text{nd})</td>
<td>24.62 (1\text{st})</td>
<td>97.49 (2\text{nd})</td>
</tr>
<tr>
<td>6p0 (P=0 kN)</td>
<td>33.77 (21\text{st} mode)</td>
<td>119.14 (40\text{th})</td>
<td>22.73 (21\text{st})</td>
<td>71.72 (40\text{th})</td>
</tr>
<tr>
<td>6p (P=2×10 kN)</td>
<td>33.63 (19\text{th})</td>
<td>119.10 (40\text{th})</td>
<td>22.85 (21\text{st})</td>
<td>72.44 (40\text{th})</td>
</tr>
<tr>
<td>7p (P=2×95 kN)</td>
<td>33.54 (19\text{th})</td>
<td>119.08 (40\text{th})</td>
<td>22.86 (19\text{th})</td>
<td>72.93 (40\text{th})</td>
</tr>
</tbody>
</table>

Table 5.2 contains the associated frequencies of the selected modes, determined also for zero post-tensioning force (state 6p0) together with the mode number. The first row of the table gives the first two natural frequencies of single, simply-supported beams (with side cantilevers according to Fig. 3.7 and without the post-tensioning equipment) as alternatives, calculated without the consideration of embedded reinforcement, for \( \tilde{f}_{\text{I,calc}} \) given in Table 5.1.

Although the difference in \( f_{1,\text{num}} \) and in \( f_{2,\text{num}} \) between beams with (state 6p0) and without (state 0) the post-tensioning equipment was explained by the changed structural system, it was noticeable that the percentage decrease in \( f_{2,\text{num}} \) (between states 6p0 and 8p) was much higher than that in \( f_{1,\text{num}} \) and, consequently, that the \( f_{2,\text{num}}/f_{1,\text{num}} \) ratio significantly decreased from \( 
\sim 4 \) (which belongs to single, simply-supported beams; see \( f_{2,\text{I,calc}}/f_{1,\text{I,calc}} \) in Table 5.1). Another remark is that considerable change in the intensity of the post-tensioning force (\( P \)) slightly influenced the natural frequencies. However, taking these small changes, \( f_{1,\text{num}} \) and \( f_{2,\text{num}} \) slightly decreased with the increase of \( P \) for the shorter (P1) beam but this trend was opposite for the longer beams. It was also
interesting that with increasing $P$ the modes of the system rearranged (the first beam mode belonged to the 21$^{st}$ mode of the system for lower $P$ then to the 19$^{th}$ mode for higher $P$). These trends in comparison with the corresponding measured natural frequencies are further analysed in Sect. 5.4.3.

Even the above results suggested that in practice when any new (structural and/or non-structural) element is added to an existing structure (e.g. for strengthening purposes), the changed modal data of the structure have to be registered as a baseline for further investigations.

### 5.3 DAMAGE IDENTIFICATION OF MODEL BEAMS

This section deals with damage identification aspects for the tested beams with respect to the identification levels discussed in Sect. 1.2.

#### 5.3.1 Detection

As discussed in Sect. 5.1.2, the incrementative form of damage indices in damage detection is helpful. The success of damage detection by natural frequency measurements depends mainly on the coincidence in shape of $\Delta f_1$ and $\Delta f_2$ curves with those of the applied damage indices. For diagrams presented in Fig. 5.1 for the test beams, a similar shape of $\Delta f_1$ or $\Delta f_2$ and of the defined indices ($\Delta l$, $\Delta w$, $\Delta a_{tot}$, $\Delta a_{res}$, $\Delta A_r$, $\Delta W_{calc}$, $\Delta a_{calc}$, $\Delta W$) indicates such damage, which is reflected by the $\Delta f_1$ and $\Delta f_2$ natural frequency shifts.

Generally good coincidence in shape of the applied non-model-based indices ($\Delta l$, $\Delta w$, $\Delta a_{tot}$, $\Delta a_{res}$) was found for most of the beams through the full deterioration process. In some cases, the non-regular deviations of some indices from other indices could be explained either by the different character (global or local) of the indices or by the unreliably appearing first cracks. As an example of the latter, consider states 1 and 2 of the P2/1 beam. The $\Delta a_{res}$ indicated high damage around midspan in state 1 (see also high $\Delta f_1$) and very little additional damage in state 2. Meanwhile, $\Delta l$ and $\Delta a_{tot}$ indicated a similar extent of damage both in state 1 and 2. According to Table 3.6, the first cracks were expected in state 1 ($M_{max}/M_r=3.18$) then the length of the cracked zone was widened in state 2 ($l$ increased from 0.5 m to 1.0 m) with almost identical $M_{max}$ ($M_{max}/M_r=3.14$). Extremely high $\Delta a_{res}$ in state 1 was caused by one or two wide cracks. These remained active and were followed by many small cracks with negligible widths in state 2. Unfortunately, no crack width measurements were made but the registered
crack pattern of the P2/1 beam also suggested this explanation. The local character of the $\Delta w$ index was the reason for its extreme value in state 1_2 of the R2/3 beam. In this state, $\Delta w$ indicated suddenly appearing, wide crack(s) which hardly widened in the further states. Parallel to this, all the other global indices were in good coincidence throughout all states (also including state 1_2).

The good coincidence in shape of the model-based indices ($\Delta A_r$, $\Delta w_{\text{calc}}$, $\Delta a_{\text{calc}}$, $\Delta W$) was mostly due to the applied common calculation models. However, due to its local character, $\Delta w_{\text{calc}}$ regularly indicated more intensive damage in state 1_1 of the R1 and R2 beams as well as in state 1/23 of the P2/3 beam, when first cracks appeared, and less intensive damage growth in the subsequent states in contrast with the other model-based indices.

In comparing the non-model-based indices with the model-based ones, generally quite good coincidence in shape could be observed until states when plastic deformations appeared. Plastic deformations in the last states for the R1 and R2 beams ($M_{\text{max}}/M_{Rm}>1.0$ in Table 3.6) were clearly addressed by the non-model-based indices (relatively high values) and simply not reflected by the model-based indices (zero or nearly zero values in the last states). The reason was that the applied models were based on linear elastic material law. Another note for the R1 beams is that model-based indices indicated small, first cracks in state 1_1 and their considerable intensification in state 1_2. Actually, wide, first cracks appeared in state 1_1 and then less intensively expanded further in state 1_2, which was clearly indicated by the non-model-based indices. This case definitely pointed out the practical advantage of non-model-based indices against the model-based ones.

When considering the coincidence of $\Delta f_1$ and $\Delta f_2$ with the defined (non-model-based and especially model-based) indices, generally similar shapes were experienced except for the domain of plastic deformations as mentioned above for the model-based indices. The best example of this was the R2/1 beam and the worst was the P2/1 beam. The $\Delta f_1$ and $\Delta f_2$ curves clearly addressed the appearance of first cracks for all beams and the plastic deformations for the related (R1 and R2) beams as well as any other damage propagation indicated by the defined indices (e.g. state 32 for the P1 beam or states 5 & 6 for the P2/2 beam).

Large and unusual changes in $\Delta f_1$ and $\Delta f_2$ as well as in $f_1$ and $f_2$ were observed in state 3 of the P2/1 beam, which was not indicated by either damage index at all. For its discussion see Sect. 5.3.4.
5.3.2 Quantification

Quantification of damage can be conducted in two areas. The first is the comparison of the extent of damage appearing in one state with the extent of damage occurring in another state of the deterioration process. This may be interesting if filtering the dynamic measurement events, which are executed regularly for e.g. maintenance reasons, for when a thorough survey on the structure becomes necessary to trace the reason of the detected damage. If using the incrementative form of both the measured frequencies and the defined damage indices then their magnitudes in each state are proportional to the extent of damage occurred in the same state. In Fig. 5.1 high amplitudes were found in states when first cracks appeared for all beams and in states when plastic deformation occurred for the R1 and the R2 beams. Intensive cracking outside these regions was also reflected by high amplitudes e.g. for state 3_2 of the P1 beam. However, note that amplitudes of curves in Fig. 5.1 (with the exception of the \( f_1 \) and \( f_2 \) curves) were drawn in percentage and scaled to the maximum of the related cumulative index (see Sect. 4.1.4), therefore, the same (in %) amplitudes being in different diagrams did not indicate the same extent of damage. The comparison of amplitudes (in %) along curves of any incrementative index was reasonable only if a single beam would have been analysed.

The second area is the quantification of the total extent of damage accumulated in the beam due to previous damage events. This further leads to damage assessment, within the frame of which the structural performance consequences of damage are analysed (see Sect. 5.4). From a damage identification aspect, the total decrease of natural frequencies started from the undamaged state (state 0) is informative. In Fig. 5.1 the \( f_1 \) and \( f_2 \) curves show the decreasing trend of natural frequencies over the full deterioration process. As shown, this decrease in e.g. \( f_1 \) was \( \sim 16\% \) for the P1 beam, between 12\% and 4\% for the P2 beams, between 35%-12\% for the R1 beams and between 15\% and 20\% for the R2 beams. These percentage decreases were not directly comparable because the applied loading phases were also different. However for the R2 beams, which were subjected to the same loading phases, the 15-20\% total decrease in \( f_1 \) and the 9-16\% total decrease in \( f_2 \) over the full deterioration process could be explained by structural differences between the beams due to hidden defects (such as variations in compactness, anchorage properties, etc.).
5.3.3 Localization

Localization of damage is generally based on the comparison of varying trends of the measured modal parameters (here in this test, the natural frequencies). As discussed in Sect. 4.1.3, a frequency shift is mostly influenced by damage located around the highest amplitudes of the associated mode shape. Therefore for this test, $\Delta f_1$ higher than $\Delta f_2$ was expected for states in which a load ($F$ in Fig. 3.7) was positioned close to midspan (states 1, 1_1, 1_2, 5) and the opposite ($\Delta f_2 > \Delta f_1$) for states with the load acting far from the midspan (states 3_1, 3_2). Note that load positions belonging to states of the same type were not identical for all beams, therefore $l$ in Table 3.6 was essential to consider the load position.

The best example for damage localization in Fig. 5.1 was the R1/1 beam. In state 1_1, $\Delta f_1$ was significantly higher than $\Delta f_2$ and, consequently, the percentage decrease in $f_1$ remained higher than in $f_2$ until state 3_1. Parallel to moving the acting loads farther from midspan, $\Delta f_2$ became higher already from state 1_2, which equalized the percentage decrease in $f_2$ with that in $f_1$ by state 3_1. Starting from 3_2, the acting loads were moved toward the midspan which intensified cracking around midspan. Consequently, $\Delta f_1$ again became higher than $\Delta f_2$ and $f_1$ decreased quicker than $f_2$ until state 5. Similar but no so obvious tendencies could be observed for the P1, P2/2, R1/3&4, and all R2 beams. For the P2/3 beam, the same tendencies as explained above for R1/1 ran in $\Delta f_1$ and $\Delta f_2$ as well as in $f_1$ and $f_2$ but the marking of states was different (the loads were positioned farthest from the midspan ($l=2.5$ m) in states 4/3_1 and 4/3_2).

5.3.4 Unusual symptoms in frequency trends

Unusual events, which could not be explained on the basis of prior load history and were not indicated by either of the damage indices at all, were observed and further traced in state 3 of the P2/1 beam. Until state 2, usual trends in both $\Delta f_1$ and $\Delta f_2$ as well as in both $f_1$ and $f_2$ were experienced. Based on the load history, wide first cracks were expected around midspan in state 1 then smaller new cracks in state 2 (the $M_{\text{max}}/M_r$ ratio was almost identical in both states). However, a surprise in state 2: the SD of $f_1$ (see Table 3.7c) tripled compared to state 1 but it was not the case for the SD of $f_2$ at all, which indicated an unusual crack pattern around midspan. In state 3, the $M_{\text{max}}/M_r (=3.12$, see Table 3.6) ratio was almost identical to states 1 & 2 and the acting loads were located a bit nearer to midspan than to supports ($l=1.5$ m). However, a negative $\Delta f_1$ and, consequently, an increase in $f_1$ (above $f_1$ of state 1 (!)) was obtained.
Meanwhile, a high (positive) $\Delta f_2$ and, consequently, a significant decrease in $f_2$ are derived from signal processing. In addition to this, starting from state 4, usual trends came again in both $f_1$ and $f_2$ and associated with considerable multiple peaks around $f_2$ (see Sect. 3.5.2) as indicated by the high SD values belonging to $f_2$ in Table 3.7c.

![Graph showing frequency spectrums](image)

**Fig. 5.3** Duplicated sub-peaks around $f_1$ in the average frequency spectrums (state 3 of P2/1)

The reason behind the above virtual hardening in $f_1$ and the parallel softening in $f_2$ in state 3 was suspected to be a special degradation process in one or a few of the first wide, active cracks around midspan (see the related part of Sect. 5.3.1) due e.g. to grains breaking off the crack faces and preventing them to close. This brought significant non-linearity into the structural response and resulted in duplicated peaks around $f_1$ in the frequency spectrums of state 3. **Fig. 5.3** shows that only the upper peak, the corresponding average frequency of which was put into Table 3.7c and **Fig. 5.1**, was observed by the average frequency spectrum derived from signals belonging to impulse excitation. This degradation presumably started already in state 2 (tripled SD) and further continued non-symmetrically toward one half of the beam then resulted in significant duplicated peaks around $f_2$ throughout states 4-7.

### 5.4 DAMAGE ASSESSMENT OF MODEL BEAMS

This section deals with the damage assessment of the tested beams. According to the concept, as noted in Sect. 5.3.2 and indicated in Sect. 1.2 by the definition of Level 3a, damage assessment may be taken as a further step following damage quantification (Level 3) in the conventional system of damage identification by Rytter (1993).

In this test, experimentally determined relationships between the measured natural frequencies (and their derivatives) and the structural performance are aimed to be established for typical deterioration states. Deterioration is assumed to be caused only by crack-related damage. Within this framework, the influence of test variables (see Sect. 2.3.1) on the extent of damage will also be discussed. The originally non-
prestressed (reinforced) beams, the originally prestressed beams as well as the reinforced beams equipped with post-tensioning will be treated separately. Due to the limited number of tested beams, no deeply analysed and statistically validated result can be expected, but the setting of basic trends and definition of likely intervals of frequency changes in the investigated, typical deterioration states is realistic. Therefore, the introduced results are considered as preliminary. An increase in the number of tested specimens would improve the reliability of the results.

5.4.1 Reinforced (non-prestressed) beams

For the originally non-prestressed (P1, P2, R1 and R2) beams, damage was simulated by flexural cracking. Starting from the uncracked condition, subsequent states with gradually intensified cracking were produced through the introduced experimental deterioration process. The assessment of the beams focused on:

a) the influence of the applied level of internal force (utilization) on the total change in \( f (\Sigma \Delta f_1 \text{ and/or } \Sigma \Delta f_2) \) and,

b) the influence of first cracks on the magnitude of \( \Delta f (\Delta f_1 \text{ and/or } \Delta f_2) \) relative to the total change in \( f (\Sigma \Delta f_1 \text{ and/or } \Sigma \Delta f_2) \) over states with dominantly elastic behaviour,

c) the influence of the type and the amount of reinforcement on the total change in \( f (\Sigma \Delta f_1 \text{ and/or } \Sigma \Delta f_2) \) at different levels of internal force (utilization).

In the following, the above relationships will be discussed on the basis of the test results given in Chapter 3.

a) Influence of the applied level of internal force on the total decrease in \( f (\Sigma \Delta f) \)

In reinforced concrete beams under pure flexure, cracking is caused by bending moments higher than the cracking moment \( (M_r) \). To measure the applied internal force level in the beam, the maximum bending moment at midspan \( (M_{\text{max}}) \) in each loading phase was proportioned to performance-related parameters such as \( M_r, M_{\text{Rd}} \) or \( M_{\text{Rm}} \). The use of the \( M_{\text{max}}/M_{\text{Rd}} \) ratio was recommended for design purposes while the \( M_{\text{max}}/M_{\text{Rm}} \) ratio was proposed for experimental purposes.

When considering the full deterioration process, the total decrease in \( f_1 (\Sigma \Delta f_1) \) and in \( f_2 (\Sigma \Delta f_2) \) was calculated generally as \( \Sigma \Delta f = f^{\text{last}} - f^0 \) where \( f^{\text{last}} \) and \( f^0 \) were the values of \( f_1 \) or \( f_2 \) in the deterioration state associated with the highest bending moment, \( M_{\text{max}} \) (maximum damage, see Table 3.6) and in state 0 (no damage), respectively. Fig. 5.4 shows \( \Sigma \Delta f_1 \) and \( \Sigma \Delta f_2 \) as a function of \( M_{\text{max}}/M_r \) and of \( M_{\text{max}}/M_{\text{Rm}} \). For the same beam types with iden-
tical loading phases (as was the case for the P2/1&2 and for the R1/1&2 beams), the average of $\Sigma \Delta f$ values were put into Fig. 5.4. The distinction between the two diagrams was explained by the fact that for identical beams with different types of reinforcement the $M_{\text{max}}/M_t$ and the $M_{\text{max}}/M_{\text{Rm}}$ ratio did not move parallel from beam to beam because of the different strengths of the applied reinforcement. The $M_{\text{max}}/M_{\text{Rm}}$ ratio became much higher for beams reinforced with prestressing steel (P1 and P2) compared to those reinforced with reinforcing steel (R1 and R2).

![Fig. 5.4 Total decrease in $f_1$ and in $f_2$ versus $M_{\text{max}}/M_t$ and $M_{\text{max}}/M_{\text{Rm}}$ ratios](image)

As shown, $\Sigma \Delta f$ had a clear increasing tendency with the increase of $M_{\text{max}}/M_{\text{Rm}}$ but had no clear tendency as a function of $M_{\text{max}}/M_t$. Another observation was that, starting from $M_{\text{max}}/M_{\text{Rm}} \approx 0.9$, $\Sigma \Delta f$ increased at a much higher rate than at lower $M_{\text{max}}/M_{\text{Rm}}$ values, which was a clear sign of the appearance of plastic deformations (for the R1 and R2 beams). Before the appearance of plastic deformations, $\Sigma \Delta f \approx 15\%$ was observed which corresponded to $\approx 15\%$ decrease in $f$ compared to the undamaged state. For the R1/4 beam, no plastic deformation was observed even for $M_{\text{max}}/M_{\text{Rm}}=1.07$ in the last deterioration state, therefore $\Sigma \Delta f$ remained below 15%. Considering that $M_{\text{Rm}}/M_{\text{Rd}} \approx 1.5$, $M_{\text{max}}/M_{\text{Rm}} \approx 0.67$ is applicable for design purposes. At this $M_{\text{max}}/M_{\text{Rm}}$ ratio, about 8% natural frequency decrease was found compared to the undamaged beam.

b) Influence of first cracks on the magnitude of $\Delta f$

As shown in Fig. 5.1, damage indices pointed out parts in the introduced deterioration process in which more intensive damage occurred than in other parts. Such damage events happened in states when the first cracks appeared (for all beams) and when plastic deformations occurred (for the R1 and R2 beams). From a practical point of view the development of first cracks is relevant; the latter is mostly precluded by the limitation of service load. Fig. 5.5 shows how effectively the appearance of first cracks can be indicated by natural frequency shifts over the practical range of applicability of structures. The columns measure the natural frequency decrease in states when cracking started to develop ($\Delta f_1^t$ and $\Delta f_2^t$) relative to the total decrease in $f$ over (the first six)
states during which beams gave dominantly elastic response ($\Sigma \Delta f_1^{0-6}$ and $\Sigma \Delta f_2^{0-6}$). The percentage values given in Fig. 5.5 were averaged for each beam type. Based on Fig. 5.1, damage indices indicated no significant plastic deformation during the first six states for all beams.

As shown, the decrease in $f$ when first cracks appeared ($\Delta f$) was found to be at least 31% of the total change in $f$ over the interval of dominantly elastic behaviour ($\Sigma \Delta f^{0-6}$) for all beam types. This demonstrated that natural frequency shifts were a good indicator of first cracks.

c) Influence of type and amount of reinforcement (test variables) on the total decrease in $f$ ($\Sigma \Delta f$)

For reinforced concrete beams under pure flexure, both crack width and crack spacing depend on the amount of reinforcement within the effective tension area of the cross-section (see Eq. (4.3)-(4.6)). Consequently, the extent of cracking is tightly influenced by the amount of tension reinforcement. Sect. 5.3 demonstrated that the values of the incrementative damage indices, many of which followed the crack-amount-based approach according to Sect. 4.1.1, and those of the $\Delta f$ natural frequency shifts resulted in similar shaped curves over the deterioration states of beams when using them for crack-related damage identification purposes. Therefore, it was supposed that natural frequency shifts might be quantified by the amount of tension reinforcement.

In practical cases, the steel ratio ($\rho=A_s/A_c$) as given in Table 3.1 is used instead of effective steel ratio according to Eq. (4.5). In the following, the influence of steel ratio is investigated on the total decrease in $f$ ($\Sigma \Delta f$) at different states of the introduced deterioration process. The deterioration states are characterized by either the $M_{max}/M_r$ or the $M_{max}/M_{Rm}$ ratio (see Table 3.6).

The first investigated state was the occurrence of first cracks. The first crack occurs when $M_{max}=M_r$. The cracking moment ($M_r$) of a cross-section is not sensitive to the properties (ratio ($\rho$), strength ($f_{pk}$ or $f_{tk}$) and bond (surface) characteristics) of the em-
bedded steel, while crack width and crack spacing are influenced by steel ratio and bond characteristics. Thus, crack-induced damage caused by first cracks is expected to be influenced by steel ratio and bond characteristics of the embedded steel and not affected by steel strength (if steel stress remains in the elastic domain after crack formulation). Fig. 5.6a shows the total change in $f$ ($\Sigma \Delta f_1$ and $\Sigma \Delta f_2$) developed until the state with $M_{\text{max}}/M_r \approx 1.25$ (immediately after first cracks appeared) as a function of steel ratio. As shown, all $\Sigma \Delta f$ values (concerning both $\Sigma \Delta f_1$ and $\Sigma \Delta f_2$) fell between 1% and 6%. Because first cracks were expected as well as observed around midspan, thus $f_1$ was affected to a higher degree than $f_2$, which resulted in $\Sigma \Delta f_1$ generally higher than $\Sigma \Delta f_2$. For beams with the same type (i.e. the same bond characteristics) of steel (trend lines in Fig. 5.6a relate to beams with the same type of reinforcement), the magnitude of $\Delta \Sigma f$ unambiguously decreased with the increase of $\rho$. This was explained by smaller crack widths associated with increasing $\rho$, which, in accordance with Sect. 4.1.1, resulted in less damage. Another observation was that, for identical $\rho$, higher $\Sigma \Delta f$ was obtained for beams with reinforcing steel (higher bond, for R1 and R2 beams) compared to those with unstressed prestressing steel (lower bond, for P1 and P2 beams). The reason was the longer transmission length of the bar with lower bond than with higher bond. Bond stresses distributing along a longer length might cause less micro-cracking in concrete beyond crack faces, which results in less damage. To check this supposition, Fig. 5.6b was drawn similarly to Fig. 5.6a but corresponding to a higher $M_{\text{max}}/M_r(\approx 3.05)$ ratio. About the same tendencies, together with a higher $\Sigma \Delta f$, which fell between 4% and 15%, were obtained.

![Fig. 5.6 Relationship between total decrease in $f$ and steel ratio at different $M_{\text{max}}/M_r$ ratios](image)

The second investigated state of the introduced deterioration process corresponded to $M_{\text{max}}/M_{Rm} \approx 0.72$. In contrast with $M_r$, $M_{Rm}$ is highly sensitive to steel strength, therefore different a multiplicator transforms $M_{\text{max}}/M_r$ into $M_{\text{max}}/M_{Rm}$ if the strength of the applied reinforcement is different. In ultimate limit state design, the tension capacity of the
cross-section counts, which is a function of steel strength. To equalize the strength difference between reinforcing and prestressing steel, the strength-modified, mechanical steel ratio ($\rho_f$) instead of steel ratio ($\rho$) was considered as follows:

$$\rho_f = \rho \frac{f_{f\text{k}}}{f_{\text{tk}}}$$

(5.1)

where $f_{\text{tk}}$ was the characteristic tensile strength of the applied reinforcing steel according to Table 3.1 and $f$ was the characteristic tensile strength of the steel embedded in the considered beam (either $f_{\text{pk}}$ or $f_{\text{tk}}$). Fig. 5.7 shows the total change in $f$ ($\Sigma \Delta f_1$ and $\Sigma \Delta f_2$) developed until the state with $M_{\text{max}}/M_{\text{Rm}} \approx 0.72$ as function of strength-modified, mechanical steel ratio.

![Fig. 5.7](image)

Fig. 5.7 Relationship between the total decrease in $f$ and the strength-modified, mechanical steel ratio at $M_{\text{max}}/M_{\text{Rm}}=0.72$

As discussed above in Sect. 5.4.1a), this $M_{\text{max}}/M_{\text{Rm}}$ ratio fairly corresponds to $M_{\text{max}}=M_{\text{Rd}}$, which is applicable for ultimate limit state design. As shown, $\Sigma \Delta f$ values around 8% (with the exception of $f_1$ for the P1 beam) were obtained with no unambiguous sensitivity to $\rho_f$. This echoed what was found in Sect. 5.4.1a) related to $\Sigma \Delta f$ at $M_{\text{max}}=M_{\text{Rd}}$. It meant that the total change in $f$ around the design level of internal force was found to be insensitive to the ratio of embedded steel for all of the investigated beams. This was explained by the fact that, for a cross-section normally reinforced with the same type of bars and being at their design capacity ($M=M_{\text{Rd}}$), the stress in the reinforcement approaches the design tensile strength of the applied steel. The anchorage of the respective tensile force results in about the same crack-induced damage.

### 5.4.2 Prestressed beams

For the originally prestressed (P2p) beams, damage was simulated by the combination of flexural cracking and wire cuts in accordance with Sect. 3.3.2.1 and Sect. 3.3.1.2. For these beams, the extent of cracking was influenced by the intensity (level) of bending moment as well as by the intensity (level) of prestress.
The applied level of internal force was measured, similarly to that in Sect. 5.4.1, by the proportion of the maximum bending moment at midspan \( (M_{\text{max}}) \) to the mean value of bending capacity \( (M_{\text{Rm}}) \) and, additionally, to the decompression moment \( (M_{\text{dec}}) \). Because wire cuts were concentrated at cut points (see Fig. 3.3), the prestressing force at cut points decreased in strict proportion to the number of cut wires, however, due to bond, the prestress remained intact at beam sections farther than the transmission length from the cut points. To measure the effective level of prestress in the whole beam in each state, the remaining prestressing force was averaged along the full length of the beam \( (P) \) and then proportioned to the prestressing force of the undamaged (no wire cuts) beam \( (P^0) \) as follows:

\[
P = \frac{2l_p \sum i n_i P_i + P^0 \left( L_b - 2l_p \sum i n_i \right)}{L_b P^0} \tag{5.2}
\]

where \( l_p \) was the transmission length according to Eq. (3.2); \( L_b \) (=4.4 m) was the total beam length according to Table 3.1; \( i \) was the mark of cut points at which the same number of wires were cut, \( n_i \) was the number of \( i \)-marked cut points along the beam length; \( P_i \) (<\( P^0 \)) was the decreased prestressing force at the \( i \)-th cut points. The difference in strategy of wire cuts between the P2p/1 as well as the P2p/2 & P2p/3 beams was introduced in Sect. 3.3.1.2.

Diagrams in Fig. 5.8a & b show the \( f_1 \) and \( f_2 \) natural frequencies (see Table 3.7e) and their deduced shifts (\( \Delta f_1 \) and \( \Delta f_2 \)) in percentage form (i.e. relative to \( f^0 \) (the \( f \) value in state 0)) for each deterioration state similarly to that in Fig. 5.1. For assessment purposes Fig. 5.8a & b also contain the effective level of prestress \( (P/P^0) \) calculated according to Eq. (5.2) and the level of bending moment measured by the \( M_{\text{max}}/M_{\text{Rm}} \) ratio for each state; both are also in percentage form. The natural frequency-related values \( (f, \Delta f) \) are drawn to the left-side vertical axis while the \( M_{\text{max}}/M_{\text{Rm}} \) and the \( P/P^0 \) ratios are drawn to the right-side vertical axis.

Using Fig. 5.8a & b, the assessment of the P2p type beams focused on:

a) the influence of prestressing on \( f \) for undamaged beams (state 0) – a comparison with the unstressed P2 type beams;

b) the influence of change in prestress on the change in \( f \) for negligible crack-related damage (P2p/1 beam);

c) the influence of intensive prior cracking on the extent of change in \( f \) due to change in prestress (P2p/2 and P2p/3 beams).
In the following, the above relationships will be discussed on the basis of test results given in Chapter 3.

a) Influence of prestressing on \( f \) for undamaged beams – a comparison with the unstressed P2 type beams

Although pre-stressing of the internal, bonded reinforcement in a beam results in the deformation of the shape, it does not influence the bending stiffness of the beam. Theoretically, in the absence of any bending stiffness change, bonded prestressing may influence the modal properties of a beam only if its effect (eccentric compression) on the beam varies during the vibration. Because bonded tendons are fully embedded into concrete, thus, if excluding tendon damage, changes neither in the intensity of prestressing (excluding the elastic stress variation in tendons due to vibration motion) nor in the position of tendons relative to beam are possible.

On the basis of this theoretical approach, no difference between natural frequencies belonging to deterioration state 0 of the P2 and the P2p type beams were expected. These beams contained identical reinforcement (see Table 3.1), were free from crack-related damage in state 0 and differed only in the presence of prestressing (P2 was unstressed; P2p was prestressed). However, according to Table 3.7c&e, the average of \( f_1 \) values and of \( f_2 \) values in state 0 was 20.95 Hz and 85.89 Hz for the P2 beams and 25.30 Hz and 96.21 Hz for the P2p beams, respectively. This showed \( f_1 \) and \( f_2 \) values 21% and 12% higher for the prestressed P2p beams than for the unstressed P2 beams. Additionally, the \( f \) values of P2p beams fell very close to both the analytical estimations calculated according to Eq. (5.1a&b) and the numerically calculated \( f \) values shown in the first row of Table 5.2.

As already suggested in Sect. 5.2.1, the difference between the corresponding measured and calculated \( f \) values of visibly undamaged (here P2) beams indicated that non-prestressed concrete structural elements might contain invisible, hidden defects that prevented tight coincidence between the measured and the calculated natural frequencies. The difference between the corresponding measured \( f \) values belonging to the undamaged states of the P2 and the P2p beams demonstrated that prestressing was helpful either in preventing these defects from occurring or eliminating their effect on the natural frequencies of the beam.

It can be concluded that the application of internal, bonded prestressing influences the natural frequencies of beams even in the absence of any tendon- or crack-related
Damage. This fact has great practical importance for the assessment of prestressed beams on the basis of on-site frequency measurements.

b) Influence of change in prestress on the change in $f$ for negligible-crack-related damage (P2p/1 beam)

One practical reason for natural frequency shift for beams prestressed with internal, bonded tendons can be the breakage of tendons. If a tendon break does not result in a crack of the cross-section of the break then a marginal change in the bending stiffness of that cross-section occurs, which results only in a marginal change in the natural frequencies. However, considering that prestressing plays a great part in neglecting the effects of invisible, hidden defects, which was suggested above in a), the decrease in prestress for undamaged (i.e. uncracked) beams may result in a higher influence on the natural frequencies. Furthermore, similar to damage along the transmission length of non-stressed reinforcement after crack formulation, local damage in bond along the transmission lengths of tendons is probable due to tendon break-induced sudden stress release, which, without any crack in the beam, may also result in adverse (decreasing) effect on the natural frequencies.

![Fig. 5.8a](image)

**Fig. 5.8a** Change in $f$ as well as in the $M_{\text{max}}/M_{\text{Rm}}$ and the $P/P^0$ ratios throughout the full deterioration process of the P2p/1 beam

The effect of decrease in prestress on the natural frequencies in the case of negligible crack-related damage was tested for the P2p/1 beam between states 3c and 55c as shown in **Fig. 5.8a**. Unfortunately, the artificial reduction of bonded prestress in the tension zone of beams was only possible by sawing wires through the concrete cover. This resulted in small artificial cracks stretching from the bottom face of the beam up to the level of cut wires (see **Fig. 3.3**), which decreased the bending stiffness of the cross-sections at cut points a bit more than a simple corrosion-induced tendon break. The gradual reduction of prestress was executed according to Sect. 3.3.1.2 and measured by the $P/P^0$ ratio. According to **Table 3.6**, the level of internal force (measured by the...
The $M_{\text{max}}/M_{\text{Rm}}$ ratio before wire cuts in states 1 and 2 was limited as $M_{\text{max}}/M_{\text{Rm}} \leq 0.53$, which assured negligible cracking ($M_{\text{max}}/M_{\text{cr}} \leq 1.03$) and corresponded to practical load levels ($M_{\text{max}}/M_{\text{Rd}} \leq 0.82$). As shown in Fig. 5.8a, the total decrease in $f$ (in $f_1$ and $f_2$) between states 3c and 55c was on average only 4% (3% in $f_1$ and 4% in $f_2$) while the effective level of prestress was reduced by 30% ($1.0 \leq P/P_0 \leq 0.7$). A greater part of this decrease in $f$ could be observed towards the end (state 55c) of the prestress reduction process. The small decrease in $f$ indicated that:

- natural frequencies were not really sensitive to small a degradation of internal, bonded prestress for beams with negligible crack-related damage;
- for greater decreases in $P/P_0$, the adverse effects of hidden defects in concrete and of local damage along the transmission lengths of cut wires on the natural frequencies became noticeable.

Note that unusual increase of $f_1$, being very similar to that discussed in Sect. 5.3.4, occurred between states 5c and 55c of the P2p/1 beam. In Sect. 5.3.4 this symptom was explained by the virtual hardening of cracks being prevented to close by e.g. grains falling off the crack faces. In this case, this supposition was further supported by the sawing process in state 51c, which might directly push falling grains into the crack of the midspan cut point.

After wire cuts were finished, the level of internal force was increased (in states 56 and 57) in order to investigate the effect of cracking on the natural frequencies separately from that of prestress reduction. The $M_{\text{max}}/M_{\text{cr}}$ ratio was increased up to 1.31, which corresponded to the upper level of practical applicability ($M_{\text{max}}/M_{\text{Rd}}=1.04$). At these ratios, the $M_{\text{cr}}$ and $M_{\text{Rd}}$ values belonged to the actual (i.e. reduced) level of prestress ($P/P_0=0.7$). As shown in Fig. 5.8a, cracking had a significant, decreasing effect on $f$ despite the reasonably high level of remaining prestress. This clearly indicated that crack-related damage could not be neglected even with relatively high prestress. The decrease in $f_1$ through states 56 and 57 was much more intensive than in $f_2$, which, as a damage location indicator, suggested that the newly formulated cracks in states 56 and 57 were located around midspan. This supposition was also supported by the position of acting forces ($l=1.5$ m) read from Table 3.6.

c) Influence of intensive prior cracking on the extent of change in $f$ due to change in prestress (P2p/2 and P2p/3 beams)

When producing significant cracking by mechanical forces in a beam prestressed with internal bonded tendons, prestressing may be unable to perfectly close the wide cracks
after load removal. Open or not-perfectly-closed cracks result in a decrease of bending stiffness compared to the undamaged state of the beam. During the vibration of such a beam, non-linearity as discussed in Sect. A1.3.1 may also appear and, together with the bending stiffness decrease, adversely affects the modal parameters (i.e. decrease the natural frequencies and lead to multiple peaks in the frequency spectrum). Meanwhile, the prestressing effect on each cross-section of the beam remains intact. If any local decrease in the prestressing force occurs thereafter, it is supposed to indirectly affect the modal parameters by allowing existing cracks and other hidden defects to reduce the structural integrity to a greater extent.

Fig. 5.8b Change in \( f \) as well as in the \( M_{\text{max}}/M_{\text{Rm}} \) and the \( P/P^0 \) ratios throughout the full deterioration process of the P2p/2 and the P2p/3 beams

This supposition was tested for the P2p/2 and the P2p/3 beams (see Fig. 5.8b). The essential difference in load history compared to the P2p/1 beam was the significantly higher level of internal force (\( M_{\text{max}}/M_{\text{Rm}}=0.92 \), see Table 3.6) applied to the visibly undamaged beams with full prestress. This resulted in a medium degree of cracking around midspan (\( M_{\text{max}}/M_c=1.78 \)) with \( ~0.2-0.3 \) mm wide cracks as well as on average natural frequency decrease of \( \Delta f_1=3.6\% \) and \( \Delta f_2=2.4\% \) in state 1. As a comparison, negligible cracking as well as \( \Delta f_1=2.5\% \) and \( \Delta f_2=1.9\% \) was obtained in the corresponding state 2 for the P2p/1 beam. As a consequence of the intensive prior cracking, the decrease in \( f \) until state 4c, when the prestress reduction process ended, became higher for the P2p/2 & P2p/3 beams than for the P2p/1 beam. Considering the introduced deterioration process between state 0 (undamaged beam) and state 4c (prestress reduction ended), the average total decrease in \( f_1 \) and in \( f_2 \) was 10.2% and 10.0%, respectively. The corresponding decreases for the P2p/1 beam were 4.8% and 6.0%, respectively. If considering only the prestress reduction process, the decrease in \( P/P^0 \) was 30% between states 3c and 55c for the P2p/1 beam and 43% between states 2c and 4c for the P2p/2&3 beams. Meanwhile, the decrease in \( f_1 \) and in \( f_2 \) during only the prestress reduction process was 2.3% and 3.5% (2.9% on average) for the P2p/1
beam and 7.2% and 5.5% (6.4% on average) for the P2p/2&3 beams, respectively. The latter 6.4% average decrease in f for the P2p/2&3 beams was more than the corresponding value for the P2p/1 beam multiplied by the ratio of extents of prestress reductions (i.e. the ratio of decreases in $P/P^0$) for the two cases as follows: $2.9 \times 43/30 = 4.2\%$.

This indicated that the degradation of internal, bonded prestress resulted in a greater extent of natural frequency decrease for beams, on which intensive cracking formed before the prestress degradation started to develop, in comparison with beams being uncracked before the prestress degradation process.

Another observation concerned the effect of loading applied in state 31 on the natural frequencies. The aim of this load was to model the service load level on a prestressed concrete beam, which was seriously damaged by cracks and tendon breaks earlier. This loading remained below the decompression moment of cross-sections with already reduced prestressing force (according to Table 3.6, the $M_{max}/M_{dec.2}$ ratio was equal to 0.73 and 0.97 for the P2p/2 and the P2p/3 beam, respectively), therefore no new cracks and no opening of existing cracks were expected in state 31. However, at least 0.9% (1.5% on average) decrease in f (including both $f_1$ and $f_2$) could be registered for the P2p/2 and the P2p/3 beams. Although no opening of existing cracks was discovered under this loading, non-negligible residual deflection at midspan (5.1 mm for the P2p/2 beam and 3.7 mm for the P2p/3 beam; see Table 4.1e) was registered after load removal. The reason behind this damage was supposed to be the gradual degradation of bond along the transmission lengths of broken tendons. This suggested that prestressed beams already damaged by existing cracks and broken, bonded tendons could also be further damaged by low-intensity service loads.

The tendencies in state 41 (loading after prestress reduction was finished) were very similar to that for the P2p/1 beam. Only the applied level of internal force was higher (i.e. far above the level of practical applicability; $M_{max}/M_{Rn}=1.54$). The load was increased close to failure ($M_{max}/M_{Rm}=1.07$), which induced significant plastic deformation in the remaining (i.e. non-cut) tendons of cut points. As a result of this, an average decrease of $\Delta f_1=14.8\%$ and $\Delta f_2=5.8\%$ occurred only in state 41, which corresponded to an average, total decrease in f as $\Sigma \Delta f_1=25.0\%$ and $\Sigma \Delta f_2=15.8\%$ considering the full deterioration process. This was fully in accordance with that experienced for non-prestressed beams in Sect. 5.4.1 (see Fig. 5.4).

Again, virtual hardening in $f_1$, being similar to that discussed in a) above for the P2p/1 beam, could be observed in state 2c of the P2p/2 beam. An identical reason for this
was assumed. No such symptom was discovered for the P2p/3 beam, which suggested no peculiarity in the structural behaviour.

5.4.3 Post-tensioned beams

Post-tensioning is widely used in practice for structural strengthening of concrete structures. By the application of post-tensioning to a few, originally non-prestressed model beams, which had already been affected by significant crack-related damage according to Sect. 3.3.2.1, the influence of post-tensioning on the natural frequencies was aimed to be investigated.

The experimental arrangement was described in Sect. 3.3.1.3 and discussed in Sect. 5.2.2. The only variable parameter of these tests was the intensity of the post-tensioning force \(P\). If assuming centric compression according to Sect. 3.3.1.3, the applied \(P\) of 20, 190 and 300 kN corresponded to a uniform compression stress of 1.3, 12.0 and 19.0 N/mm\(^2\) over the cross-section, respectively.

Distinctions between post-tensioned beams were made according to the level of internal force (measured by \(M_{\text{max}}/M_{\text{Rm}}\)) applied to them in their last loading state (i.e. state 5 for the P1, R1 and R2 type beams as well as state 7 for the P2 type beams, see Table 3.6) immediately before to the application of post-tensioning. This \(M_{\text{max}}/M_{\text{Rm}}\) ratio was primarily used for evaluation purposes when analysing the influence of post-tensioning on the natural frequencies. For this reason, further discussion is limited to (P1 and R2/1) beams representing the extreme values of the \(M_{\text{max}}/M_{\text{Rm}}\) ratio. State 5 of the P1 beam represented the lowest value of \(M_{\text{max}}/M_{\text{Rm}}\)=0.88 and that of the R2/1 beam corresponded to \(M_{\text{max}}/M_{\text{Rm}}=1.01\). The \(f_1\) and \(f_2\) values belonging to states 5 of the P1 and R2/1 beams were taken from Table 3.7a and \(d\), marked by \(f_{1,m5}\) \((f_{1,m5}\) and \(f_{2,m5}\,\) respectively), and assigned to \(P=0\) in Fig. 5.9.

Note that numerical frequency analysis based on second order, linear elastic model was performed in Sect. 5.2.2, which focused on mode changes (i.e. natural frequency shifts and rearrangement of modes) due to an increase in the post-tensioning force. The resulting \(f_{\text{num}}\) \((f_{1,\text{num}}\) and \(f_{2,\text{num}}\)\) natural frequencies included in Table 5.2 were graphically introduced in the diagrams of Fig. 5.9.

Fig. 5.9 shows the measured \(f_m\) natural frequencies \((f_{1,m}\) and \(f_{2,m}\)) in Hz as functions of \(P\) for the P1 and the R2/1 beam. In order to help simultaneous representation of \(f_1\) and \(f_2\, f_1\) values were drawn to the left-side vertical axis of diagrams in Fig. 5.9 and \(f_2\) were drawn similarly to the right-side axis. The percentage values of the measured \(f_m\) and \(f_{m5}\) natural frequencies in Fig. 5.9 were scaled to those corresponding \(f_{\text{num}}\) values, which
belonged to \( P=0 \) (state 6p0 in Table 5.2). Due to technical difficulties, state 6p0 \( (P=0) \) was tested and, consequently, the related \( f_m \) natural frequencies were measured only for the P1 beam.

![Diagram](image)

Fig. 5.9 Change in \( f \) as function of the post-tensioning force \( (P) \) for different levels of \( M_{\text{max}}/M_{\text{Rm}} \) before post-tensioning

(a) For lower \( M_{\text{max}}/M_{\text{Rm}} \) before post-tensioning

(b) For higher \( M_{\text{max}}/M_{\text{Rm}} \) before post-tensioning

Although it was pointed out in Sect. 5.2.2 on undamaged beams by numerical models that the simple installation of the post-tensioning equipment to beams with zero post-tensioning force \( (P=0, \text{state } 6p0) \) led to a slight decrease in \( f_{\text{num}} \) (see Table 5.2), while the \( f_{m5} \) natural frequencies measured before the installation \( (f_{1,m5}=15.80 \text{ Hz } (70%); f_{2,m5}=65.23 \text{ Hz } (91%); \text{state } 5) \) fell far below \( f_{\text{num}} \) \( (f_{1,\text{num}}=22.68 \text{ Hz } (100%); f_{2,\text{num}}=71.62 \text{ Hz } (100%)) \) calculated in state 6p0 of the undamaged R2/1 beam. However, if comparing \( f_{m5} \) to \( f_m \) (the \( f_m \) value belonging to state 6p0 might be extrapolated from \( f_m \) values belonging to \( P>0 \) e.g. as follows: \( f_{1,m}<14.13 \text{ Hz } (62%); f_{2,m}<50.88 \text{ Hz } (71%) \)) for the R2/1 beam, \( f_{m5} \) safely remained above \( f_m \). According to Fig. 5.9, these tendencies were similar but not so clear for the P1 beam, for which the \( M_{\text{max}}/M_{\text{Rm}}(=0.88) \) ratio was significantly smaller in state 5 than that \( (M_{\text{max}}/M_{\text{Rm}}=1.01) \) for the R2/1 beam. In that context, greater extent of damage in state 5 (before the installation of post-tensioning) for the R2/1 beam was considered as the obvious reason for the proportionally higher difference between \( f_m \) and \( f_{m5} \) as well as between \( f_m \) and \( f_{\text{num}} \) in state 6p0 for the R2/1 beam compared to those for the P1 beam. Notwithstanding, the decreasing effect of installation of the post-tensioning equipment on \( f \), which was pointed out numerically in Sect. 5.2.2, was justified by the \( f_{m5}>f_m \) relation (between natural frequencies measured before \( f_{m5} \)) and after \( f_m \) the installation).

It was interesting to analyse the variation of \( f_m \) as a function of the intensity of \( P \). As shown in Fig. 5.9, until an extremely high intensity of \( P \) (in states 6p and 7p), the measured \( f_m \) frequency unambiguously increased for both types of beam then (in state 8p) slightly decreased for the P1 beam and further increased for the R2/1 beam. This
tendency was in contrast to that of $f_{num}$, which, according to Table 5.2, monotonously decreased for the shorter P1 beam and increased for the other longer beams. Additionally, the variation rate of $f_{num}$ (for both $f_{1,\text{num}}$ and $f_{2,\text{num}}$) with $P$ was much less than that experienced for $f_m$. Note that any change in $f_{num}$ was considered as a clear consequence of the increase in $P$ because mass, geometry and stiffness were intact in the calculation model. The rapid and clearly-increasing tendency of $f_m$ for low and medium degrees of post-tensioning (in states 6p and 7p) suggested that the existing crack-related damage of beams influenced $f_m$ to a much greater extent than the intensifying post-tensioning. Since existing cracks could be considerably decreased (theoretically closed) by post-tensioning, a gradual increase in bending stiffness of the already damaged beams was expected with the increase in $P$ until cracking was minimized. Owing to no change in mass and in geometry, this increasing bending stiffness resulted in increasing $f_m$ natural frequencies for both types of beam. It was pointed out in Sect. 5.4.1 that large plastic deformations in bars prevented cracks to perfectly close for the R1 and the R2 type beams. Therefore, in the case of the P1 beam, for which no plastic deformation occurred in state 5 due to the relatively low $M_{\text{max}}/M_{\text{Rm}}(=0.88)$ ratio, the bending stiffness belonging to the uncracked beam could actually be approached by the application of high-intensity post-tensioning force between states 7p and 8p. In state 8p, in the absence of a further bending stiffness increase, the $f_m$ values were influenced only by $P$ and consequently decreased according to the calculated tendency of $f_{num}$. However, in the case of the R2/1 beam, approaching the bending stiffness belonging to the uncracked beam remained unrealistic even for high-intensity post-tensioning due to large plastic deformations of bars incurred in state 5 ($M_{\text{max}}/M_{\text{Rm}}=1.01$). Consequently, the increase of bending stiffness due to intensifying $P$ did not stop before state 8p, which resulted in a further increase of $f_m$.

The lower $f_m/f_{num}$ ratio of the R2/1 beam for all intensities of post-tensioning force was considered as another consequence of plastic deformation in bars. As shown in Fig. 5.9 for the P1 beam associated with a lower $M_{\text{max}}/M_{\text{Rm}}(=0.88)$ ratio before post-tensioning, the measured $f_m$ natural frequencies exceeded the corresponding $f_{num}$ values from a relatively low post-tensioning force. Meanwhile, for the R2/1 beam associated with a high $M_{\text{max}}/M_{\text{Rm}}(=1.01)$ ratio before post-tensioning, the $f_m$ values remained far below the corresponding $f_{num}$ values. Since all parameters, except for the length, of post-tensioning were identical for each beam, the large difference in the $f_m/f_{num}$ ratio between the P1 and the R2/1 beams throughout the full post-tensioning process could only be explained by the difference in bending stiffness between the two beams. This
again suggested that the cracking-induced bending stiffness decrease of the beams could not be eliminated even by high intensity post-tensioning if previously-occurred, intensive plastic deformation existed in the bars. The above observations indicated that the effectiveness of post-tensioning in closing existing cracks of beams can be justified by frequency measurements. This has great importance from the point of view of the practical applicability of post-tensioning for structural strengthening purposes.
PRACTICAL APPLICATION OF FREQUENCY MEASUREMENTS
6.1 GENERAL

This chapter deals with the applicability of frequency measurements for the in-situ dynamic investigation of a concrete highway bridge. The chapter does not focus on the dynamic analysis of the structure, but demonstrates that frequency measurements can be executed also on existing structures without significant disturbance of their normal service.

6.1.1 Aspects of practical application

As discussed in Sect. A1.1 condition control systems for civil engineering structures are based on either occasionally executed inspections or regular monitoring. There are dynamic-based, on-line monitoring systems, which provide continuous control of modal parameters of the structure. However, their installation has to be considered already in the design period of the structure and, furthermore, their operation has to be continuously supervised by the maintaining staff.

This method, the very simple version of which is introduced in this chapter, is intended for intermittent application to existing structures. The aim of the method is to provide monitoring of the first few natural frequencies of the structure and to alert if changes in these frequencies are detected, which may indicate structural problems. The clear advantage of this method is that it requires little and easily installable equipment, consumes short time for in-situ activities, can be applied on different structures as many times as needed without disturbing their normal service. All these result in cost-efficiency.

Monitoring is essential for such civil engineering structures that are exposed to many, various deterioration effects. Bridges have priority from this point of view. The majority of bridges have a simple, girder-type structural system. The simpler the structural system of a structure, the easier the differentiation of its modes and, consequently, the less measurement as well as computation effort is needed for its identification and assessment. The introduced method has been applied to a concrete highway bridge built with a continuous, girder-type structural system. The aim of identifying and assessing existing crack-related damage of the tested bridge was desirable but unrealistic, considering that very limited information on the first natural frequency was available from previous investigations (Illéssy, 1980).

The natural frequencies of a structure are usually determined under free vibration if service conditions make it possible to induce free vibration. For bridges subjected to high traffic, the induction of free vibration requires a traffic restriction, which results in
significant cost and many organizational measures. For the introduced in-situ measurement, the excitation of the bridge was induced by the normal road traffic.

6.1.2 Assumption on the excitation effect caused by road traffic

If neglecting structural damping, the differential equation of motion for beams under forced vibration as well as its general solution is as follows:

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = P(t) \quad \Rightarrow \quad y(x,t) = Y_{\text{hom}}(x,t) + Y_{\text{inhom}}(x,t,P(t)) \quad (6.1)$$

where $EI$ is the bending stiffness of the beam, $\mu$ is the specific weight, $y$ is the deflection, $x$ is the longitudinal abscissa, $t$ denotes time and $P(t)$ means the time-dependent excitation effect. In this test, $P(t)$ was produced by normal road traffic. Because the excitation force caused by the road traffic is non-regular in time and, consequently, a priori non-specifiable, it may be assumed that the occurrence of signs of excitation in the vibration signals of the structure under normal road traffic is stochastic. If the in-situ recorded time signal of the bridge is put in place of $y(x,t)$ in Eq. (6.1) then its homogeneous part corresponds to free vibration and the inhomogeneous part is assumed to correspond to the excitation effect. Therefore, a statistic-based signal evaluation procedure, which allows for this stochastic character of the excitation effect, is able to filter the non-regular excitation signs (inhomogeneous part) out of the vibration signals and to enlarge the regularly occurring modal effects (homogeneous part). The modal response of the structure is always included in free vibration.

6.2 INTRODUCTION OF THE IN-SITU TEST

The aim of the conducted test was to measure the first few natural frequencies of the investigated bridge and to reproduce the respective mode shapes. In this section, the components and the arrangement of the measuring system, the introduction of the signal evaluation procedure and the applicability of the test results will be presented.

6.2.1 The investigated structure

The test was carried out on a four-span, concrete highway bridge arranged as a conventional Gerber-system with a statically indeterminate, two-span, post-tensioned internal part as shown in Fig. 6.1. The load carrying system had a voided slab-type cross-section of constant depth along its full length. The vibration signals of the struc-
ture were simultaneously recorded by Hottinger B3-5 type vibration detectors in three measurement locations and numerically recorded by appropriate devices.

![Fig. 6.1 View of the test bridge](image)

The arrangement of measurement locations was determined on the basis of the expected mode shapes. To help in selecting the appropriate measurement locations for structures with a complex shape and/or structural system, the determination of expected mode shapes in advance is recommended by simple, finite element calculation models, which are not applied later in the assessment process.

![Fig. 6.2 Scheme of the test bridge and the measurement locations](image)

Owing to the natural frequency of the vibration detector \(f_{\text{det}}\) itself, deflection was registered by the detector for vibration of the bridge with frequencies ranging below \(f_{\text{det}}\) and acceleration was registered for vibration with frequencies ranging above \(f_{\text{det}}\). Vibration signals were derived from the in-situ numerical recording of either deflection or acceleration as a function of time.

### 6.2.2 Excitation of the structure

For excitation purposes normal road traffic was used, which kept the structure under continuous vibration and resulted in the following:
• the frequency spectrum of the excitation force depended on the characteristics of the crossing vehicles (weight, speed, suspension, etc.).
• due to the short time intervals between crossing vehicles, there was no real possibility (especially for dense traffic) for the development of “clear” free vibration periods of the bridge. Thus, direct mode shape measurements (i.e. measurement of the vibration amplitudes simultaneously at as many locations on the structure as needed to reliably reproduce mode shapes) were unrealistic.
• only natural frequencies falling into the frequency range of the excitation effect could be determined.
• statistical evaluation of measurement data was necessary to filter the recorded vibration signals. During this process, the relative weight of the irregularly occurring effects was decreased while that belonging to the regularly occurring modal effects was increased (see Sect. 6.1.2).

Considering the above, it was assumed that the frequency range of the excitation effect was sufficiently wide to cover the first few natural frequencies of the bridge. The intensity of the excitation force was of the same order as for ordinary static tests and, consequently, assumed to be sufficiently high along this frequency range (5-20 Hz).

6.2.3 Statistic-based signal evaluation
The evaluation of the in-situ recorded time signals of all three measurement locations was carried out identically.
In the first evaluation step, parts of time signals were transformed into frequency spectrums by discrete, fast Fourier transformation. The length (in time) of these time signal parts were chosen according to the intended resolution of the frequency spectrums. Two-level signal processing was conducted. The reason for this was the intention to analyse how effectively could the stochastic excitation effect be statistically eliminated from the frequency spectrums by changing the length of the applied time signal parts. On the first level, ~0.1 Hz frequency resolution of spectrums was aimed to be achieved. On the second level, the resolution was increased up to 0.02 Hz. In the latter case, the number of data in each time signal part was set to 2048, which resulted in 40.96 s long time signal parts. This required 34 minutes of time signal in total to be recorded simultaneously at each measurement location.
In the second evaluation step, the above individual frequency spectrums were averaged and which resulted in average frequency spectrums. Fifty individual frequency
spectrums were involved in the averaging process. By calculating these average spectrums:

- the relative weight of the stochastically occurring (excitation) frequencies decreased and that of the permanently present natural frequencies increased.
- if increasing the number of the individual frequency spectrums involved in the averaging process, which required longer in-situ recorded time signals, the reliability of natural frequencies read from the average frequency spectrum could be improved. The reliability was measured by the standard deviation of frequencies associated with the maximum ordinates in the individual spectrums.

![Fig. 6.3 Average frequency spectrums of measurement locations](image)

**Fig. 6.3** Average frequency spectrums of measurement locations

*Fig. 6.3* shows that exactly the same numerical values for the first four natural frequencies were obtained from the separate evaluation of signals recorded simultaneously at the three measurement locations. The first natural frequency ($f_1$), as the only available modal data from previous investigations, was determined by field test as equal to 7.88 Hz in 1980 (*Illéssy*, 1980). Our measurement resulted in $f_1$ very close to this value. No information was available whether resurfacing, change of bearings or any structural intervention was made during the elapsed period. Direct comparison of $f_1$ values would have been justified if information on these events were considered.

Since the ordinates of average frequency spectrums are of no sign, no information on phase differences between the vibrating measurement locations could be obtained. Thus, when using average spectrums, mode shapes associated with the natural frequencies cannot be reproduced.
6.2.4 Identification of mode shapes

In order to make visible the phase differences between each two measurement locations during the vibration, the sum and the difference of recorded time signals belonging to the considered two locations were composed. Then the average frequency spectrums of these "sum" and "difference" time signals were calculated (and noted by (1+2), (1-2), ...(2-3), respectively). For any two measurement locations, which vibrated in the same phase in a mode, the "sum" time signal amplified the measured $y(x)$ vibration signal while the "difference" time signal minimized it. On the contrary, for other two measurement locations, which vibrated in the opposite phase in a mode, the "sum" time signal minimized the vibration signal while the "difference" signal amplified it. The phase spectrums belonging to all possible measurement location pairs were obtained by calculating the differences of the above average frequency spectrums composed from the "sum" and "difference" time signals according to (Kálló, 1996). The amplitudes in these phase spectrums, as shown in Fig. 6.4, had either positive or negative sign. The sign of the amplitude at one of the natural frequencies indicated the phase difference between the considered two measurement locations in the mode associated with the considered natural frequency. This made the reproduction of mode shapes associated with the already determined natural frequencies possible and, what is more important, enabled them to be compared with the calculated mode shapes (see Sect. 6.2.1).
Based on the above sign rule, the phase spectrums in Fig. 6.4 suggested that the first and the second mode were bending modes and that the fourth mode was a torsional mode. The identification of the third mode, whose shape seemed to be a complex one, was difficult. To clarify the shape of the third mode as well as the reason for the relatively wide frequency range with uncertain peak in the spectrums between the second and third natural frequency, computer models are helpful.
7

SUMMARY AND
NEW SCIENTIFIC RESULTS
7.1 SUMMARY OF THE THESIS

This thesis deals with vibration-based, crack-related damage identification and assessment of concrete model beams under experimental conditions. The research was motivated by the fact that existing vibration-based damage identification methods in the civil engineering field focus primarily on detection, localization and maybe quantification of damage, but do not address structural performance, which is of particular importance for owners to plan structural interventions or to estimate remaining service life. The idea behind the research is that any change in the modal data of a structure indicates a change in one or more of the influencing parameters (mass, geometry, support conditions, stiffness, external forces). If excluding intentional alteration of a structure and its operating conditions, damage-caused changes in the influencing parameters can be revealed by dynamic measurements during the service life.

This research was carried out on simply-supported beams, focused only on flexural crack-related damage and used the first two natural frequencies and their shifts as damage indicators. Damage was modelled either by cracking for reinforced concrete beams or by the combination of cracking and tendon cuts for prestressed beams. Damage was measured by the extent of cracking and by the extent of reduction in the effective prestress due to tendon cuts. To model structural strengthening, post-tensioning was applied to a few, already deteriorated reinforced concrete beams. To reduce the effect of model uncertainties coming from modelling the structural behaviour of cracked beams under vibration, damage assessment was carried out without the development or use of dynamic models.

Chapter 1 gives a summary on damage identification in civil engineering by briefly introducing existing vibration-based methods through a detailed literature review. A comparison of methods is provided according to the information content of the investigated parameter on the structure, practical applicability and sensitivity to damage. Chapter 2 clarifies the concept and the goals of the research. Chapter 3 introduces the conducted laboratory test by describing the test beams, the execution of the artificial deterioration process and the frequency measurements as well as the applied signal processing and signal evaluation procedures. Chapter 4 defines, classifies and determines calculated and measured damage indices used for crack-related damage identification purposes. Chapter 5 establishes and concludes relationships between the first two natural frequencies and the structural performance-related parameters of test beams for assessment purposes. Chapter 6 demonstrates
the practical applicability of frequency measurements with a simple test carried out on a concrete highway bridge under service conditions.

7.2 PRACTICAL APPLICABILITY AND POSSIBLE FURTHER DEVELOPMENT OF THE RESEARCH

The introduced experimental, vibration-based damage identification and assessment method consisted of a natural frequency measurement technique and a crack-related damage assessment procedure. The measurement technique is available at low cost for intermittent or continuous observation of natural frequencies of reinforced, prestressed and post-tensioned concrete structures. Its application is mainly intended for girder-type bridges, for which normal road traffic can be used for excitation purposes. Using the relationships established between the measured natural frequencies and the above performance-related parameters, any change in the structural performance of the investigated structure during the service life can be detected, localized and quantified on the basis of measured changes in the natural frequencies.

Further development of the introduced method is possible in two main directions. The first is the expansion of the range of measured dynamic characteristics in order to increase the information content of on-site measurements on dynamic behaviour. This is recommended only if the required effort (cost and time) of the on-site measurements can be kept within reasonable limits. From this point of view, the inclusion of damping parameters in this range is recommended. The second direction is either the further improvement of the existing relationships between the measured dynamic characteristics and the structural performance, or the definition of new ones. In this area, the conducting of sensitivity studies, the provision of a statistical basis behind the relationships as well as the extension of the concept of damage beyond flexural cracks is desirable.

7.3 ACKNOWLEDGEMENT

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7.4 LIST OF THE NEW SCIENTIFIC RESULTS

The new scientific results of the presented work are summarized as follows.

New result 1

Based on a literature review, it was concluded that the existing dynamic-based damage identification methods focus mainly on detection, localization and sometimes quantification of damage but do not closely address structural performance-related issues, which limits their practical applicability.

To overcome this, a new damage identification level (Level 3a) was defined. Beyond damage identification, assessment methods fulfilling Level 3a are able to provide information on the structural performance. It was pointed out that the effective practical applicability of these methods requires:

a) the definition of performance-related damage indices that are based on previous static models and able to identify damage as well as to measure its extent (identification);

b) the establishment of relationships between the measured dynamic parameters and the performance-related structural properties (assessment).

Related publications: [1],[3],[4],[5],[6],[7],[8],[9],[10],[14],[15],[17]

New result 2

Cracking-induced deterioration processes were modelled in experimental conditions on simply supported, reinforced and prestressed concrete beams under flexure. Meanwhile the first two natural frequencies of beams were determined at gradually increasing damage levels. Damage was modelled by:

a) gradually intensified cracking due to defined, four-point mechanical loading for the reinforced beams;

b) the combination of a) and artificially-made tendon cuts for the prestressed beams;

c) the application of external post-tensioning (simulated strengthening effect) to a few, already deteriorated reinforced concrete beams.

The extent of damage was measured by the extent of cracking and by the number of tendon cuts. Alternatively, single mechanical impact and a periodic signal were used for excitation purposes during the natural frequency measurements.

Related publications: [1],[3],[4],[5],[6],[7],[9],[10],[11],[12],[13],[14],[15],[17],[18]
New result 3
A computational procedure for the evaluation of numerically-recorded acceleration-time signals was developed. By the use of this, the first two natural frequencies of the test beams were determined. The procedure was based on the statistical analysis of discrete Fourier-transformed individual frequency spectrums. Using this technique, the following were pointed out:

a) Both the average frequency spectrum (average spectrum method) and the average value of frequencies associated with the maximum ordinate in the individual spectrums (average frequency method) were solely able to indicate the first two natural frequencies with an acceptably low (<1%) coefficient of variation for the states of the beams in which the bending stiffness distribution remained symmetric along the beam length.

b) For states where the bending stiffness distribution became non-symmetrical due to a non-symmetrical crack pattern, multiple peaks occurred in the relevant frequency spectrums. The value of the standard deviation associated with the average frequency clearly indicated the presence of multiple peaks. If one of them was dominant, then the combined use of the average spectrum method and the average frequency method was required to estimate the true natural frequency as the average of the frequencies associated with sub-peaks.

c) The first two natural frequencies could reliably (with a relative difference less than 1.5%) be determined from vibration signals derived from either impulse or periodic excitation input, if no multiple peaks existed in the related frequency spectrums. If multiple peaks existed, then the relative difference increased significantly (>>1.5%).

Related publications: [2],[6],[8],[16],[19]

New result 4
Static-based damage indices which reflected cracking-related damage and were in accordance with a) of New result 1 were defined as follows:

- Growth of "total length of cracks"
- Growth of total and residual deflection at midspan
- Growth of crack width at midspan
- Growth of "total of crack sections"
- Growth of calculated midspan deflection
- Growth of calculated crack width at midspan
- Growth of internal strain energy
Their values were determined for all deterioration states of the test beams. Their classification was based on:

- their information content on the structure (global or local indices);
- the absence or the presence of any calculation model behind the index itself and their involved parameters (non-model-based (measured) or model-based (calculated)).

Depending on their intended purpose in damage identification, the indices were interpreted in both incrementative (for detection purposes) and cumulative (for quantification purposes) forms. Successful conformity checks were carried out between the corresponding non-model-based and model-based indices.

_Related publications: [1],[3],[4],[5],[6],[7],[9],[10],[11],[12],[13],[14],[15],[17],[18]

**New result 5**

By the use of the defined damage indices, damage identification as well as assessment of test beams were carried out.

The defined indices were able to detect and to quantify the introduced artificial, crack-induced damage in the beams. The identified damage was confirmed by the measured natural frequency shifts.

Within the assessment, numerical relationships were established between the natural frequency shifts and:

- for the reinforced concrete beams
  - the internal force level;
  - the extent of cracking immediately after the occurrence of first cracks;
  - the type and amount of reinforcement;
- for the prestressed beams
  - the internal force level;
  - the extent of degradation in prestress;
- for the post-tensioned beams
  - the intensity of the post-tensioning force.

_Related publications: [5],[15],[17]
New result 6
An in-situ frequency measurement and the respective signal evaluation were carried out on an operational reinforced concrete highway bridge. The aim of the test was to verify that reliable (with low standard deviation) determination of the first few natural frequencies of the structure was possible under the service load of the bridge. For excitation purposes the normal road traffic was used. The acceleration-time vibration signals were recorded simultaneously in three measurement locations. It was found that:

a) the excitation force of the normal road traffic was sufficiently high and smooth over the target 5-20 Hz frequency range of the first four natural frequency of the bridge. By the use of 34 min. long in-situ recorded acceleration-time signals at each measurement location, this frequency range could be covered by discrete Fourier transformed frequency spectrums with 0.02 Hz resolution;

b) the phase spectrums computed as combinations of average frequency spectrums determined from sums and differences of acceleration-time signals belonging to measurement locations of selected measurement location pairs were capable of indicating the phase differences between the measurement locations of the selected pairs in each mode. This made it possible to reproduce the mode shapes associated with the measured natural frequencies.

Related publications: [2],[6],[8],[16],[19]
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Publications referred in the new scientific results (Sect. 7)

**Book, sections in book**


**Paper in collected volume**


**International journal paper**


**Domestic journal paper in English**


**International conference paper**


REFERENCES


Hungarian conference paper


Technical Report

APPENDIX
A1 ASPECTS OF CONDITION CONTROL FOR CIVIL ENGINEERING STRUCTURES

This section summarizes the main aspects of the existing condition control systems applied to civil engineering structures then provides background to their comparison.

A1.1 Inspection versus monitoring

Considering the frequency of activities carried out within the framework of a condition control system for CE structures, they can be carried out occasionally and regularly in time (Fig. A1.1). The occasionally-performed condition control is mentioned as “inspection” and the regularly-performed condition control is mentioned as “monitoring”.

![CONDITION CONTROL of CE structures](image)

Occasional INSPECTION

Regular MONITORING

Intermittent

Continuous

Fig. A1.1 Overview of condition control methods regarding the frequency of their application

However, not only the occasionally-performed condition controls are covered by the concept of inspection but all of the other, intermittently-performed investigations (which are not based on the comparison of results obtained from current investigations to those from previous ones to evaluate the change in the structural condition (e.g. serviceability, reliability)) are classified as inspection. Without this comparison, at every inspection when the performance of the structure is to be assessed, all the characteristics of the structure, which are needed to calculate the current performance, have to be measured or estimated. The result coming from one of these occasional inspections can be considered as a snapshot taken of the structure at a particular moment.

Often these inspections are carried out when the presence and sometimes the location of the damage is obvious (e.g. in cases of truck collisions with bridges). At these times, the goal of the inspection is to evaluate the effect of damage on the current performance of the structure by comparing it to the designed performance.

In other cases, intermittently-performed inspections are carried out to compare one or more parameters of a structure to the designed values. The evaluation of the structural performance is only based on the results of the current inspection. Generally this is the
case for bridges in many countries, for instance in Hungary, when the responsible bridge authority is officially obliged to carry out inspections on bridges after every given period - depending on the importance of the bridge - in order to verify the adequate reliability of their structures. At these inspections, if no obvious damage is known, generally the deflections of the structure are measured in typical points due to appropriately positioned static trucks (static load test, see Sect. A1.3.2) and compared to the corresponding calculated values.

Without the mentioned comparison and without any more sophisticated damage detector and localizing technique, these simple inspections are not able to detect and to localize hidden damage in CE structures.

If the condition control activities are performed regularly and based on the comparison of the current measurements or their results to those of the previous investigations, a monitoring process is occurring. Naturally, in most cases, an assessment method supplements the monitoring process, by which the system is capable of damage localization as well as structural condition evaluation.

A monitoring process can be carried out continuously in time by the application of online measuring techniques using different kinds of sensors and/or videos, which are either embedded in the (concrete) structure or externally fixed to it (Harichandran and Hong, 2003). When using sensors, either an electronic or a wireless communication system is necessarily installed between the sensors and the data acquisition instrument (Schulz et al., 1998).

The 165.1 m long Taylor Bridge (1997), which spans the Assiniboine River in Canada and is fully made of prestressed concrete with fibre reinforced polymers as reinforcement, is equipped with 66 fibre optic sensors coupled with conventional embedded electric strain gauges, which communicate with the engineers through telephone lines (Korany and Rizkalla, 2000). The Tsing Ma Bridge (1997), which crosses the 1377 m wide Ma Wan Channel in Hong Kong, is equipped with 600 sensors to monitor the structure permanently (Farrar et al., 1999; Chan et al., 1998) (Fig. A1.2).
The earliest monitoring systems work intermittently rather than continuously in time. These are not capable of on-line data transfer however the essence of operation is similar to those for the continuously working systems. The measurements are usually repeated daily, weekly, monthly or yearly depending on the needs of the owner or the engineers.

### A1.2 Diagnostic techniques

The diagnostic (inspection/monitoring) techniques can be categorized as local or global techniques (Fig. A1.3).

#### A1.2.1 Local diagnostic techniques

In this context, a local technique is considered as a group of activities which focuses on the affected structural part(s) only and detects damage by often measuring the change in a particular quantity. Based on this change, the method is able to detect and sometimes characterize the severity of the damage. The selection of the measured quantity...
Appendix

depends on the applied measurement technique. It means that the vicinity of damage to be quantified has to be known beforehand, otherwise a full scan through the whole structure is needed to detect damage first. The crucial point is that a local diagnostic technique does not take into account the global behaviour (response) of the structure in the structural evaluation even if it is:

- repeated as many times per investigation as needed to perform a full scan through the entire structure and
- used for regularly scanning (monitoring) the structure.

The local diagnostic techniques can be classified as non-instrumental or instrumental techniques.

The only obvious non-instrumental technique is the visual inspection. It is heuristic and fully based on the experience and the knowledge of the applying person. There is a lack of close correlation between visual appearance and structural reliability of a structure. Visual inspections find signs of corrosion and damage such as cracks, spalls, local brittle failures, chemical attacks, etc. only when those become visible. The influence of damage on the structural reliability, which can be considerably different for steel or concrete structures, is very difficult to establish based on these inspections. The causes of the observed damage can also not be clearly identified. Thus, the judgement of the structure is also based mainly on experience.

Based on Schickert (1997), Fig. A1.3 lists the most frequently applied, instrumental local damage detecting techniques in the sequence of their usefulness in the civil engineering field. The developed techniques and their applied methods are generally used to measure different properties of material such as strength, thickness, water content, moisture content, density, porosity, thermal conductivity, permeability, surface properties, etc. and, in several cases, those of structural parts such as cracking, fractures, integrity, delamination, presence of air and liquid voids, holes or hidden objects, position of reinforcement, corrosion activity (carbonation, chloride-attack), stress intensity, etc. Similar review on the instrumental local damage detecting techniques for grouting of prestressing tendons is included in IMT (2003).

A complete COST action (COST Action 521) dealt with the corrosion and the corrosion monitoring of steel embedded in concrete (COST, 2003) by local techniques. It particularly focused on the corrosion state monitoring of reinforcing steel by using embedded sensors and electrochemical techniques, determining the chloride threshold as well as on particular maintenance methods (electrochemical methods, surface applied inhibitors, surface treatments and coatings, durability of patch repairs). The COST Action
Appendix

534 (New Materials and Systems for Prestressed Concrete Structures) mainly focused on the non-destructive monitoring of prestressed concrete structures with the use of acoustic emission techniques (Golaski, 2004; de Wit, 2004; Watson, 2004), ultrasonic methods (Forde, 2004) and electrical isolation (Elsener, 2004a, 2004b; Vedova and Evangelista, 2004). Magnetic methods are also frequently used to locate prestressing tendon breaks in different types of prestressed concrete structures including bridge decks and parking structures (Hillemeier and Scheel, 2004). Significant efforts are made in many countries to develop and improve techniques as well as testing methods to detect and locate local damage in post-tensioning tendons (Poston and West, 2004; Miyagawa et al., 2004).

It is important to see from the literature that the above instrumental local techniques are being apparently used more and more frequently in practice for monitoring purposes rather than as a damage locating tool for occasionally-performed state controls. These techniques involve significant cost and, in spite of this, the current performance (reliability) and the remaining service life (Level 4) of the whole structure, which are very meaningful pieces of information for both the owner and the engineers, can not be assessed on the basis of these investigations.

A1.2.2 Global diagnostic techniques (methods)

Based on the foregoing, a global diagnostic method is considered if the global behaviour of the structure is measured in-situ in an appropriate way and the structural (condition) evaluation (damage assessment) is based on these results. The key issue is the recording and the consideration of the response of the whole structure influenced by deterioration. This makes it possible to define deterioration mechanisms, to make distinctions between different damage with respect to their risk and to clearly understand how local damage affects the performance of the whole structure.

In using a global technique, the location of a possible deficiency does not need to be known beforehand. Since global diagnostic methods record the response of the whole structure at given points rather than performing local scans around the damage-affected structural part, they have to be supplemented by evaluation algorithms or postprocessing tools, which transform the recorded data into meaningful quantities, diagrams, graphs, etc., which more comprehensively and visibly show the structural performance for both the engineers and the owner. Fig. A1.3 lists the most frequently used global diagnostic techniques (Catbas and Aktan, 2002) with a reference in brackets to the applied evaluation tools.
Consequently, damage assessment can only be carried out with the application of a global diagnostic technique. By doing so, the influence of both the affected and the unaffected (by damage) structural parts on the global response of the structure can be taken into account in the assessment, which is fundamental to qualifying structures as a whole and in performing reliability analyses. The reliability analyses are necessary preconditions to carry out Level 4 of the proposed damage identification procedure in all cases. Often the quantification of damage at Level 3 also requires a damage assessment based on the global response of the structure (Enright and Frangopol, 1999).

A1.3 Damage assessment for civil engineering structures

There are many terms in literature used for the process which provides information on the structural performance of a CE structure based on the analysis of global response data. The real content of processes described by the expressions such as:

- damage assessment
- condition assessment
- (structural) condition evaluation
- (structural) health monitoring

are nearly identical, therefore they will be mentioned as synonyms henceforth in this thesis.

As mentioned in Sect. 1.2, besides being a useful tool in carrying out Level 2, 3 and especially Level 3a of a damage identification procedure, the main goal of damage assessment is to point out how damage affects the structural performance of the whole structure. According to Sect. A1.2.2, a damage assessment method requires the application of a global diagnostic technique together with an appropriate evaluation algorithm. These are briefly indicated in Fig. A1.3.

A1.3.1 Linear versus non-linear damage assessment

Depending on the preliminary assumptions concerning the global behaviour (force-displacement response) of the investigated structure, the method of damage assessment can be classified as *linear* on *non-linear*. It means, for instance, that for a linear method, the expressions applied by the algorithm of the assessment procedure assume a linear(-elastic) relationship between the external forces and the displacements in the global response of the structure.

It is important to underline the global character of the structural behaviour from the point of view of the above classification. In the case of constant geometric and physical (mass, stiffness, damping) properties, invariable support conditions and usual materi-
als, in which the behaviour does not show significant hysteretic or non-linear character, the linear-elastic structural behaviour of an undamaged CE structure is generally assured by the nearly linear-elastic relation between the external forces and the displacements (global response) in that displacement domain, in which the structure works under service conditions. However, damage may lead to non-linearities in the global response of the structure by inducing change in one or more of the geometric or physical properties, the support conditions and possibly the material behaviour (law).

When deciding whether a linear or non-linear method of damage assessment should be used, the questions of crucial importance are:

- whether the global behaviour of the structure is linear before the damage
- whether the possible damage is significant enough to change the previously linear-elastic global response of the structure to non-linear after its occurrence.

Before answering these questions, a clear distinction has to be made between the following cases:

a) The first and most frequent case is, when a previously assumed linear-elastic structural behaviour remains linear-elastic after the occurrence of the damage. This case is defined as a "linear damage situation".

b) The second one is when the non-linear character of the previously-assumed linear-elastic global structural behaviour is caused exactly by the occurrence of the damage. This case is defined as a "non-linear damage situation" (Doebling et al., 1996).

c) The third one is when the global response of the structure, independently of any possible damage, can not be assumed to be linear-elastic because of the presence of significant non-linear effects in the global (structural) and/or possibly in the local (material law) behaviour.

The most obvious example is when cracks appear in a previously uncracked, reinforced concrete beam during a monotonic load history. The occurrence of cracks causes a significant singularity in the global (force-displacement) response, but for usual cases, the global response of this structure through the full load history, which remains in the service domain, can be modelled by assuming a bilinear global force-displacement relation. If the cracks are considered as damage in terms of the assessment and the occurred cracks do not close any longer under service conditions then, even if the load is not monotonic, this situation for small size cracks generally can be taken into account in Case a) as a linear damage situation and is generally assessed by linear methods. However, for large size cracks, often non-linearities occur, the case of which belongs to Case b).
More complex examples are cited below. In many cases, the global behaviour of cracked reinforced concrete beams subjected to altering flexure contains non-linear effects due to the behaviour of cracks.

For a simply reinforced concrete beam under flexure of varying intensity, when the cracks are continuously open with varying width (Fig. A1.4a), the non-linear behaviour comes from the friction effects between the crack surfaces. However, according to experience, this non-linear effect in the global response can be neglected if the stress in the tension reinforcement is far below the yield stress. If damage occurs in this beam and this damage does not result in significant non-linearity in the global response then this case belongs to Case a). If not, the case belongs to Case b).

For the same structure, if the bending moment alternates (Fig. A1.4b) between sufficiently large intensities (e.g. for slender continuous structures subjected to heavy moving loads), the cracks may close in one extreme deformation phase and open in the opposite phase. Between these deformation phases in the same cycle, the bending stiffness of the beam around the cracks changes considerably which results in non-linear force-displacement response of the structure, while the linearity in the material behaviour (between the internal stresses ($\sigma$) and deformations ($\varepsilon$)) is generally fulfilled. This case belong to Case c).

Furthermore for cracked prestressed concrete beams, even when the varying bending moment does not alternate (Fig. A1.4c), the effective prestress may close the cracks in one extreme deformation phase, therefore the continuously closing and opening cracks under service conditions result in the same non-linearity in the global force-displacement response of the beam as for the above case. This case also belongs to Case c).

Similar non-linear effects in the global response may occur in the case of fatigue-induced cracks for steel structures.

Another obvious case for non-linear force-displacement response is the occurrence of plastic strain in steel structures and in reinforcing steel of concrete structures. How-
ever, these situations are very rare in practice for ordinary CE structures because the intensity of service loads is generally much lower than those which cause plastic strains in the material. For field tests including also the initial load test, the applied load also remains below that level. For typical bridges in Hungary, approximately 80% of the design load is applied for the initial static load tests, which for concrete structures may approach that level from below. For moving load tests, a single 200-250 kN truck, which has a mass far below the design load, rolls across the bridge.

In practice, the primary goal of monitoring CE structures is the detection of the onset of deterioration or the unexpected malfunctions due mainly to environmental actions and long-term effects such as ageing and fatigue (Level 1). According to experience, the significance of early damage to the global behaviour of a real structure is generally low, therefore the assumption of a linear relationship between the external forces and the displacements can be appropriate in most cases. This makes it possible to apply linear damage assessment methods by assuming Case a) even if the global linear behaviour of the structure is not strictly linear-elastic, because of, for instance, generally non-linear soil-foundation-structure interaction or many local non-linearities, which may come from local damage or localized malfunctions (e.g. malfunctions in connections or at bearings) (Catbas and Aktan, 2002). Additionally, this linearity is even more apparent if the applied load (excitation) during the diagnostic phase of the assessment is low and, therefore, unable to cause significant non-linearities or hysteretic effects in the global response of the structure.

A1.3.2 Damage (condition) indices

Generally, the result of a damage assessment procedure is a parameter called a “damage index” (or “condition index”). In optimum cases, it contains all the information (or the most important ones) on the structural behaviour, which have been recorded during the diagnostic phase. This index is not necessarily a numerical value; it can be represented by, for instance, a set of numerical values having physical content, influence lines, patterns of typical properties, etc. The damage-assessment-based monitoring of a CE structure, which, in this context, means an intermittently repeated damage assessment procedure or, in other words, an intermittent recalculation of the applied damage index, focuses on the change in the applied damage index.

In order to improve the applicability and the efficiency of these indices in assessing CE structures, the following qualities should be assured on the highest possible level (Catbas and Aktan, 2002):
The experimental and postprocessing requirements for determining an index as well as the tolerance of this index to measurement and postprocessing errors (robustness) should be minimal.

A damage index based on the global response of the structure should be sensitive to the occurrence and the accumulation of damage anywhere in the structure as well as it should be insensitive to ambient effects, which are not related to damage.

A damage index should remain valid throughout the service conditions.

While the damage index is related to global structural behaviour, it should be useful to detect (Level 1), to localize (Level 2) and to quantify (Level 3) damage.

The damage index should provide meaningful information on the structural performance and it should be the basis for making management decisions.

In detecting damage, a basic damage index (baseline), preferably related to the undamaged structure can be very useful. While monitoring is based on intermittent or continuous comparison of the current and the previous damage indices, often the basic damage index relates to the initial, definitely undamaged state of the fully equipped structure just before the beginning of its intended use. For existing (older) structures, where this is not realistic or where a known change in the geometric properties, the support conditions, the mass of the whole structure and/or the mass- and/or stiffness-distribution due to e.g. performed renovation works has occurred, a thoroughly scanned and fully assessed state of the changed structure can also be a baseline. If the above changes are not known beforehand, they will likely be identified as damage by the assessment method when using the initial state as a baseline.

Depending on whether linear or non-linear damage assessment is performed, the resulting damage indices can be classified as linearized or non-linearized. Taking into account the fact that in most cases linear damage assessment methods are used (see Sect. A1.3.1), generally linearized damage indices are defined. A brief summary on the most frequently applied, linearized damage indices will be introduced below.

**Static load test**

The most similar and widely used but circuitous, expensive and time-consuming way of conducting linear damage assessment is the measurement of the typical deflections of a structure at particular points after positioning preweighed (truck) loads into preliminarily determined locations (e.g. static track load test for bridges (Farkas and Szalai, 1998)). In order to collect a sufficient amount of information on the global deflection response of the structure, the measurements have to be repeated several times with
different load configurations. The numerical values of the structural properties (stiffness distribution, rigidity of supports mainly in vertical direction) can be calculated based on the measured deflection values. If a baseline finite element model is used, the model properties can be adjusted until the measured and the calculated deflection values coincide. The monitoring process can be realised by intermittently repeating these tests and comparing the corresponding resulting information on the global behaviour of the structure. For concrete bridges, damage such as the occurrence of unexpected cracks can be identified as, for instance, a change in the stiffness around a local structural part. For damage assessments carried out in practice by using such static load tests, the damage indices are defined as typical deflection lines belonging to a given load configuration rather than individual deflection values of particular points.

The damage assessment can be performed in a similar way by measuring strains at particular points of the structure instead of deflections (Catbas et al., 2000, Lenett et al., 1999) because the installation of the equipment for deflection measurements are often problematic due to difficult accessibility. For concrete structures with expected cracks around the intended measurement points, generally the deflection values are registered rather than strains because of measurement difficulties. However, for steel structures, when cracks are rarely expected, mainly strain measurements are conducted.

**Moving load test**

A powerful damage assessment can be conducted by using unit influence lines as a damage index determined generally from recorded displacement (mainly deflection) and/or strain responses during moving load tests (Catbas et al., 1998). The method is well-known and widely used for railway bridges (Illésy, 1977). If the geometric and the weight properties (the weight of each axle and the distance between the axles) of the moving vehicle (truck or train) are known and the synchronized recording of the load position and the measured response (deflection or strain) of the bridge is carried out, then the measured response (e.g. along a line parallel to the bridge longitudinal axis) can be decomposed into a normalized influence line as if a unit concentrated load travels over the bridge (Fig. A1.5) (Goble et al., 1991). Usually the speed of the vehicle is set as low as possible in order to eliminate the dynamic amplification from the recorded response. In opposite cases the dynamic components of the measured response have to be separated before the decomposition. In the possession of the normalized (unit) influence lines, they can be extracted for any set of moving loads (truck or train) by scaling them to the current axle weights, shifting them by the distance between the axles.
axles and finally summing them up. By using the latter procedure, the magnitude of the response of the structure due to any known set of moving loads can be preliminarily predicted and checked on site (Catbas and Aktan, 2002).

The monitoring process can be realised by intermittently determining and comparing the corresponding unit influence lines or by using the normalized influence lines for predicting the measured response of the structure at each time for the applied set of loads.

**Vibration test**

Damage assessment can also be conducted by modal-analysis-supported vibration tests. These assessment methods and the accompanying diagnostic tools focus on the dynamic characteristics of the investigated structure such as the natural frequencies, the corresponding mode shapes and sometimes the damping parameters. Depending on the applied method, several damage indices have been defined. The classification and the essence of these methods as well as several examples for their practical application are reviewed in Sect. 1.3 and Sect. 1.4.

**A1.3.3. Environmental effects**

In conducting damage assessment on CE structures, environmental effects, especially temperature changes, may have significant impact on the results thus on the damage indices. As mentioned before, the introduced damage assessment methods are based
on the preliminary assumption that the global response of the structure changes only due to the possible and identifiable damage. However, the temperature effects may significantly influence the boundary (at supports) as well as the continuity (at joints) conditions of the structure or even strains may be accumulated in particular elements of statically indeterminate structures due to uneven temperature changes, effects of radiation or temperature shocks (high change in temperature within a very short time) (Catbas and Aktan, 2002). Sometimes the influence of these effects on the damage indices may be more significant than the effect of a possible damage (Aktan et al., 1994). Therefore, it is extremely important to perform the diagnostic phase of the damage assessment under approximately the same ambient conditions. This can be easier if an additional, long-term and possibly continuous (not intermittent) observation of the temperature changes is included in the monitoring program.

A2 EXCITATION TECHNIQUES IN VIBRATION-BASED CONDITION CONTROL

Within a vibration-based damage identification process, many issues have to be taken into account which are on the periphery of civil engineering interest but may considerably influence the results. These issues mainly relate to the design and the execution of the vibration test (excitation, measuring equipment, type-selection and positioning of sensors) as well as the evaluation of the recorded data (signal processing, Fourier analysis). Among them, only the excitation considerations will be treated in the following (Kovács, 1998a).

How to force the structure into vibration with some kind of excitation effect is a critical issue to consider before conducting vibration tests. The main characteristics of an excitation effect are the wideness of the frequency range \( f \) and the magnitude of the excitation effect \( F \) over the applied frequency range.

Two types of excitation effects have to be distinguished. They are caused by either mechanical or ambient effects so the vibration tests are accordingly called “forced” or “ambient” vibration test. The substantial difference between them is that mechanical equipment is needed to induce “forced” vibration while no equipment is required to induce “ambient” vibration. Consequently, the characteristics of the excitation effect (input data) are generally known for mechanical excitation forces and unknown for ambient excitation effects.
\section{A2.1 Ambient excitation}

The ambient excitation effects are generally sourced from background noises, wind, rain, waves or traffic. Because of their stochastic character, a statistical analysis (at least averaging) has to be included in signal processing in order to obtain appropriately smooth resulting spectra. The applicability of these ambient vibration tests is limited when the frequency range of the excitation effect is not wide enough to cover all the natural frequencies of the structure which are intended to be measured.

The excitation effect called “white noise” has only theoretical importance (Fig. A2.1). In this case, the magnitude of the excitation force is constant over the full, infinitely wide frequency range. The simulation of white noise in practice is difficult.

\begin{figure}
\centering
\begin{subfigure}[b]{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{white_noise.png}
\caption{white noise}
\end{subfigure} \hspace{1cm}
\begin{subfigure}[b]{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{wind.png}
\caption{wind}
\end{subfigure}
\caption{Frequency range ($f$) versus excitation force ($F$) for different types of excitation}
\end{figure}

In some cases, the background noises generally caused by operating machines indoor conditions or especially the excitation effect of wind in outdoor conditions (Fig. A2.1b) may approximate the white noise effect by producing approximately constant excitation force ($F$) over a limited frequency range ($f_w$). Under certain wind conditions, the magnitude of this effect may be sufficiently high to force slender CE structures (e.g. cable stayed bridges, offshore wind turbines) into vibration with significant amplitudes. (This wind-induced excitation effect led to the disaster of the Tacoma Narrows Bridge in 1940.) The use of this type of excitation is obvious for structures built against wind (e.g. wind barriers on bridges, \textit{Strukelj et al., 2005}), however, many vibration tests have also been performed on bridges, buildings and various CE structures (dams, tunnels) by using wind-induced excitation (\textit{Wenzel et al., 2003}).

Rain- or wave-induced vibration is relevant only for specific structures. Significant rain-induced vibration may occur for very slender structural elements such as cables of cable stayed bridges. Wave-induced vibration tests have been carried out on offshore oil platforms (\textit{Coppolino and Rubin, 1980}) and wind turbines many times.

For highway bridges, normal road traffic may be used as an excitation effect. The wide frequency range of this effect comes from the non-uniform running properties of differ-
ent vehicles crossing the bridge. By the use of a statistical analysis during signal processing, the relative weight of the natural frequencies can significantly be amplified in the averaged spectrums (Mazurek and DeWolf, 1990, Raghavendrachar and Aktan, 1992, Kálló, 1997, Kovács et al., 1998b). During the in-situ measurements, no restriction or interruption of traffic is generally needed, which makes these tests cost-effective. Therefore, these tests are often considered as parts of long-term monitoring programs. A considerable number of papers deal with this topic in literature.

A2.2 Excitation by mechanical forces

Excitation characteristics similar to that for wind can be achieved by using impact hammers with changeable heads (e.g. PCB hammer). The excitation effect is induced by an impact with this hammer. Depending on the applied head, the frequency range of the excitation force (an example is shown in Fig. A2.2a) can coincide with the frequency domain which includes all natural frequencies of the structure and are to be measured. Therefore, due to the previously known and adjustable characteristics of the input excitation data, these hammers are the most sufficiently and most frequently applied excitation devices for controlled vibration tests in both field and laboratory conditions (Raghavendrachar and Aktan, 1992, Mazurek and DeWolf, 1990).

In comparison to impact hammers, less than perfect excitation effects arise from simple mechanical impulses. Theoretically, the magnitude of the amplitude ($A$) of the so called “Dirac-$\delta$” effect is infinite ($A=\infty$); furthermore the duration of the amplitude-changes for both the Dirac-$\delta$ and the ideal diminishing impulse equals zero ($\Delta t=0$) as shown in Fig. A2.2b and Fig. A2.2c. The shape of the frequency spectrum corresponds to the $\sin(x)/x$ function for the Dirac-$\delta$ effect and to the $1/x$ function for the ideal diminishing impulse, therefore, the magnitude of the excitation force over the frequency scale is proportional to the $\sin(x)/x$ function for the Dirac-$\delta$ effect and to the $1/x$ function for the ideal diminishing impulse. Consequently, the intensity of the excitation force is much higher in the lower frequency domains than in the higher domains. For real structures and especially for bridges, these effects are simulated and approximated by simple mechanical impacts with $A\neq\infty$ and $\Delta t\neq0$ represented by objects falling on or off the structure to be excited, respectively (Illéssy, 1982).
Sometimes exciters inducing periodic excitation force (e.g. machines with rotating eccentric mass) are used for both laboratory and field tests. In these cases the revolution of the exciter can be changed and, consequently, can coincide with one of the natural frequencies of the structure in order to cause resonance effects in the structural response (Illéssy, 1982, Kovács and Farkas, 2000).

A3 LITERATURE REVIEW OF VIBRATION-BASED DAMAGE IDENTIFICATION METHODS

A3.1 Damage identification based on changes in the basic modal parameters

A3.1.1 Methods based on changes in the natural frequencies

A3.1.1.1 Model-based methods

*The forward problem*

In the 1970s and the early 1980s many authors analysed offshore structures (oil platforms and light stations) by numerical analysis. They modelled local damage by partly
or fully cutting structural elements. Depending on the modelled damage location, 1-2% shifts in the natural frequencies were found to be indicative of damage (Coppolino and Rubin, 1980). Cracks had little influence on global stiffness (Begg et al., 1976). Many in-situ tests resulted in the conclusion that it was important to analyse natural frequency shifts parallel to the associated mode shapes when trying to locate damage (Kenley and Dodds, 1980).

Cawley and Adams (1979) detected single damage from frequency shifts on aluminium and carbon-fibre-reinforced-plastic plates. They defined an error term that related to the ratio between frequency shifts belonging to a pair of modes measured on the real structure and calculated from a numerical model. The potential single damage was modelled by local stiffness reductions; no possible multiple damage was considered. The damage location was indicated when the considered frequency pairs minimized the error form. Many mode pairs were taken into account. This sensitivity-based damage localization procedure needed considerable computing time and capacity to produce ratios of frequency shifts for all possible damage locations. The accuracy of the method depended on the quality of the numerical model. Similar work has been performed by Penny, et al. (1993).

Ismail et al. (1990) pointed out by experiments that a crack in a concrete beam, which periodically opens and closes, causes a smaller drop in the natural frequencies than a continuously open crack. It concludes that the ratio between permanent and variable loads also influences the magnitude of the crack-induced frequency shifts.

Brincker et al. (1995a) used statistical analysis to detect damage in concrete beams with different steel ratios by ordering so-called “significance” indicators to every modal frequency, as follows:

\[
(S_i)_i = \frac{f_i^u - f_i^d}{\sqrt{(\sigma_{f_i}^u)^2 + (\sigma_{f_i}^d)^2}} \tag{A3.1}
\]

where \( (\sigma_{f_i}) \) is the experimentally-determined standard deviation (SD) of the \( i \)-th natural frequency. The \( u \) and \( d \) indices refer to the undamaged and the damaged states, respectively. This indicator significantly increases for natural frequencies measured with low SD that indicates the presence of a possible damage compared to the undamaged state. A similar significance indicator was defined for the measured damping ratio. The author also defined a “unified significance indicator” by summing the above frequency and damping significance indicators over several measured modes. It was pointed out that the significance indicators were sensitive to damage but were not capable of locating them. This technique was insensitive to the type of excitation (input signals).
Choy et al. (1995) developed an iterative procedure to detect and locate damage in simply supported beams and for beams on an elastic foundation. Damage was modelled by reducing the Young’s modulus of one element of the beam. They adjusted the Young’s modulus of the considered element until the calculated natural frequency matched the corresponding measured one. This procedure was repeated for each element and for many natural frequencies. The location of the damage was obtained from the intersection of the Young’s modulus reduction versus element location curves plotted for each natural frequency (Fig. A3.1). This procedure has been extended for localizing multiple damage.

![Fig. A3.1](image1)

Fig. A3.1 Results of the damage locating iterative procedure developed by Choy et al. (1995)

However, without any information on the mode shapes associated with the measured natural frequencies, this procedure is not able to distinguish between damage situated in symmetric locations in symmetric structures as also shown in Fig. A3.1.

The inverse problem

Armon et al. (1994) applied a method to detect and localize cracks in beams using a rank-ordering of modes according to the changes in the associated natural frequencies. The method is based on the recognition that a forming crack often modifies the sequence of the fractional natural frequency shifts of a structure. A computer-simulated result using the first four modes is shown in Fig. A3.2 for a cantilever beam.

![Fig. A3.2](image2)

Fig. A3.2 Fractional changes in natural frequencies versus crack location (Armon et al., 1994)
The authors declared that although rank-ordering was not sensitive to the crack size (consequently it could not be used for damage quantification) it was also insensitive to small uncertainties in boundary conditions and temperature changes as well as required moderate measurement accuracy. The method was experimentally studied on cantilever Perspex and aluminium beams.

Stubbs and Osegueda (1990a, 1990b) used sensitivity analyses for beams to point out how local damage, which was defined as reductions in the stiffness of one of the elements forming the structure, influences the natural frequencies. The damage localizing procedure is built on the definition and the use of sensitivity matrices, which describe the sensitivity of the frequency changes to changes in the element stiffness (and mass). This method has been improved by including the effects of changes in damping. The matrices were calculated by FEM, which demanded a considerable amount of computation and the use of an accurate, previously-built numerical model.

Based on the work by Cawley and Adams (1979), Stubbs and Osegueda (1990a, 1990b) developed another damage detection method by including the sensitivity of natural frequency change to changes in stiffness of local elements into the error term. This improved method proved to be more accurate compared to their previous one when the number of elements were much higher than the number of measured natural frequencies.

Another damage localizing method has been developed by Hearn and Testa (1991) and applied to a four-member welded steel frame subjected to cyclic loading and wire ropes under constant tension. In both cases, concentrated, significant damage cases were produced (fatigue crack and saw cut) in the verification tests. This method, again similarly to the work by Cawley and Adams (1979), focused on the ratios of the natural frequency shifts. They started from the eigenvalue problem of free vibration

\[(K - \omega^2 M)\phi = 0\] (A3.2)

where \(K\) and \(M\) are the global stiffness and mass matrices, respectively; \(\omega^2\) is the diagonal matrix containing the natural frequency squares and \(\phi\) is the normalized mode shape vector. Assuming that damage does not change the mass properties of the structure and neglecting the second-order terms, the perturbation of Eq. (A3.2) results in a relationship between the change in the \(i\)-th natural frequency and the damage-induced change in the global stiffness as follows:

\[
\Delta\omega_i^2 = \frac{\phi_i^T \Delta K \phi_i}{\phi_i^T \phi_i} \quad (A3.3)
\]
where $\Delta K$ is the change in the global stiffness matrix due to damage. If decomposing $K$ into individual element stiffness matrices $k_N$ and if the element deformations $\varepsilon_N(\phi_i)$ are computed from the mode shapes of the undamaged structure, then Eq. (A3.3) can be written as

$$
\Delta \omega_i^2 = \frac{\sum_{N}^{N} \varepsilon_N(\phi_i)^T \Delta k_N \varepsilon_N(\phi_i)}{\phi_i^T M \phi_i}
$$

(A3.4)

where $N$ denotes the location of the individual element in the whole structure. Furthermore, assuming that only one (the $N^{th}$) element is affected by damage (local damage) and that this damage is limited to one (the dominant) component of the element stiffness matrix, the change in the stiffness matrix of the $N$-th element can be represented by a scalar damage function, as follows:

$$
\Delta k_N = \alpha_N k_N
$$

(A3.5)

where $\alpha_N$ is a scalar and represents the fractional change of the components in $k_N$. Mathematically it means that all elements in $k_N$ are decreased ($\alpha_N < 1$) but it is assumed that the influence of the $N$-th element on the change in natural frequency is primarily the function of only one of the components of $k_N$ while the contribution of other components of $k_N$ to this function is negligible. Therefore, the application of the above scalar damage function is a reality. Using Eq. (A3.5), Eq. (A3.4) becomes simpler as

$$
\Delta \omega_i^2 = \frac{\varepsilon_N(\phi_i)^T k_N \varepsilon_N(\phi_i)}{\phi_i^T M \phi_i}
$$

(A3.6)

If considering the ratios of the $i$-th and $j$-th natural frequency shifts, $\alpha_N$ can be eliminated as follows:

$$
\frac{\Delta \omega_i^2}{\Delta \omega_j^2} = \frac{\varepsilon_N(\phi_i)^T k_N \varepsilon_N(\phi_i)}{\varepsilon_N(\phi_j)^T k_N \varepsilon_N(\phi_j)}
$$

(A3.7)

Eq. (A3.7) shows the characteristic influence of each element on the changes in natural frequencies that can be calculated from the modal properties of the undamaged structure. However, in calculating the frequency shifts with Eq. (A3.4), the procedure does not take into account the changes in the element deformations $\varepsilon_N(\phi)$ due to damage. For field vibration tests, damage localization is based on the comparison of the above ratios of frequency shifts to the corresponding ratios obtained from the measured frequency shifts. To make this process automatic, the authors defined an error term, whose minimum value indicated the location of the damage.
Richardson and Mannan (1992) tracked damage by calculating changes in the global stiffness matrix. They determined a sensitivity equation for each mode based on the orthogonality properties of the damaged and the undamaged structure. Assuming again that mode shapes did not change due to damage, the difference between natural frequency squares of the damaged \((\omega_{i,d}^2)\) and the undamaged \((\omega_{i,u}^2)\) structure for the \(i\)-th mode shape resulted in

\[
\phi_i^T \Delta K \phi_i = \omega_{i,d}^2 - \omega_{i,u}^2 \tag{A3.8}
\]

where notations are the same as for Eq. (A3.3). To calculate \(\Delta K\) in Eq. (A3.8), the mode shape and the natural frequency of the undamaged structure and the natural frequency of the damaged structure are required. However, these equations often become undetermined.

Brincker et al. (1995b) applied a so-called “auto-regressive moving average” (ARMA) procedure to the measured acceleration-time histories of reinforced concrete offshore platforms in order to estimate modal properties (frequencies and damping). The \([n,m]\) order ARMA model expresses the present structural response as a linear combination of \(n\) past responses as well as the present and \(m\) past inputs as follows:

\[
y(t) = \sum_{i=1}^{n} c(i) y(t-i) - \sum_{i=1}^{m} d(i) e(t-i) + e(t) \tag{A3.9}
\]

where \(y(t)\) is the acceleration-time response at the present time \(t\), \(y(t-i)\) are the responses at previous times, \(c(i)\) and \(d(i)\) are the auto-regressive (AR) and the moving average (MA) parameters, respectively, \(e(t)\) and \(e(t-i)\) are the present and the past inputs assumed to be white noise. The AR and MA parameters are iteratively obtained by minimizing an error term based on the measured and the predicted (by Eq. (A3.9)) responses. The roots of the polynomial can be related to the modal properties (frequencies and damping).

Koh et al. (1995) presented a recursive method for damage localization in multi-storey steel frame buildings using frequency measurements. The degrees of freedom of the structure were reduced based on static considerations. Damage was modelled by local stiffness reduction in particular columns.

Narkis (1994) presented analytical, closed formulas for determining the crack location \((e)\) in beams based on the measured shifts of any two natural frequencies \((\Delta f_i\) and \(\Delta f_j)\). The parameter of these formulae is

\[
R_y = \frac{\Delta f_i / f_i}{\Delta f_j / f_j} \tag{A3.10}
\]
where both the $i$-th and the $j$-th indices relate to either axial or flexural modes depending on which type of vibration is considered. For simply supported beams under flexure, this formula is as follows:

$$e = \frac{1}{\pi} \arccos \left( 1 - \frac{R_i}{2} \right)$$  \hspace{1cm} (A3.11)

For the same beam under axial vibration (no restraint of the axial force at one end and axially fixed other end):

$$e = \frac{2}{\pi} \arcsin \left( \sqrt{R_j} - 1 \right)$$  \hspace{1cm} (A3.12)

Similar equations can be defined for a combination of any two modes.

Morassi (2001) improved Narkis’ (1994) analytical results by using the knowledge that for beams, the change in a natural frequency due to a single crack may be represented as the product of two terms, of which the first is proportional to the severity ($K$) and the second depends solely on the location ($S$) of the damage. Thus, this method was intended for both damage localization and quantification of its severity. The modelled crack was simulated by a linear spring with stiffness $K$ connecting to the two segments of the beam. He found that for beams with free ends under axial vibration, the ratio between the changes in the $2m$-th and the $m$-th natural frequencies explicitly determined the position variable ($S$) (for $m=1$, this formula matched with the corresponding result achieved by Narkis (1994)) and the changes in the $2m$-th and the $m$-th natural frequencies allowed to explicitly determine the spring stiffness $K$. The theoretical results were confirmed by the results coming from axial vibration tests of suspended steel beams with free-free ends. The beams were damaged by different notches. The author noted that some of the results were also valid for beams under flexure.

Messina et al. (1998) localized damage in structures by defining Damage Location Assurance Criterion (DLAC) for single damage cases and Multiple Damage Location Assurance Criterion (MDLAC) for multiple damage cases, as follows:

$$DLAC(j) = \frac{\left( \Delta f^T \delta f_j \right)^2}{\left( \Delta f^T \Delta f \right)^2}$$  \hspace{1cm} (A3.13a)

$$MDLAC(\delta D) = \frac{\left( \Delta f^T \delta f(\delta D) \right)^2}{\left( \Delta f^T \Delta f \right)^2}$$  \hspace{1cm} (A3.13b)

where $\Delta f$ is the measured natural frequency change vector of the structure with a single damage of unknown size and $\delta f_j$ is the calculated natural frequency change vector for a damage of known size at location $j$. For the multiple damage case, $\delta D$ is the change of
the global stiffness matrix containing the individual stiffness reductions of one or more elements in any locations and $\delta f(\delta D)$ contains the calculated natural frequency changes due to $\delta D$. This part of their work, which integrated the measured and the (analytically or numerically) calculated changes in the natural frequencies into one indicator value, was only an improvement of the original results achieved by Cawley and Adams (1979). The DLAC and MDLAC values are very similar to the Modal Assurance Criterion (MAC) value used for comparing mode shapes (see Sect. A3.1.2.1). However, the authors developed this method and defined coefficients for estimating the size of the damage both for single and multiple damage cases. The reliability of the MDLAC was confirmed by analytical examples as well as by experiments carried out on an aluminium frame model.

A3.1.1.2 Non-model-based methods

Kato and Shimada (1986) conducted ambient vibration tests on a statically preloaded, full scale prestressed concrete bridge. The structure was subjected to gradually increasing, controlled static loads in four loading steps up to failure. They found small change in the vibration characteristics while the prestressing tendons were in the elastic stage even when cracking occurred. This was explained by the cracks closing together by the effective prestressing after the load was removed. After the tendons exceeded the elastic limit a sudden decrease in the natural frequencies was observed, which continued as the failure progressed. It was concluded that this observation offered the possibility of discovering tendon weaknesses in prestressed concrete bridges. Salawu (1997b) defined a Global Integrity Index (GI) for detecting changes in the structural integrity (mass or stiffness) of CE structures as follows:

$$GI = \sum r \left[ a_r \left( \frac{\omega_{Dr}}{\omega_{0r}} \right) \right]$$  \hspace{1cm} (A3.14)

where $\omega_{Dr}$ and $\omega_{0r}$ are the natural frequencies associated with the mode $r$ and referred to the damaged and the undamaged structure, respectively. Values of $GI$ other than 1.0 indicated a change in the structural integrity. The $a_r$ is a weighting factor of the $r$-th mode that depended on the dominance of the considered mode in the dynamic response of the structure. It was suggested that at least (the most dominant) three modes should have been considered with the corresponding $a_r$ values equal to 0.7, 0.2 and 0.1 in the sequence of their dominance. The most dominant modes were selected by using the Modal Sensitivity Value (MSV) defined as:
\[
MSV(\phi_{0i}, \phi_{Di}) = \sqrt{\frac{\sum (\phi_{0i})^2}{\sum (\phi_{Di})^2} - \sum \frac{\omega_i^2}{\sum \omega_i^2}}
\]

where \( \phi_{0i} \) and \( \phi_{Di} \) are the elements of the \( r \)-th mode shape vector at measurement point \( i \) belonging to the undamaged and the damaged state, respectively. The highest MSV indicates the most dominant mode. For selecting dominant modes, the use of the MSV is alternative to the use of the Modal Assurance Criterion (MAC, see Sect. A3.1.2.1). Because only natural frequencies are included in Eq. (A3.14), theoretically, one measurement point is sufficient. The author completed this method by defining a Local Integrity Index (LI) for each measurement point to locate and quantify local damage (see Sect. A3.1.2.1).

A3.1.2 Methods based on changes in the mode shapes and in their derivatives

A3.1.2.1 Methods using displacement mode shapes

Model-based methods

Yuen (1985) defined mode shape and mode-shape-slope parameters by combining the mode shape amplitudes and the associated natural frequencies of the damaged and the undamaged structure, and examined their changes due to numerically simulated stiffness reductions (as models of localized damage) in each structural element for a cantilever beam. Then these predicted changes were compared to the corresponding measured changes to determine the damage location in the investigated structure. This method can be considered as a classical solution for the forward problem (Sect. A3.1.1.1) using mode shape data.

An example for the solution of the inverse problem was given by Rizos et al. (1990) who presented an analytical method, which was used to predict the position and the depth of a single, open crack along a cantilever steel beam. The authors established a non-linear system of equations for the harmonically vibrating beam divided into two parts by a bending spring, which represented an open crack, the spring constant of which was calculated on the basis of the crack-strain-energy function. Having the experimentally measured amplitude-time function of the beam forced into harmonic vibration at one of the natural frequencies, the analytical solution resulted in the crack parameters (location and depth) as unknowns. The mode shapes were recorded at only two locations. The differences between the computed and the precisely checked crack parameters remained below 8%. The authors noted that the analytical model can be
substituted by an appropriate finite element model. It was also concluded that the method lacked accuracy for small cracks (where the crack depth was smaller than 1/10-th of the cross-section depth) and for more complex structures (with multiple cracks). Therefore, for real structures this method was intended only to locate and quantify single cracks.

**Non-model-based methods**

MAC indicates correlation between two modes including mode shape amplitudes gained from each measurement point:

$$MAC\left(\phi_{A_{i,q}}, \phi_{B_{i,r}}\right) = \frac{\sum_{i=1}^{n} (\phi_{A_{i,q}} \phi_{B_{i,r}})^2}{\sum_{i=1}^{n} (\phi_{A_{i,q}})^2 \sum_{i=1}^{n} (\phi_{B_{i,r}})^2}$$  \hspace{1cm} (A3.15)

where $\phi_{A_{i,q}}$ ($\phi_{B_{i,r}}$) are the element of the $A$-th ($B$-th) set of mode shape vectors for the $q$-th ($r$-th) mode belonging to the $i$-th measurement point and $n$ is the total number of measurement points. If MAC is used as a damage indicator, the considered mode shapes in Eq. (A3.15) belong to the same mode measured before and after the assumed damage event. The MAC value can vary between 1.0 and 0.0, where the 1.0 value indicates full correlation and the 0.0 value indicates no correlation between the considered mode shapes. Because mode shape amplitudes gained from all measurement points are included into Eq. (A3.15), MAC can be considered as a global damage indicator. Therefore, often MAC proves to be insensitive to local damage.

COMAC contains local information on the vicinity of the considered measurement location because it includes two sets of amplitudes of all recorded mode shapes at the $i$-th measurement point as follows:

$$COMAC(A,B,i) = \frac{\left(\sum_{r=1}^{m} (\phi_{A_{i,r}} \phi_{B_{i,r}})\right)^2}{\sum_{r=1}^{m} (\phi_{A_{i,r}})^2 \sum_{r=1}^{m} (\phi_{B_{i,r}})^2}$$  \hspace{1cm} (A3.16)

where $\phi_{A_{i,r}}$ ($\phi_{B_{i,r}}$) is the element of the $A$-th ($B$-th) set of mode shape vectors for the $r$-th mode belonging to the $i$-th measurement point and $m$ is the number of the recorded mode shapes. If COMAC is used as a damage indicator, the considered sets of mode shape amplitudes in Eq. (A3.16) are measured before and after the assumed damage event. The COMAC value can also vary between 1.0 and 0.0, where the 1.0 value indicates full correlation and the 0.0 value indicates no correlation between the considered sets of mode shape amplitudes.
Allemang (2003) gave a thorough overview on the theoretical background and the experience of application of MAC together with many other related assurance criteria used in the field of experimental modal analysis. Salawu and Williams (1995) used MAC and COMAC to compare the mode shapes of a multispans reinforced concrete highway bridge before and after a local structural repair. Artificial excitation of the bridge was achieved by a special excitation system giving a periodic random signal. The authors concluded that for an appropriate measurement point arrangement MAC can be used to indicate the existence of any change in the mode shapes (Level 1) as well as to select mode shapes which are mostly influenced by this change. However, due to their local character, COMAC values give better information on the location of this change (Level 2) than MAC values, therefore COMAC values can effectively be used for damage detection and localization as well. The authors also suggested that the quantification (Level 3) of the change detected and localized by the use of MAC and COMAC should have been examined by other routine, local non-destructive techniques.

A similar damage localization approach for CE structures was introduced by Salawu (1997b) as an improvement of the Global Integrity Index (GI) method (see Sect. A3.1.1.2). The author noted that due to its global nature, GI (similarly to MAC) was not sensitive enough to local defects, therefore the Local Integrity Index (LI) was defined at the \( i \)-th measurement point and included mode shape data into the LI index as follows:

\[
LI_i = \sum_r a_r \left( \frac{\omega^{Dr}_{ri}}{\omega^{Or}_{ri}} \left( \frac{\phi^{Dr}_{ri}}{\phi^{Or}_{ri}} \right)^2 \right)
\]

where \( \phi^{Dr}_{ri} (\phi^{Or}_{ri}) \) is the element of the \( r \)-th mode shape vector belonging to the \( i \)-th measurement point for the damaged and the undamaged structure, respectively. Existence of any damage (loss of integrity) at point \( i \) is likely if \(|LI_i - 1| > 0\) and unlikely if \(|LI_i - 1| = 0\). For practical applications, values of \(|LI_i - 1|\) for points most likely to be damage locations are much larger than those for points in undamaged areas. To define mode shapes and to locate the defective areas with the \( LI_i \) index, a sufficient number of measurement points is required. For practical applications, authors suggested to calculate GI first. If damage (loss of integrity) is detected by the GI then \( LI_i \) should be calculated at a few widely spaced points. The GI and \( LI_i \) indices can be calculated very simply without extensive computation effort. The GI and \( LI_i \) indices were verified by a numerical example and an experimental test conducted on a steel beam deteriorated by saw cuts.
Dong et al. (1994) studied the applicability of mode shapes based on direct strain measurements (strain mode shapes) according to Eq. (A3.18), instead of curvature mode shapes (see Sect. A3.1.2.2) calculated from displacement mode shapes, by defining a new parameter formulated as:

\[
\{ \Delta \phi \} _i = \{ \phi \} _i^d \left( \frac{\omega _i^u}{\omega _i^d} \right)^2 - \{ \phi \} _i
\]

where \( \{ \phi \} _i^d \) and \( \{ \phi \} _i^u \) are the amplitudes of the \( i \)-th strain mode shape for the undamaged and the damaged beam, respectively, while \( \omega _i^u \) and \( \omega _i^d \) are the associated natural frequencies. The authors proved that, thanks to the combination of the mode shape changes with the associated natural frequency shifts, this parameter was more sensitive to damage than the equivalent indices computed only from displacement mode shape data.

Topole and Stubbs (1995) introduced a damage prediction method based on matrix structural analysis using limited modal information on the damaged structure. Instead of previous knowledge of modal information of an undamaged structure, only estimates of masses and stiffnesses based on geometrical data of the initial structure were required. They found, and numerically verified on a 10-storey model building, that structural damage, assumed to be a local reduction in stiffness, could be correctly detected, located and quantified from available measured information of approximately \( \sqrt{NE} \) modes, where \( NE \) denotes the number of structural elements. They suggested that the experimentally measured modal data could be more efficiently used for this case than in methods using MAC according to Eq. (A3.15).

A3.1.2.2 Methods based on mode shape curvature

Pandey et al. (1991) first computed the displacement mode shapes of a cantilever and a simply supported beam numerically by finite element analysis and then compiled the corresponding curvature mode shapes by calculating the curvature at each element \( (\rho _{q,i}) \) by using the central difference approximation as follows:

\[
\rho _{q,i} = \phi _{q,i}'' = \frac{\phi _{q-1,i} - 2\phi _{q,i} + \phi _{q+1,i}}{h_q^2}
\]

where \( \phi _{q-1,i}, \phi _{q,i} \) and \( \phi _{q+1,i} \) are the corresponding amplitudes of the \( i \)-th displacement mode shape in the vicinity of the \( q \)-th element and \( h_q \) is the length of this \( q \)-th element. Damage was numerically modelled by reducing the Young’s modulus \( (E) \) in the corresponding element(s) of the beam and then identified and quantified by the comparison between the curvature mode shapes for the undamaged and the damaged structure.
They found that curvature mode shapes are much more sensitive to local damage than either displacement mode shapes (Fig. A3.3) or even the MAC and the COMAC values.

A very similar work has been conducted by Ratcliffe (1997) for beams with discrete mode shapes (recorded at discrete points). He showed that the Laplacian operator itself formulated similarly to Eq. (A3.19) as

$$\mathcal{S}_i = (\phi_{i+1} + \phi_{i-1}) - 2\phi_i$$  \hspace{1cm} (A3.20)

and, when applied to a discrete displacement mode shape, was able to successfully indicate the presence and even the location of damage (modelled by stiffness reduction at one of the discrete points) for severe damage cases without the knowledge of the mode shape of the undamaged structure. For less severe damage cases, difference functions, defined as the difference between the Laplacian and a cubic polynomial fitted to the Laplacian locally around each discrete point, provided the necessary information on the damage location. The method was demonstrated on a finite element beam model and was experimentally verified on a suspended steel beam. Because the Laplacian represented the local curvature of the displacement mode shape and the surface strain was proportional to the curvature according to Eq. (1.1), the author suggested replacing the Laplacian by the experimentally measured strain mode shapes for damage identification purposes on real structures.

Salawu and Williams (1994) demonstrated that when using experimental data the curvature change did not typically give a good damage indication. They emphasized the importance of selection of which mode shapes are to be used in the analysis.
Appendix

A3.1.2.3 Methods based on strain energy

Stubbs et al. (1992) defined damage indices ($\beta_p$) for each element of a beam under flexure on the basis of change in the fractional strain energy along the element length. The strain energy was not directly calculated but the curvature of the measured mode shape was used for the definition of the damage index. This $\beta_p$ for the $p$-th element can be written as:

$$\beta_p = \left( \frac{\sum_{i=1}^{m} \mu^u_{ip}}{\sum_{i=1}^{m} \mu^d_{ip}} \right)$$  \hspace{1cm} (A3.21)

where

$$\mu^u_{ip} = \left( \frac{\int_a^L \left[ \phi^u_i(x) \right]^2 \text{dx}}{\int_0^L \left[ \phi^u_i(x) \right]^2 \text{dx}} \right)$$ \hspace{1cm} (A3.22a)

$$\mu^d_{ip} = \left( \frac{\int_a^L \left[ \phi^d_i(x) \right]^2 \text{dx}}{\int_0^L \left[ \phi^d_i(x) \right]^2 \text{dx}} \right)$$ \hspace{1cm} (A3.22b)

Here $\{\phi^u_i(x)\}_i$ and $\{\phi^d_i(x)\}_i$ are the $i$-th displacement mode shape functions for the undamaged and the damaged beam, respectively, $a$ and $b$ are the endpoints of the $p$-th element and $m$ is the total number of elements. Beam elements with the highest magnitudes of $\beta_p$ are indicated as damaged. This “damage index method” can be considered as an improvement on the former inverse model-based method presented by Stubbs and Osegueda (1990a, 1990b) (see Sect. 1.4.1.1.1) but it does not require previously computed sensitivity matrices as a baseline.

Shi, et al (1998) further improved the above method and introduced it in a more general form by defining the Modal Strain Energy (MSE) of an element in a structure as follows:

$$MSE_{ij} = \Phi_i^T K_j \Phi_i$$ \hspace{1cm} (A3.23a)

where $\Phi_i$ was the $i$-th mode shape vector and $K_j$ was the $j$-th elemental stiffness matrix. The proposed damage index for damage localization is called the Modal Strain Energy Change Ratio (MSECR) and is based on the change in the MSE values between the damaged ($d$ upper index) and the undamaged ($u$ upper index) states of the structure:

$$MSECR_{ij} = \frac{MSE^u_{ij} - MSE^d_{ij}}{MSE^d_{ij}}$$ \hspace{1cm} (A3.23b)

If several modes are considered, the average of the summation of $MSECR_{ij}$ values normalized with respect to the largest $MSECR_{i,\text{max}}$ of each mode becomes more usable damage localization indicator:

$$MSECR_j = \frac{1}{m} \sum_{i=1}^{m} \frac{MSECR_{ij}}{MSECR_{i,\text{max}}}$$ \hspace{1cm} (A3.23c)
where \( m \) denotes the considered number of modes. For damage quantification purposes in practical applications Shi, et al (2000) extended their method by a MSE sensitivity analysis. This approach required the elemental stiffness matrices, the analytical (or numerically calculated) mode shapes and the incomplete measured mode shapes as input data. First the incomplete measured mode shapes are expanded by the use of the analytical mode shapes, then the damage coefficient \( \alpha_j \), which measures the fractional change in the \( j \)-th elemental stiffness matrix, is calculated by an iterative process. The applicability of MSECR values for damage localization and that of the latter sensitivity analysis for damage quantification were demonstrated on numerical examples (cantilever and fixed beams) and on an experimental two-storey steel frame.

**A3.1.3 Methods based on changes in damping parameters**

Tests carried out on an ordinary steel-concrete composite bridge by Farrar and Jauregui (1998), on concrete beams with artificially made crack-modelling cuts by Casas and Aparicio (1994) and on a continuous, reinforced concrete, voided slab bridge before and after a structural repair by Salawu and Williams (1995) indicated no clear relationship between local damage growth and damping changes. It seemed for each case that damage influenced the damping ratios associated with different modes in different ways. Salane and Baldwin (1990) found from the results of a fatigue test conducted on a three-span, steel-concrete composite bridge that the damping ratio initially increased then subsequently decreased with increasing fatigue damage for all of the first three investigated modes. However, when dynamically testing a similar, one-span composite laboratory bridge model and cutting the bottom flange of one of the two girders in a section, the modal damping ratio decreased for the first three modes. Zonta et al. (2008) suggested that damping changes in concrete structures were closely associated with friction mechanisms (i) at the concrete interfaces during closing and opening of cracks and (ii) in the bond area between the concrete in tension and the tension reinforcement. Consequently, no significant damping changes should be expected for structures where no significant cracking occurs due to damage. This was observed on a five-storey experimental building designed to resist against earthquake according to the capacity-based design theory (appropriately distributed, dissipative plastic hinges with no friction damping effects and definitely linear post-earthquake behaviour outside the plastic hinges).

Curadelli et al. (2008) used the continuous wavelet transform to identify the instantaneous damping coefficient on one numerical (three-bay, six storey reinforced concrete frame excited by a known earthquake time history) and two experimental (a shock-
excited, simply-supported reinforced concrete beam and a one-bay, six-storey aluminium frame excited on a shaking table) structures before and after damage. They found that damping increased parallel to damage for each case and can be a useful damage index for structures showing growing non-linearity due to increasing damage.

A3.2 Damage identification methods based on dynamically measured flexibility

The flexibility-based methods can be classified on the basis of the procedure applied in indicating changes in the flexibility matrix of the structure.

A3.2.1 Differentiation of methods

The methods based on the simple comparison of changes in the flexibility matrix and its derivatives deal mainly with the non-model-based estimation of the flexibility coefficients from only on-site measured data. Raghavendrachar and Aktan (1992) computed 11 scaled mode shapes and then directly transformed them into flexibility without assuming a mass matrix after a multi-reference impact test on a three-span RC slab bridge. They also observed that torsional modes and local modes with frequencies comparable to those of global modes influenced the flexibility significantly. Therefore, flexibility proved to be a sensitive indicator of both global and local damage. The results have been confirmed by a FE model. Similar work has been performed by Pandey and Biswas (1994) and (1995) on the analytical and the experimental models of steel beams with different support conditions.

Later this method was improved by demonstrating on mainly bridge superstructures that virtually “loading” the flexibility matrix by different load patterns and calculating the “deflected” shapes by selecting and summing the corresponding elements of the flexibility matrix results in new, more powerful and engineer-like damage indices such as “uniform load surface” (Zhang and Aktan, 1995) and “bridge girder condition indicator” (Catbas and Aktan, 2002)).

The unity check method uses the inverse relationship between the flexibility \([G]\) and the stiffness \([K]\) matrices to define an error matrix \([E]\) as follows

\[
[E] = [G^2] [K^0] - [I]
\]  
(A3.24)

where the information on the damage is collected in \([G^2]\) and the \([K^0]\) is fixed to the undamaged (or a previous baseline) structure. The positions of the elements in \([E]\) that differ from the unity and the magnitude of these differences relate to the position and the degree of damage in the structure. This method was introduced by Lin (1994).
Appendix

The stiffness error matrix method is an improvement of the unity check method. Here the effect of measured flexibility changes between the damaged and the undamaged (or the previous baseline) structures are transformed into stiffness changes as follows

\[ [E] = [K'] ([G^d] - [G^u]) [K'] \] (A3.25)

which is measured by the computed stiffness error matrix \([E]\).

A3.3 Matrix update methods

A3.3.1. Mathematical background

The governing rule is the minimization of the “modal force error” \((\{E\}_i)\), which comes from the structural equation of motion given as:

\[ [M] \ddot{\{x\}} + [C']\dot{\{x\}} + [K']\{x\} = \{f(t)\} \] (A3.26)

and its eigenvalue equation for free vibration given as:

\[ ((\lambda^d_i)^2[M] + (\lambda^d_i)[C'] +[K']) \{\phi^d_i\} = \{0\}, \] (A3.27)

where \(\lambda^u_i\) and \(\{\phi^u_i\}\) are the \(i\)-th natural frequency and the associated mode shape while \([M^d]\), \([C^d]\) and \([K^d]\) are the mass, the damping and the stiffness matrix (property matrices) of the undamaged structure, if the measured modal data of the damaged structure \((\lambda^d_i\) and \(\{\phi^d_i\}\)) are substituted into Eq. (A3.27) as follows:

\[ ((\lambda^d_i)^2[M^d] + (\lambda^d_i)[C^d] +[K^d]) \{\phi^d_i\} = \{E\}_i \] (A3.28)

In fact, this modal force error represents the harmonic force excitation of the undamaged structure, which would have to be applied with frequency \(\lambda^d_i\) to get \(\{\phi^d_i\}\) mode shape response. The iteration ends, and consequently the update is finished, if such \([M^d]\), \([C^d]\) and \([K^d]\) matrices are found, which fulfill Eq. (A3.27) written for the undamaged structure as follows:

\[ ((\lambda^d_i)^2[M^d] + (\lambda^d_i)[C^d] +[K^d]) \{\phi^d_i\} = \{0\} \] (A3.29)

The solution is based on the definition of the damaged model matrices as the undamaged model matrices minus a perturbation as follows:

\[ [M^d] = [M^u] - [\Delta M] \]
\[ [C^d] = [C^u] - [\Delta C] \] (A3.30)
\[ [K^d] = [K^u] - [\Delta K] \]

After substituting Eq. (A3.30) into Eq. (A3.29) and moving the perturbation terms to the left hand side of the equation, then the right hand side will be the same as the left hand side of Eq. (A3.28) and so the modal error force can be written as follows:

\[ ((\lambda^d_i)^2[\Delta M] + (\lambda^d_i)[\Delta C] +[\Delta K]) \{\phi^d_i\} = \{E\}_i \] (A3.31)
The developed methods generally apply constraints regarding the property matrices in order to simplify the solution of Eq. (A3.31). The most frequently applied constraints are the preservation of symmetry in the \([\Delta M]\), \([\Delta C]\) and \([\Delta K]\) matrices, the use of the same zero/non-zero pattern in the \([M']\), \([C']\) and \([K']\) matrices as in \([M]\), \([C]\) and \([K]\) as well as the preservation of matrix positivity for the \([\Delta M]\), \([\Delta C]\) and \([\Delta K]\) matrices (e.g. \(\{x\}^T[\Delta M]\{x\} \geq 0\)).

**A3.3.2. Differentiation of methods**

The elaborated matrix update methods differ in the mathematical procedures that are applied to solve Eq. (A3.31).

The **optimal matrix update methods** directly compute either the property matrices of the damaged structure or the perturbation matrices given in Eq. (A3.30) and give them in a closed form. The problem is generally formulated as a Lagrange multiplier or a penalty-based optimization. These methods are thoroughly reviewed by Zimmermann and Smith (1992).

The **sensitivity-based update methods** solve a first-order Taylor-series that minimizes a defined error function of the perturbation matrices (\([\Delta M]\), \([\Delta C]\) and \([\Delta K]\)). The essence of this theory is the determination of a modified parameter vector (consisting of material and/or geometrical parameters) such as

\[
\{p\}^{(n+1)} = \{p\}^{(n)} + \{\delta p\}^{(n+1)} \tag{A3.32}
\]

in which the parameter perturbation vector \(\{\delta p\}^{(n+1)}\) is computed from the Newton-Raphson iteration when minimizing an error form, which is typically selected as the modal force error according to Eq. (A3.28). These sensitivity-based techniques are summarized by Hemez and Farhat (1995).

The **eigenstructure assignment methods** are based on the determination of a fictitious term interpreted as a matrix perturbation to the undamaged model, which minimizes the modal force error. If this term (e.g. the perturbation of the \([K]\) stiffness matrix) is selected such that the modal force error between the FE models of the undamaged structure and the measured, damaged structure is equal to zero, then “best achievable mode shape vectors” (\(\{d\}i\)) can be determined as functions of the measured mode shape vectors (\(\{d\}i\)). The location and the severity of damage is concluded from the relationship between \(\{d\}i\) and \(\{d\}i\). These methods are overviewed by Lim (1995).

Due to the strong mathematical basis and the refined numerical techniques used, it is obvious that these methods require significant computation capacity. For this reason the **hybrid methods** were introduced by combining a computationally efficient optimal
matrix update method to locate the vicinity of the damage in the structure with a sensitivity-based method to fix the damaged point by updating only small number of parameters. Doebling, et al (1997a) experimentally analysed which modes of the damaged structure have to be used when updating the FE model if the computation capacity is limited. They found that modes storing the highest strain energy over the entire (damaged) structure are the most indicative of damage and that selection of modes based on maximum strain energy always provides more accurate model update than using modes with lowest modal frequency. These results came from the test of a steel truss.

### A3.4 Non-linear damage identification methods

Lin and Ewins (1990) localized non-linearity in structures by using model update techniques by neglecting the effect of structural damping. By conducting modal tests at two different response levels \(a\) and \(b\), Eq. (A3.29), with the inclusion of Eq. (A3.30), can be written as follows:

\[
((\lambda_i^a)^2[M^d + \Delta M] + [K^d + \Delta K]) \{\phi_a\}_i = \{0\} \tag{A3.33a}
\]

\[
((\lambda_i^b)^2[M^d + \Delta M] + [K^d + \Delta K + \Delta K_n]) \{\phi_b\}_i = \{0\} \tag{A3.33b}
\]

where \(\lambda_i^a\), \(\{\phi_a\}_i\), and \(\lambda_i^b\), \(\{\phi_b\}_i\) are the \(i\)-th natural frequency and the associated mode shape at response levels \(a\) and \(b\), respectively. The \([\Delta M]\) and \([\Delta K]\) matrix perturbations represent the FE model update and \([\Delta K_n]\) represents the non-linearity in stiffness (here the non-linearity in mass is neglected). If considering \(\{\phi_b\}_i\) as the perturbation of \(\{\phi_a\}_i\) and that the modelling errors \((\lambda_i)^2[\Delta M]\) and \([\Delta K]\) are of the same orders as \([\Delta K_n]\) then after multiplying Eq. (A3.33a) and Eq. (A3.33b) by \(\{\phi_b\}_i^T\) and \(\{\phi_a\}_i^T\), respectively, the following result is obtained:

\[
[\Delta K_n] \{\phi_b\}_i \langle \phi_a\}_i^T = [A] \tag{A3.34}
\]

where \([A]\) is a known matrix calculated from the property matrices of the undamaged model ([\(M\], [\(K\)]) and the modal data measured at two response levels on the damaged structure \((\lambda_i^a\), \(\{\phi_a\}_i\), and \(\lambda_i^b\), \(\phi_b\)_i\)). This means that the non-linear term \([\Delta K_n]\) may be determined from \([A]\) and the measured modal data.