A compliant and redundantly actuated 2-DOF 3RRR PKM: Less is more

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ABSTRACT

The development of a compliant (or flexure-based) manipulator with redundant actuation has been considered before, showing that the redundancy can be exploited to increase the support stiffness and reduce actuator loads. In this previous design the manipulator's workspace has been defined to encompass all kinematically accessible end effector positions. In this paper we reconsider the design philosophy. It appears that limiting the workspace ("less") ultimately results in a better performance in a larger area ("more") as before.

The dynamic performance of the manipulator is evaluated with a flexible multibody model. The links are assumed to be rigid. The SPACAR software is used as its flexible beam element can describe the non-linear behaviour of the flexure joints well at rather large deflections. This numerically efficient model is well-suited for design optimisation which aims at the largest workspace area while assuring a minimal parasitic natural frequency and limiting the local stresses throughout the full workspace. Furthermore, the simulations show that preloading of the flexures results in smaller required actuator torques.

An optimised design has been build. A control system has been synthesised that handles the actuator redundancy by minimising the 2-norm of the driving torques. It is demonstrated that the setup's behaviour is similar to the model and that in particular the preloading significantly lowers the required actuator torques.

Keywords: Flexure-based mechanisms, Redundantly actuated parallel kinematic manipulator (PKM), Non-linear beam elements, Experimental system identification, Actuator torque balancing

1 INTRODUCTION

In [1] a compliant and redundantly actuated 2-DOF 3RRR parallel kinematic manipulator (PKM) (Figure 1(a)) has been introduced as "best of both worlds" for precision applications. Being a compliant mechanism, or more precisely a flexure-based mechanism, deterministic behaviour can be realised because of the low level of friction, hysteresis and backlash [2, 3]. Being also a redundantly actuated PKM, it combines the advantages of PKM, i.e. the high stiffness, low inertia and large accelerations, with an improved handling of singularities and optimised actuator loading made possible by the redundancy [4, 5]. Simulations indeed demonstrated advantages of combining both concepts. The flexure hinges in compliant manipulators inherently show a reduced support stiffness for large joint angles. In a PKM with a redundant link this reduction can be limited. Furthermore, the redundant actuation offers a possibility to combine load balancing techniques with preloading of the compliant joints to reduce the actuator efforts needed to keep the end effector (EE) stationary at any position different from the equilibrium position [1].

A PKM with "classical" joints can be operated throughout the complete kinematically admissible range [6]. Mimicking this behaviour with flexure joints is a challenge as the required joint angles are quite large even for advanced joint concepts that emerged in recent years [7]. Hence

Figure 1. Designs of planar 2-DOF 3RRR PKM with compliant joints.

we investigate in this paper how a similar performance can be obtained throughout the same or larger workspace area as before (“more”) when the joint angles are limited (“less”). This can be accomplished by using longer links, which come with the drawback of an increased mass, but can still be beneficial if this is compensated by an increased support stiffness due to less required joint rotations. The optimisation of this trade-off is presented in this paper as well the experimental validation with an implementation of a control scheme.

2 DESIGN OPTIMISATION

In this section, the steps involved in the mechatronic design of the manipulator are addressed. The system to be manufactured should be a proof-of-concept that demonstrates its main characteristics. High accuracy and high performance are not the main targets in order to limit the costs. E.g. metal flexures are expected to show most ideal behaviour. However, 3D printing is used to realize the mechanical components. Some larger parts are printed in PLA with the FDM process which is relatively cheap. The flexure hinges require a higher accuracy for the thin leaf springs and are printed in Nylon with the SLS process. Rotational motors are used to actuate the system, although the bearings in these motors to some extent sacrifice the compliant behaviour of the manipulator.

2.1 Kinematic analysis and definitions of the workspace

The three arms of the 3RRR PKM are assumed to be similar and the actuators are located at the corners of an equilateral triangle, see Figure 1. At first a simplified kinematic model is used to determine the reachable workspace and the required joint rotations. The rigid links are connected with ideal rotational joints. Two important geometrical parameters are the total length L of each arm and the distance R of each actuator to the centre of the triangle. The workspace reachable by the EE is bounded by three circular arcs with radii L of which an example is shown in red and labelled “Defⁿ 1” in Figure 2. This is the workspace that has been considered in [1]. The worst case dynamic performance is found in the corners of this area where two arms are fully

Figure 2. Workspace definitions.

Figure 3. Rotation ranges of the joints.

stretched. In these locations the support stiffness will be lowest as several joints are at or close to their extreme rotation angles.

The blue curves, labelled “Defⁿ 2”, in Figure 2 present a first alternative workspace definition. Instead of trying to move to all reachable locations, the corners are cut off e.g. by limiting the workspace to the enclosed circle. It can be shown that using longer arms, i.e. larger L , and restricting the joint rotations, a larger workspace area can be reached with the same ratio between mass and support stiffness as before [8]. Although this is an improvement, it appeared that for controlled EE motion still difficulties arose from singular behaviour near the three locations on the enclosed circle where one of the arms is fully stretched.

Hence a third definition of the workspace is presented with the green curves, labelled “Defⁿ 3”, in Figure 2. This workspace is a circle which is some fixed offset radius R_o smaller than the maximum enclosed circle such that the (near) singularities are avoided. This results in even more reduced ranges of the rotations for the joints as presented in Figure 3. The joint angles in this figure are computed for a manipulator where the total arm length L is split into equal halves for the upper and lower arm respectively. The required rotational ranges are presented as functions of the so-called (linear) workspace ratio, which is defined as

$$r_{ws} = \sqrt{A_{ws}/A_{fp}} \quad (1)$$

where A_{ws} is the workspace area and A_{fp} is the triangular area of the manipulator’s footprint. It can be seen that the third definition of the workspace requires smaller joint angles to reach the same area. A drawback is that the arm length increases even more compared to “Defⁿ 2” which could result in less support stiffness as will be examined next in a dynamic analysis.

2.2 Parametric model

A more detailed flexible multibody model of the manipulator is used to obtain an optimal design of the system to be manufactured with 3D printing. First the joint concepts are detailed. Next relevant dimensional parameters are defined that are optimized.

In the previous design [1], the shoulder and elbow joint are butterfly joints [9]. This joint type is used at the shoulder location for its small pivot shift compared to many other flexure joints. This is also favourable in the current design as at the shoulder the rotation is driven by a motor with a fixed rotational axis. For the elbow joint it is essential that the flexure can handle rather rotations as can be seen in Figure 3. Although other, more complex joint types [7] may offer a larger range of motion, the butterfly hinge will also be adequate for the present concept.

In the previous design, the basic working principle of the butterfly hinge was also used in the wrist joint to connect the three arms. A disadvantage is the need of an intermediate body that can give rise to additional rotational DOF at the EE, which can result in unwanted parasitic vibrations or

Figure 4. Schematic representation of the Tri-Cartwheel joint.

Figure 5. Design parameters.

even loss of the redundancy. Hence a so-called Tri-Cartwheel joint, see Figure 4 is proposed for this connection. This hinge shows a larger pivot shift and more stresses will appear in the deformed leaf springs, but for the considered range of motion this appears to be acceptable.

For design optimization the overall model of the manipulator with the joints is expressed in a set of parameters, see Figure 5. To limit the computer time needed for the optimization, the number of parameters is limited to a reasonable number as will be explained next. Some parameters are fixed by design or it is known beforehand that some extreme value is the most likely outcome, see Table 1(a). Table 1(b) lists the varying parameters with lower and upper bounds.

Table 1. Design parameters of the 3RRR RA-PKM as defined Figure 5.

(a) Fixed parameters			(b) Variable parameters with upper bound (ub), lower bound (lb) and optimal value (opt, Section 2.6).				
Parameter	Value	Unit	Parameter	ub	lb	opt	Unit
ξ_s	22.5	deg	A_s	9	20	12.0	mm
ξ_e	30	deg	D_s	10	60	16.9	mm
R	230.5	mm	A_e	9	20	15.9	mm
t	0.4	mm	D_e	10	60	30.5	mm
$h_{s/e}$	40	mm	D_w	10	60	56.4	mm
h_w	50	mm	L_r	1.1	1.8	1.34	
			β_s	-90	0	-0.1	deg
			β_e	-90	0	-71.3	deg
			β_w	-30	30	20.0	deg
			R_o	0	50	25.9	mm

At manipulator level, the link lengths and the location of the actuators strongly affect the workspace ratio as defined in Eq. (1). The radius R at which the actuators are positioned, see Figure 5(a), is fixed to the value used in [1]. The link lengths L_1 and L_2 of upper and lower arm, respectively, are taken identical and are described by a single design variable, the length ratio L_r defined as

$$L_r = \frac{L}{R} = \frac{L_1 + L_2}{R} = \frac{2L_1}{R} = \frac{2L_2}{R}. \quad (2)$$

A final parameter that affects the (effective) workspace is the offset radius R_o that was introduced to exclude singularities from the workspace labelled “Defⁿ 3” in Figure 2.

At hinge level, the leaf springs are characterized by their length, width h and thickness t . The latter two dimensions are fixed for all joints. It is known that the thickness t of the leaf springs tends to be minimized as it reduces stress build-up without affecting the support stiffness too much. Hence $t = 0.4$ mm which is close the minimal required thickness for the used SLS process.

For the optimal performance of the Butterfly hinges, it has been found that the clearance angles ξ_s

and ξ_e as indicated in Figure 5(b) need to be minimized [10]. Hence these angles are fixed to the minimal estimated required clearance angle for the expected range of motion.

Finally, the stiffness properties of the joints depend on the other dimensions listed in Table 1(b) as well as the hinge orientation angles β_s , β_e and β_w . These angles describe the respective hinge orientations from a global point of view, see Figures 5(b) and (c).

2.3 2-DOF dynamic model

A low order 2-DOF model is derived first. It should capture the main low frequent dynamic behaviour and will be used for the control synthesis. The low order model can be obtained relatively straightforwardly e.g. using the Euler-Lagrange equation for the kinematic model of Section 2.1 supplemented with (link) mass and (joint) stiffness properties. The result can be expressed in the usual way as

$$\tilde{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}) = \mathbf{A}^T(\mathbf{q})\boldsymbol{\tau}, \quad (3)$$

where $\mathbf{q}$ are the two independent coordinates for which it is convenient to take the EE coordinates x_{EE} and y_{EE} . Matrix $\tilde{\mathbf{M}}$ is the configuration dependent (reduced) mass matrix; $\mathbf{C}$ accounts for the Coriolis terms; $\mathbf{Q}$ represents the (non-linear) stiffness contribution and the transpose of the Jacobian matrix $\mathbf{A}$ transforms the three actuator torques $\boldsymbol{\tau}$ into effective forces on the EE.

2.4 Stiffness balancing with preloaded joints

The term $\mathbf{Q}$ in Eq. (3) is directly linked to the finite stiffness of the flexure joints for rotation in the compliant direction. In the neutral configuration of the manipulator with the EE in the centre, this term is zero. To position the EE at locations near the extremity of the workspace, this term can be quite large and even result in actuator saturation making these locations unreachable. Lowering the stiffness is mostly not possible, but as shown in [1] preloading of the joints can result in lower required actuator torques. Although preloading isn't possible for the Nylon flexures, metal clock springs can be installed at the hinges to realize preloading.

A quick way to evaluate the effectiveness of these springs makes use of an analysis of the stored potential energy. Knowing the kinematic configuration of the manipulator, Section 2.1, the potential energy stored in all flexure joints can be computed as a function of the EE position, see Figure 6(a). With zero potential energy in the neutral configuration, the energy stored increases towards the boundary of the workspace.

Similarly, the energy stored in the preloaded clock springs can be evaluated. It appears that this energy can decrease when moving towards the boundary with adequate settings for preload and

Figure 6. Potential energy stored in the flexure joints (left), balancing clock springs (middle) and combined (right).

stiffness. Figure 6(b) shows an example where the minimum energy level in the workspace is arbitrarily set to zero. Combining these plots give Figure 6(c) which illustrates that for the manipulator with these balancing clock springs still some positive stiffness is observed, but less compared to the unbalanced case. In this way it will be easier to reach the full workspace with limited actuator torques.

2.5 Controller synthesis

Reconsidering the equation of motion (3), it can be seen that any specific motion can be accomplished by applying the effective force on the EE as follows from the right-hand side of this equation. In other words, the actuator torques should be such that the term $\mathbf{A}^T(\mathbf{q})\boldsymbol{\tau}$ equals the left-hand side of this equation. Because of the redundant actuation there is no unique solution. Let's consider e.g. the stationary positioning of the manipulator in which case only the stiffness has to be handled, i.e.

$$\mathbf{A}^T(\mathbf{q})\boldsymbol{\tau} = \mathbf{Q}(\mathbf{q}), \quad (4)$$

from which it can be seen that with one valid solution for $\boldsymbol{\tau}$, all other solutions can be found by adding any vector from the null space of the Jacobian matrix $\mathbf{A}^T$. A common procedure is to select the set of torques $\boldsymbol{\tau}$ that minimize a specific norm. Considering the 2-norm, the solution equals

$$\boldsymbol{\tau} = (\mathbf{A}^T(\mathbf{q}))^\dagger \mathbf{Q}(\mathbf{q}), \quad (5)$$

where $()^\dagger$ indicates the (Moore-Penrose) pseudo-inverse of the matrix.

It can be noted that in view of possible actuator saturation, the use of the ∞ -norm is more adequate as this minimizes the largest required actuator torque. However, it was found that the benefit from this norm was relatively small and hence the simpler computation of the 2-norm will be implemented in the set-up.

For controlled motion, the required effective EE force is generated by a control system, where feedback and feedforward control can be combined. Feedforward makes use of system knowledge as the equation of motion (3) is evaluated in real-time for a prescribed trajectory $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$. Relevant parameters of the manipulator need to be estimated. In this paper only relatively slow motions are considered and hence only stiffness feedforward will be applied, i.e. Eq. (4).

In addition feedback control is used to account for model inaccuracies in the feedforward control and disturbances. PID-control will be implemented following the approach of [11].

2.6 Design optimization

The workspace for the test-setup is optimized for the parametric model outlined in Section 2.2. Ten design parameters have been defined in Table 1(b). Material properties are given in Table 2 for the materials introduced at the beginning of this Section 2.

To assure the optimization doesn't result in deteriorated dynamic performance, constraints are imposed that require the parasitic natural frequencies to be at least as high as in the previous design [1]. Table 3 lists all constraints, which also impose that the stresses are below the maximum allowable stress, Table 2, with a safety factor of 1.5. Furthermore, the actuator limits have to be taken into account. The upper arms are actuated with Maxon EC45 Flat 70W motors that offer a maximum nominal torque of 0.128 Nm.

The MATLAB script `fminsearchcon` is used for the optimization, which implements a direct search method where non-linear inequality constraints are included by means of a penalty function [12]. As the goal is to obtain the maximum workspace, the inverse of the workspace ratio r_{ws} of Eq. (1) is minimized. As the constraints include natural frequencies and stresses these are evaluated in the neutral configuration as well as in two critical EE locations on the border of the workspace. For this purpose a flexible multibody model of the manipulator has been defined in the SPACAR software package [13]. All leaf springs are modelled with non-linear flexible beam

Table 2. Material properties of the test setup.

Material	Property	Value	Unit
PolyLactic Acid (PLA)	Tensile strength	45	MPa
	Young's modulus	3.5	GPa
	Density	1250	kg/m ³
Nylon (PA2200)	Tensile strength	45	MPa
	Young's modulus	1.7	GPa
	Density	930	kg/m ³

Table 3. Non-linear inequality constraints.

Property	Value	Unit
Maximum stress	≤ 30	MPa
First parasitic eigenfrequency	≥ 45	Hz
Second parasitic eigenfrequency	≥ 50	Hz
Balanced driving torque	≤ 0.1	Nm
Non-balanced driving torque	≤ 0.4	Nm

elements as this has been proven to be an accurate and efficient approach [1, 7, 10]. The obtained optimal values for the parameters are included in Table 1 and a CAD model of the manipulator is shown in Figure 1(b). Figure 7(a) illustrates the first natural frequency throughout the workspace. Clearly it satisfies the constraint and it can be verified that the other constraints are not violated either. The parasitic natural frequency doesn't vary much which shows that the support stiffness remains rather constant. The accompanying vibration mode in the neutral configuration is presented in Figure 7(b) from which it is clear that this is an out-of-plane vibration.

(a) First parasitic frequency (in Hz).

(b) First parasitic vibration mode.

Figure 7. First parasitic vibration.

Evaluating the optimized design, it appears that it results in a further increase of the workspace area while the first simulated parasitic natural frequency is kept above 45 Hz. More specifically, the workspace ratios as shown in Figure 2 are respectively $r_{ws} = 0.1922$ in the original “Defⁿ 1” (red) [1], $r_{ws} = 0.2167$ for “Defⁿ 2” (blue) [8], but can now be increased to $r_{ws} = 0.3532$ (green).

3 EXPERIMENTAL RESULTS

3.1 System identification

The dynamic behaviour of the actual manipulator has been characterized with system identification where a multi-sine excitation is used to estimate the frequency response of Figure 8. In agreement with the equation of motion (3), the redundant actuation and the sensing are transformed to two degrees of freedom being both in-plane forces F_x , F_y and translations x_{ee} , y_{ee} of the EE. This experiment is performed at 14 locations in the workspace (red curves). The blue curves indicate averages of these measurements.

In the theoretical equation of motion, the translations in x and y directions of the EE near the neutral configuration can be modelled as decoupled motions. In the FRF plots this is confirmed as the diagonal terms are larger than the off-diagonal cross-coupling terms.

The first parasitic natural frequency appears to be about 500 rad/s or 80 Hz. This frequency is

Figure 8. Identified frequency response functions (FRF) in 14 different EE locations (red) and average (blue).

higher than expected in Section 2.6. A possible cause could be a larger stiffness, which to some extent can be explained as the model doesn't account for flexible shaft couplings that are installed between the actuators and upper arms. This higher natural frequency could be beneficial for the controller stability. Unfortunately however, the FRF plots also show an anti-resonance (or zero pair in the transfer function) near 30 rad/s or 5 Hz. Even worse, in some of the FRF curves this zero pair is combined with a phase lag of -180° instead of the usual phase lead of 180° , which indicates these are non-minimum phase zeros. Such zeros will make it hard or impossible to achieve a high control bandwidth. Although these anti-resonances and accompanying zeros can be simulated in the SPACAR models, their existence was not included in the design optimization.

3.2 Controller tuning

The observation in the previous section of a mainly diagonal plant can be used to apply a diagonal controller as well. In this way the settings of the PID-controller are found quite straightforwardly [11]. However, to handle the destabilizing effects from the anti-resonances described in the previous section, the PID-controller is supplemented with a notch filter. Setting the filter frequency at about 495 rad/s or 79 Hz, a stable closed-loop system could be obtained with a open-loop crossover frequency of about 18 rad/s or 3 Hz.

3.3 Reference tracking

To demonstrate the controlled motion of the EE a circular reference trajectory is defined close to the boundary of the workspace, see Figure 9(a). An initial linear segment is added to move the EE from the neutral configuration to the circle. The actual motion is included in the plot. It shows reasonable tracking accuracy with some occasional small spikes. These may be caused by cogging effects of the (direct-drive) motors or other disturbances which are only weakly suppressed as the controller bandwidth is rather limited. Also noticeable is an increased noise level in the lower right corner of the circle. This is due to quantisation noise of the digital encoders that measure the rotation with a resolution of 512 lines per revolution, so 2048 counts per revolution. In this experiment only two encoders were used, which results in an output singularity in this region. This can be avoided by exploiting the redundant sensing. Figure 9(b) presents the actuator torques during the motion, which will be discussed in more detail next.

Figure 9. Tracking of a circular reference path.

3.4 Stiffness balancing

As outlined in Section 2.4 the preloading of the flexure joints plays an important role in reducing the actuator torques needed to cope with the stiffness of the joints. The balancing of the actuator torques is investigated by positioning the EE throughout the workspace with and without preloading of elbow and shoulder joints. Figure 10(b) shows that with preloading the maximum torques of all actuators stay well within the imposed limit. Without this preloading it can be seen in Figure 10(a) that the torque limits are already exceeded before the edges of the workspace are reached. Hence only a smaller area is shown.

Figure 10. Experimentally determined actuator torques for the unbalanced (a/top) and balanced (b/bottom) manipulator.

4 CONCLUSION

This paper shows a new design approach for a planar 2-DOF 3RRR parallel manipulator with redundant actuation and compliant joints, i.e. flexure joints. The goal was to maximize the workspace area in which a similar or better performance could be realized as before [1] in terms of the first parasitic natural frequency. It turned out to be beneficial to limit the workspace to a circle that avoids (near) singular manipulator configurations. In this way the ranges for the joint rotations are restricted such that the joint support stiffnesses of the deformed flexures are reduced less. This allows for the use of longer link lengths which in the end results in a significant increase of the workspace area. In addition a new concept is proposed for the wrist joint connecting the three arms in the end effector. A flexible multibody model has been built where the leaf springs in the flexure joints are modelled with the non-linear beam elements of the SPACAR software. The parameters in this model have been optimized to obtain the largest workspace area while satisfying constraints regarding the parasitic natural frequencies, local stresses and actuator limits.

A hardware implementation of this manipulator has been realized. Although the mechanical parts of this setup have been manufactured with relatively low-cost 3D printing, it demonstrates some of the main features like the sufficiently high parasitic natural frequency. However, it also appeared that the closed-loop performance is considerably limited by the existence of anti-resonances or transfer function zeros that could have been predicted by the SPACAR flexible multibody models, but were not taken into account during the design. A further increase of the workspace area could result from applying a more advanced joint type for the elbow joints allowing larger rotations. For true high performance the flexure hinges should be manufactured from different material like e.g. metal. Also the use of frictionless actuators like pure torque motors should be considered. Attention should be paid to using sufficiently accurate sensors and taking advantage of the sensor redundancy. Finally, in order to handle higher velocities the stiffness or position dependent feed-forward control should be supplemented with the acceleration, velocity and position dependent inertia effects of the equation of motion (3).

The setup in particular proved that preloading of the shoulder and elbow joints results in a significant reduction of the required actuator torques in agreement with the modelled prediction.

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