A tailor-made 1D hydrodynamic modelling solution for flood forecasting on the Danube catchment

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Abstract. The Hungarian Hydrological Forecasting Service has a long history in model based hydrological forecasting on the Danube catchment. An extensive set of semi-distributed rainfall-runoff models with advanced snow calculation serve as a basis for the operational daily forecasting completed with discrete linear cascade models of non-integer and integer cascade numbers solved based on the state-space approach for accurate flow routing. In this paper we present an additional module of the forecasting system, a tailor-made one dimensional hydrodynamic model. The model is developed for fast and robust simulation with assimilation observed data to provide support for flood management decisions and more accurate operative water level prediction. The model is based on the Verwey variant of the Preissmann scheme solved by the double sweep algorithm. It handles complex river networks with dry beds. The data assimilation algorithm provides a longitudinally valid update of the simulated water levels resulting in a continuous water surface.

1 Introduction

Prevention and protection from floods have become a major issue, where forecasting plays an indispensable role. Long term hydro-meteorological data shows the increasing number of rainfall and flood related natural hazards. Besides the decreasing return frequencies of high-flow events, the intensity of weather-related hazards have also been increasing, which results in high economic damages.

Forecasting of floods refers to the thorough understanding of catchment-scale hydrologic processes. Flood protection usually requires detailed hydraulic analysis which is not derivable from hydrological transformations. Hydrodynamic modelling has advanced rapidly since the publication of the first numerical methods. Nowadays it is a well published scientific area with a wide range of commercial, open-access and open-source models available even as part of complete operational systems dealing with data management and dissemination of results. As the probability of intense hydrological events, e.g. flash floods and drought, increases forecasting models are tested at their extremes where simplifications and generalised solutions easily fail. Customised and tailor-made solutions are highly favourable for satisfactory operational safety and precision.

Hungary is settled in the Danube basin. The floods of the Danube River are well documented by the International Commission for the Protection of the Danube River (ICPDR). Flood protection and sustainable flood risk

management are among the key tasks along the Danube River. Flood forecasting is an important part of decision making. Hungary has a dual approach in flood forecasting. The Hungarian Hydrological Forecasting Service (HHFS) is responsible for daily operative water level forecast for 6 days of lead time with the time step of 6 hours. The forecasts are publically available. The calculations are made by semi-distributed hydrological models and flood routing with different technics. On the other hand Regional Water Directorates are responsible for forecast on their own river reaches. These forecasts are generally casual and only available for decision makers. The methodology behind these forecasts are various. Linear regression is generally applied throughout the country but it has a shorter lead time. There are five commercial model based flood forecasting solutions. An overview of the systems is presented on Figure 1.

The flood routing of HHFS is based on cascade models since the early 1970s. Today this is an enormous system of cascades embedded in a complex operative forecasting framework. Linear cascade models are fast in calculation and usually satisfactory in accuracy but do not describe detailed hydraulics that are often required for decision support.

In this paper we present an overview of the one dimensional hydrodynamic model we developed to increase the accuracy of daily water level predictions. We are currently in the first phase of implementation introducing the hydrodynamic simulation on the Hungarian reach of the Danube River. The Danube sub-

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basin cascade models are presented on Figure 2. Hydrodynamic calculations are carried out from Nagybajcs to Mohács stations on the Danube River, including also varying length of the downstream reaches of its tributaries.

Figure 1. Map of the Danube catchment and the Hungarian hydrological forecasting systems.

Figure 2. The system of cascade models on the Danube catchment.

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2 Materials and Methods

2.1. Discrete linear cascade model

The flood routing methodology consists of two approaches. The first is a large scale hydrological routing with linear cascades. The models of integer number of cascades are solved based on the state-space approach for accurate flow routing. The general equation of the cascade model is written as:

$$Q(t) = \int_0^t I(\tau)h(t-\tau)d\tau \quad (1)$$

Where $Q(t)$ is the outflow discharge [m^3/s], $I(\tau)$ is the input discharge, $h(t-\tau)$ is the response or transfer function, $(t-\tau)$ is the lag of the cascades. The response function in discrete form is written as:

$$h_n(t) = \frac{1}{K} \left(\frac{t}{K} \right)^{n-1} \frac{1}{(n-1)!} e^{-t/K} \quad (2)$$

Where n is the number of cascades, K is storage coefficient of one cascade. This discrete form is a proper approach for rainfall-runoff simulation (Nash 1960), but

does not give satisfactory result for hydrograph transformation. The continuous linear cascade model, or also known as the Kalinin-Milyukov-Nash cascade (Kalinin & Milyukov 1957, Szilágyi & Szöllösi-Nagy 2008) provides the proper solution for river reaches.

The state-space framework offers a compact and elegant solution for the continuous case. The continuity equation in matrix notation is written as:

$$\frac{dx(t)}{dt} = Fx(t) + Gu(t) \quad (3)$$

Where $x(t)$ is the stored water volume, $u(t)$ is the inflow, F is the $n \times n$ Toeplitz-band state matrix, and G is the input vector. The outflow equation in matrix notation is written as:

$$y(t) = Hx(t) \quad (4)$$

Where H is an n dimensional row vector. Measurements are available in discrete (Δt) time steps, thus discrete solution of the state-space equations are necessary (Szöllösi-Nagy 1982). The state equation in discrete form is written as:

$$x_t = \Phi(\Delta t)x_{t-\Delta t} + \Gamma(\Delta t)u_{t-\Delta t} \quad (5)$$

Where $\Phi(\Delta t)$ is the state transition matrix, and $\Gamma(t)$ is the input transition vector. Details of the derivation are presented by Szilágyi & Szöllösi-Nagy (2008), the final forms are written as:

$$\Phi(\Delta t) = \begin{bmatrix} e^{-\Delta tk} & 0 & 0 & \dots & 0 \\ \Delta tk e^{-\Delta tk} & e^{-\Delta tk} & 0 & \dots & 0 \\ \frac{(\Delta tk)^2}{2!} e^{-\Delta tk} & \Delta tk e^{-\Delta tk} & e^{-\Delta tk} & 0 & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{(\Delta tk)^{n-1}}{(n-1)!} e^{-\Delta tk} & \frac{(\Delta tk)^{n-2}}{(n-2)!} e^{-\Delta tk} & \dots & \Delta tk e^{-\Delta tk} & e^{-\Delta tk} \end{bmatrix} \quad (6)$$

$$\Gamma(\Delta t) = \begin{bmatrix} (1 - e^{-\Delta tk})/k \\ [1 - e^{-\Delta tk}(1 + \Delta tk)]/k \\ [1 - e^{-\Delta tk}(1 + \Delta tk + \frac{(\Delta tk)^2}{2})]/k \\ \vdots \\ (1 - e^{-\Delta tk} \sum_{j=0}^{n-1} \frac{(\Delta tk)^j}{j!})/k \end{bmatrix} \quad (7)$$

This model gives an analytically solvable, fast and robust solution that is able to provide highly satisfactory results. It is an unsteady approach and also describes the diffusion of the hydrograph, but does not take backwater effect into account. Therefore unsteadiness is only partially valid, temporally changing discharge values are present but water levels are estimated based on steady state rating curves. Szilágyi & Laurinyecz (2014) published a study on backwater effect.

2.2. One dimensional hydrodynamic model

The one dimensional hydrodynamic model of the HHFS is based on the fully dynamic Saint Venant equations discretised by the Verwey variant of the Preissmann scheme and finally solved with the double

sweep method (Preissmann 1961, Abbott 1979, Cunge et al. 1980, Abbott & Basco 1989).

The continuity equation is written as:

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} \frac{1}{B} = q \quad (8)$$

The momentum equation is written as:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\beta \frac{Q^2}{A} \right) + gA \left(\frac{\partial h}{\partial x} - S_0 + S_f + S_e \right) = 0 \quad (9)$$

Where h is the water level [m], Q is the discharge [m^3/s], β is the momentum correction coefficient [-], B is the width of water surface [m], A is the wetted area [m^2], Δx is the distance of cross sections [m], Δt is the time step [s], g is the gravitational acceleration [$\approx 9.80665 \text{ m/s}^2$], S_0 is bottom slope [-], S_f is friction slope [-], S_e is the slope of the energy line due to contraction and expansion [-], K is the conveyance [m^3/s], R is the hydraulic radius [m], q is the lateral inflow [$\text{m}^3/\text{s/m}$].

The conveyance, the friction slope and the energy slope are written as:

$$K = kAR^{\frac{2}{3}} \quad (10)$$

$$S_f = \frac{Q|Q|}{K^2} \quad (11)$$

$$S_e = \frac{K_{ec}}{2g} \frac{\partial (\frac{Q}{A})^2}{\partial x} \quad (12)$$

Where K_{ec} is the expansion or contraction coefficient [-].

The discretisation of the partial derivatives in the Preissmann scheme is as follows:

$$\frac{\partial f}{\partial x} \approx \theta \frac{f_{i+1}^{j+1} - f_i^{j+1}}{\Delta x} + (1 - \theta) \frac{f_{i+1}^j - f_i^j}{\Delta x} \quad (13)$$

$$\frac{\partial f}{\partial t} \approx \psi \frac{f_i^{j+1} - f_i^j}{\Delta t} + (1 - \psi) \frac{f_{i+1}^{j+1} - f_{i+1}^j}{\Delta t} \quad (14)$$

$$\bar{f} \approx \frac{\theta}{2} (f_{i+1}^{j+1} + f_i^{j+1}) + \frac{1 - \theta}{2} (f_{i+1}^j + f_i^j) \quad (15)$$

Where θ and ψ are weighting coefficients [-] with values ranging from 0 to 1.

The Verwey variant is a compact and robust approximation of the non-linear terms of the momentum equation. The convective acceleration is discretised as:

$$\frac{\partial}{\partial x} \left(\beta \frac{Q^2}{A} \right) \approx \beta \left[\theta \frac{Q_{i+1}^j \times Q_{i+1}^{j+1}}{\Delta x \cdot A_{i+1}^{j+1/2}} - (1 - \theta) \frac{Q_i^j \times Q_i^{j+1}}{\Delta x \cdot A_i^{j+1/2}} \right] \quad (16)$$

The bottom friction is discretised as:

$$gA \frac{Q|Q|}{K^2} \approx g \left[\psi \cdot A_{i+1}^{j+1/2} \frac{Q_{i+1}^{j+1} |Q_{i+1}^j|}{(K_{i+1}^{j+1/2})^2} + (1 - \psi) \cdot A_i^{j+1/2} \frac{Q_i^{j+1} |Q_i^j|}{(K_i^{j+1/2})^2} \right] \quad (17)$$

The double sweep method is a standard, widely applied solution for linearized systems. The discretised general equations can be rearranged to a pentadiagonal matrix of the unknown variables $(Q_{i+1}^{j+1}, h_{i+1}^{j+1}, Q_i^{j+1}, h_i^{j+1})$, and all the known variables $(Q_{i+1}^j, h_{i+1}^j, Q_i^j, h_i^j)$ can be rearranged to a vector. In general form the linearized system is written as:

$$\bar{P} \cdot \bar{f}_i^{j+1} = \bar{E}_i^j \quad (18)$$

Where $\bar{f}_i^{j+1} = [Q_i^{j+1}, h_i^{j+1}]^T$ and $\bar{E}_i^j = [E1_i, E2_i]^T$, $\bar{P}$ is the coefficient matrix. The double sweep method introduces two new formulas for the calculation of Q and h :

$$Q_i^{j+1} = F1_i h_i^{j+1} + G1_i \quad (19)$$

$$h_i^{j+1} = P1_i Q_{i+1}^{j+1} + Q1_i h_{i+1}^{j+1} + R1_i \quad (20)$$

Where $F1$, $G1$, $P1$, $Q1$, and $R1$ are recurrence variables derived from the linearized coefficients. Details are available in Cunge et al. (1980) and Abbott & Basco (1989).

The double sweep method is a robust solution algorithm. The only drawback is its linear behaviour that handles single channels with confluences that are monotonically ranked from the upstream boundaries to the downstream boundary conditions. The forward sweep is not able to handle splitting channels, where parallel branches may occur. To overcome this we simply go back to the Kirchhoff laws of junctions and by basic hydraulic calculations we estimate a flow split that results in a uniform water level in each branches at the junction. A governing downstream branch is also selected based on the highest specific force to carry on with the backward sweep.

Dry channel modelling is required for complex simulations, therefore we introduced an area preserving Abbott-slot (Abbott & Basco 1989) algorithm with complex shaped slots to ensure smooth transition between wet and dry conditions.

Supercritical flow conditions rarely occur in natural rivers but may occur in a complex river network with dry branches and hydraulic structures. Its spatial and temporal scales are lower compared to the ones of general 1D modelling, so high energy and water level changes happen when the flow regime is changing. The spatial and temporal resolution of the models are generally fixed and

optimised for a certain task that is large scale and fast hydrodynamic simulation in our case. To prevent the unexpected failure of the solution due to supercritical condition we introduced the local partial inertia method (Fread & Lewis 1998). This approach suppresses the convective acceleration term of the momentum equation close to unity (or higher) Froude numbers thus preserves stability in mixed flow conditions.

The fully dynamic simulations are computationally demanding, they include a great number of equations and generally numerical approximation requires iterations. In one dimension it is still considered to be fast, but compared to the analytical solution of the cascade model it is much slower. To maximise the simulation speed we rejected the method of interim databases and temporary files, all data is read directly from its database and all raw and computed inputs and results are stored in the memory. Moreover the simulation algorithm is written for parallel operation on all the available CPU cores. As a result the model is a tailor-made solution for certain tasks and proved to be more efficient than the commercial or readily available more general solutions.

The assimilation of observed data is a key factor in operative water level forecasting. Stochastic methods such as the Kalman filter require a great number of repetitive simulations that is not preferred when flood management needs quick response. Deterministic methods such as the weighting functions of the 1D MIKE model (DHI 2014) means only local updating of the model results that invalidates a major advantage of hydrodynamic models, namely that results are available along the entire model domain at each calculation points. Therefore we decided to introduce an updating method that preserves longitudinal continuity. Another key factor in data assimilation is the type of the observed data. Discharges are usually calculated from water level observations by rating curves along the Hungarian Danube reach. These rating curves are of steady state thus a major simplification would be introduced to the model by updating simulated unsteady discharges to these values. Since water levels are directly observed and are also the basis of flood management in Hungary, we decided to update only water levels in the model.

The equation for water level in the double sweep method (Eq. 20.) offers an elegant way for updating. The recurrence variable $R1$ is directly related to the simulated water levels and its modification is in straight ratio to the results. The double sweep method usually requires two or more iterations so the correction is also getting better at each iteration step.

$$R1_{i,da} = R1_i + (h_{i,obs}^{j+1} - h_{i,sim}^{j+1}) \quad (21)$$

Since $R1$ is a recurrence variable, a local modification spreads both upstream and downstream in the model domain resulting in a continuous water surface. However this update is not a hard rule, so the final water level not be equal to the observed one but it will be as close as possible also representing the flow characteristics of the simulation. Therefore we call it a soft assimilation. Finally the direct update of the local water levels is still an option, but after

the soft assimilation the error is significantly smaller so the continuity of the water surface is only effected by minor jumps.

The error forecast during the lead time is also an important feature. The update happens based on the same algorithm but the error is equal to the error observed at the time of forecast or is calculated by a function based on the flow regime.

3 Results

3.1. Simulation performance

We compared the performance of our model to the different versions of MIKE 11 and MIKE 1D. It is important to note that the MIKE model is a widely applied software with numerous functions that were not used during the performance test, while our model is a simple task-oriented software without any unused algorithms. The test model was setup based on the Tisza River from Kisköre-alsó to Szeged with cross-sections at every 300 meters. The model contained 731 sections with pre-calculated hydraulic parameters at 50 levels, and the simulation period was 1 year with the time step of 10 minutes, resulting in 52560 calculation steps. The statistics of the test runs are shown in Table 1.

Model	Simulation time [min:sec]	CPU	RAM	SSD
OVSZ 1D (single thread)	2:07	16%	708 MB	0.0 MB/s
OVSZ 1D (parallel comp.)	0:49	78%	704 MB	0.0 MB/s
MIKE 11 Classic (low-order)	1:52	15%	65 MB	2.7 MB/s
MIKE 1D (low-order)	1:23	39%	74 MB	3.7 MB/s
MIKE 11 Classic (high-order)	3:11	14%	28 MB	0.7 MB/s
MIKE 1D (high-order)	1:18	39%	126 MB	4.1 MB/s

Table 1. Comparison of model performance.

3.2. Data assimilation

Results of the data assimilation are also presented based on the test model described in chapter 3.1. and we are also compared them to MIKE 11 results (

Figure 3).

The results with the low-order MIKE 11 approach differ from other results due to a simplified description of bed roughness (DHI 2014). High order MIKE 11 results slightly differ from our results at higher water levels. This occurs when the water level is higher than the highest geometry point of the cross-section so the hydraulic parameters are only estimated and the applied estimation method is different.

The soft assimilation corrects more than 80% of the error, while the added hard assimilation results in the observed values at the data assimilation points and has an effect only upstream since water levels are calculated in the backward sweep.

Figure 3. Simulation results without data assimilation (blue), with soft assimilation (purple), with soft+hard assimilation (green, equals to observed values), and with MIKE 11 low-order (black), MIKE 11 high-order (red).

3.3. Operative water level prediction

The model based operative water level prediction on the Hungarian reach of the Danube River has started on 01/01/2020. The necessary boundary condition are the

mix of observations and forecast from the cascade model. Initial condition are derived from observations. A task-oriented graphical user interface (GUI) has been developed for the model (Figure 4) to aid the effective overview of input data and results.

Figure 4. GUI of the model.

4 Discussion

We developed a fully dynamic one dimensional hydrodynamic model for water level prediction on the Hungarian Danube reach. The developed model shows superior simulation performance due to the parallelization of its calculation algorithm and optimised data management. A longitudinally valid deterministic data assimilation and error forecasting method was also introduced to meet the requirements of operative water level prediction.

In the current phase of the development the model structure is continuously refined to better describe the flow characteristics along the Danube River. The calibration and validation of the parameters are also in progress, but the final parameter set will be obtained only when the model structure is finalised.

The cascade model has an average error of 3-20 cm on the first day of lead time and 35-55 cm on the sixth day. Lower values are achieved at stations with simple flow characteristics and higher values are for stations with complex hydraulics, often effected by hydraulic structures or river confluences (backwater effect). The ultimate target of the 1D model is to exceed these statistics and provide accurate results where the cascade model shows poor performance or is not even applicable.

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